NDA (MATHS) MOCK TEST - 66 (SOLUTION)

1. (B) The given equation represents a real sphere, if

$u^2 + v^2 + w^2 > d$ [by defination]

2. (A) From option (a),

Let d = $5i - j - 5k \Rightarrow |d| = \sqrt{51}$

Then, $\cos\theta_1 = \frac{a.b}{|a||d|}$

$= \left|\frac{\frac{(i-2j+2k)}{3}.(5i-j-5k)}{1.\sqrt{51}}\right|$

$= \left|\frac{\frac{5}{3}+\frac{2}{3}-\frac{10}{3}}{\sqrt{51}}\right| = \frac{1}{\sqrt{51}}$

Similarly,

$\cos\theta_2 = \frac{b.d}{|b||d|}$

$= \left|\frac{\frac{(-4i-3k)}{5}.(5i-j-5k)}{1.\sqrt{51}}\right|$

$= \left|\frac{-4+3}{\sqrt{51}}\right| = \frac{1}{\sqrt{51}}$

And, $\cos\theta_3 = \frac{c.d}{|c||d|}$

$= \left|\frac{j.(5i-j-5k)}{1.\sqrt{51}}\right|$

$= \left|\frac{-1}{\sqrt{51}}\right| = \frac{1}{\sqrt{51}}$

Here, $\theta_1 = \theta_2 = \theta_3 = \cos^{-1}\left(\frac{1}{\sqrt{51}}\right)$

So, the vector $5i - j - 5k$ makes an equal angles with three vectors a, b and c.

3. (B) We know that,

$|a \times b|^2 + |a . b|^2 = (|a|^2 \times |b|^2)$

$\therefore$ $64 + |a . b|^2 = (4 \times 25)$

$\Rightarrow$ $|a . b|^2 = 36$

$\Rightarrow$ $a . b = 6$

4. (B) $\because$ $|a + b| = |a - b|$

$\Rightarrow |a + b|^2 = |a - b|^2$

$\Rightarrow |a|^2 + |b|^2 + 2|a|.|b|$

$= |a|^2 + |b|^2 - 2|a|.|b|$

$\Rightarrow$ $4|a|.|b| = 0$

$\Rightarrow$ $a \perp b$

$\Rightarrow$ a is perpendicular to b.

5. (B) $\because$ a = i – 2j + 5k

b = 2i + j – 3k

$\therefore$ b – a = 2i + j – 3k – i + 2j – 5k

= i + 3j – 8k

and (3a + b) = (3i – 6j + 15k) + (2i + j – 3k)

= 5i – 5j + 12k

Hence, (b – a).(3a + b) = (i + 3j – 8k). (5i – 5j + 12k)

= 5 – 15 – 96

= –106

6. (D) Points A, B and C are collinear, if

$(a \times b) + (b \times c) + (c \times a) = 0$

[by property]

7. (D) Since, a = i + j + k

b = i – j + k

c = i + j – k

$\therefore$ a × (b + c) + b × (c + a) + c × (a + b)

$\left(\begin{array}{l}\because \; a \times b = -b \times a \\ \quad b \times c = -c \times b \\ \quad c \times a = -a \times c\end{array}\right)$

= a × b + a × c + b × c + b × a + c × a + c × b

= a × b – c × a + b × c – a × b + c × a – b × c = 0

8. (B) Required even = $A \cap B \cap \overline{C}$.

9. (C) Month 1, CV = $\frac{\sigma}{x} \times 100$

$= \frac{2}{30} \times 100 = 6.67$

Month 2, CV = $\frac{3}{57} \times 100 = 5.26$

Month 3, CV = $\frac{4}{82} \times 100 = 4.88$

Month 4, CV = $\frac{2}{28} \times 100 = 7.14$

Hence, month 3, the sales are most consistent.

10. (D) We know that by Baye's theorem conditional probability is calculated.

11. (B) $\because$ $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{6}$

But $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \frac{1}{6} = \frac{P(A \cap B)}{\frac{1}{4}}$

$\Rightarrow P(A \cap B) = \frac{1}{24}$

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{1}{24}}{\frac{1}{3}} = \frac{1}{8}$

12. (B) Since, A and B are mutually exclusive and exhaustive events, therefore

$P(A \cap B) = 0$, $P(A \cup B) = 1$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 1 = P(A) + 3P(A)$

$[\because P(B) = 3P(A)]$

$\Rightarrow P(A) = \frac{1}{4}$

$\therefore P(B) = \frac{3}{4}$ $[\because P(A) + P(B) = 1]$

Hence, $P(\overline{B}) = 1 - P(B)$

$= 1 - \frac{3}{4} = \frac{1}{4}$

13. (D) $\because$ $n(S) = 36$

E = Sum of the faces equals or exceeds.

= {(5, 5), (4, 6), (6, 4), (5, 6), (6, 5), (6, 6)}

$\therefore$ n(E)= 6

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

14. (D) $\because$ $np = 4$ and $npq = \frac{4}{3}$ [given]

$\therefore 4q = \frac{4}{3} \Rightarrow q = \frac{1}{3}$

$\therefore p = 1 - \frac{1}{3} = \frac{2}{3}$ $(\because p + q = 1)$

$\Rightarrow n = \frac{4 \times 3}{2} = 6$

Now, $P(X \geq 5) = {}^6C_5 p^5 q^1 + {}^6C_6 p^6 q^0$

$= {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + {}^6C_6\left(\frac{2}{3}\right)^6$

$= \frac{6 \times 32}{3^6} + \frac{64}{3^6} = \frac{256}{3^6} = \frac{2^8}{3^6}$

15. (C) $\because$ H = 21.6 and a = 27

We know that

$H = \frac{2ab}{a+b} \Rightarrow 21.6 = \frac{2 \times 27 \times b}{27 + b}$

$\Rightarrow 583.2 = 54b - 21.6b$

$\Rightarrow b = \frac{583.2}{32.4} = 18$

16. (B) Average marks of A

$= \frac{71 + 56 + 55 + 75 + 54 + 49}{6}$

$= \frac{360}{6} = 60$

and SD =

$\sqrt{\frac{121 + 16 + 25 + 225 + 36 + 121}{6}}$

$= \sqrt{\frac{544}{6}} = 9.52$

Also, average of marks B

$= \frac{55 + 74 + 83 + 54 + 38 + 52}{6}$

$= \frac{356}{6} = 59.33 \cong 59$

and SD =

$\sqrt{\frac{16 + 225 + 576 + 25 + 441 + 49}{6}}$

$= \sqrt{\frac{1532}{6}} = \sqrt{255} \cong 16$

Now, $CV_A = \frac{9.52}{60} \times 100 = 15.87$

and $CV_B = \frac{16}{59} \times 100 = 27.12$

Thus, the average scores of A and B are not same but A is consistent.

17. (D) $n = 50, x = 3550, n_1 = 30, x_1 = 4050$ and $n_2 = 20$.

We know that

$nx = n_1x_1 + n_2x_2$

$\Rightarrow 50 \times 3550 = 30 \times 4050 + 20x_2$

$\Rightarrow 177500 - 121500 = 20x_2$

$\Rightarrow x_2 = 2800$

Hence, average salary of women = ₹ 2800.

18. (D) $\because \bar{x} = \frac{7+9+11+13+15}{5} = \frac{55}{5} = 11$

Now,

$$SD = \sqrt{\frac{(7-11)^2 + (9-11)^2 + (11-11)^2 + (13-11)^2 + (15-11)^2}{5}}$$

$$\because SD = \sqrt{\frac{(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{16+4+0+4+16}{5}}$$

$= \sqrt{8} = 2.8$ (Aprox)

19. (B) $\because$ n(S) = 52 and n(E) = 4

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

20. (B) Since, monthly salary = ₹ 15000 and sector angle of expenses = 15°

$\therefore$ Amount $= \frac{15^\circ}{360^\circ} \times 15000$

= Rs. 625

21. (C) $\because \sum_{i=1}^{n}(x_i - 2) = 110$

$\therefore x_1 + x_2 + ... + x_n - 2n = 110$

$\Rightarrow x_1 + x_2 + ... + x_n = 2n + 110$

and $\sum_{i=1}^{n}(x_1 - 5) = 20$

$\Rightarrow x_1 + x_2 + ... + x_n - 5n = 20$

$\Rightarrow x_1 + x_2 + ... + x_n = 5n + 20$

From Eqs. (i) and (ii), we get

$5n + 20 = 2n + 110$

$\Rightarrow 3n = 90$

$\Rightarrow n = 30$

Now, mean $= \frac{x_1 + x_2 + ... + x_n}{n}$

$$= \frac{5 \times 30 + 20}{30} = \frac{170}{30} = \frac{17}{3}$$

22. (C) $\because f(x) = x|x|$

If $f(x)_1 = f(x_2)$

$\Rightarrow x_1|x_1| = x_2|x_2|$

$\Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one.

Also, range of $f(x)$ = co-domain of $f(x)$.

$\therefore f(x)$ is onto.

Hence, $f(x)$ is both one-one and onto.

23. (A) $\because f(x) = \frac{x}{1+|x|}$

$$= \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x} & x \geq 0 \end{cases}$$

$$\therefore \text{LHD} = f'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{\frac{-h}{1+h} - 0}{-h}$$

$$= \lim_{h \to 0} \frac{1}{1+h} = 1$$

$$\text{RHD} = f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{h}{1+h} - 0}{h}$$

$$= \lim_{h \to 0} \frac{1}{1+h} = 1$$

$\because$ LHD = RHD

$\therefore f(x)$ is differentiable at $x = 0$.

Hence, $f(x)$ is differentiate in $(-\infty, \infty)$.

24. (A) $\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \left(\frac{dy}{dx}\right)_{\text{at } x=0}$

$$= \left(\frac{d}{dx}(ax^n)\right)_{\text{at } x=0} (\text{an } x^{n-1})_{\text{at } x=0} = 0$$

25. (C) We know that

$(AB)^n = A^n B^n$ is true only when AB = BA

26. (A) $(ABA)^T = A^T B^T A^T = ABA$

($\because A^T = A, \; B^T = B$)

27. (A) $(A + B)^2 = (A + B)(A + B)$

$= A^2 + AB + BA + B^2$

$= A^2 + 2AB + B^2$ ($\because$ AB = BA)

28. (A) Given that, A and B are two non singular square matrices.

So, its inverse i. e, A^{-1} and B^{-1} must be exist.

we have, AB = A

(A^{-1}) operating in left side on both sides, we get

$A^{-1}(AB) = (A^{-1})(A)$

$\Rightarrow (A^{-1}A)\; B\,(A^{-1}A)$ ($\because AA^{-1} = I$ and BI = B)

$\Rightarrow$ IB = I

$\Rightarrow$ B = I = Identity matrix

29. (D) $\because 3A^3 + 2A^2 + 5A + I = 0$

$\Rightarrow 3A^3A^{-1} + 2A^2A^{-1} + 5AA^{-1} + IA^{-1} = 0$

$\Rightarrow 3A^2 + 2A + 5I + A^{-1} = 0$

$\Rightarrow A^{-1} = -(3A^2 + 2A + 5I)$

30. (C) $$\frac{d}{dx}\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2$$

31. (B) $$\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x(x^2 - ab) + b(ab - ax) + b(a^2 - ax)$$

$= x(x^2 - ab) + ab^2 - abx + a^2b - abx$

$= x(x^2 - ab) + ab^2 + a^2b - 2abx$

$= x(x^2 - ab) + ab(a + b) - 2abx$

32. (D) If each element in a row of a determinant is multiplied by the same factor r, then the value of the determinant is multiplied by r.

33.(B) $$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$$

$$\Rightarrow abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

Applying $R_1 + R_2 + R_3 \rightarrow R_1$

$$abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$$\Rightarrow abc \begin{vmatrix} 1+0 & 1+0 & 1+0 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$$\Rightarrow abc \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & \frac{1}{b} \\ 0 & -1 & \frac{1}{c}+1 \end{vmatrix} = \lambda$$

$$abc \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = \lambda$$

$abc = \lambda$

34. (B) $$A = \begin{vmatrix} 2a & 3r & x \\ 4b & 6s & 2y \\ -2c & -3t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$$

$$= 2 \times 3 \begin{vmatrix} a & r & x \\ 2b & 2s & 2y \\ -c & -t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$$

$$= 2 \times 3 \times 2 \times -1 \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$$

$\lambda = -12$

35. (B) tan (–585°)

= tan (–585° + 720°)

= tan 135°

= tan (90° + 45°)

= –tan 45°

= –1

36. (C) $\sec\theta + \tan\theta = 4$ ... (i)

As we know that,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{4}$ ... (ii)

On adding Eqs. (i) and (ii), we get

$2\sec\theta = 4 + \frac{1}{4} = \frac{17}{4}$

$\therefore \sec\theta = \frac{17}{8}$

$\Rightarrow \cos\theta = \frac{8}{17} = \frac{b}{h}$

$p = \sqrt{289-64}$

$= \sqrt{225} = 15$

$\sin\theta = \frac{p}{h} = \frac{15}{17}$

Solutions (Q. Nos. 37-39)

Given that, sin (A + B) = 1, where A, B $\in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin(A+B) = \sin\frac{\pi}{2} \Rightarrow A + B = \frac{\pi}{2}$ ... (i)

and $\sin(A-B) = \frac{1}{2} \Rightarrow \sin(A-B) = \sin\frac{\pi}{6}$

$\Rightarrow A - B = \frac{\pi}{6}$ ... (ii)

37. (B) On adding Eqs. (i) and (ii), we get

$2A = \frac{2\pi}{3} \Rightarrow A = \frac{\pi}{3}$ and $B = \frac{\pi}{6}$

38. (C) Now, tan (A + 2B). tan (2A + B)

$= \tan\left(\frac{\pi}{3}+\frac{\pi}{3}\right).\tan\left(\frac{2\pi}{3}+\frac{\pi}{6}\right)$

$= \tan\left(\frac{2\pi}{3}\right).\tan\left(\frac{5\pi}{6}\right)$

$= \tan\left(\frac{\pi}{2}+\frac{\pi}{6}\right).\tan\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

$= \left(-\cot\frac{\pi}{6}\right)\left(-\cot\frac{\pi}{3}\right)$

$= (\sqrt{3}).\frac{1}{\sqrt{3}} = 1$

39. (B) Now

$\sin^2 A - \sin^2 B = \sin^2(\pi/3) - \sin^2(\pi/6)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

40. (D) $\cos\left(\frac{\pi}{9}\right) + \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{5\pi}{9}\right) + \cos\left(\frac{7\pi}{9}\right)$

$= \cos(20^0) + \cos(60^0) + \cos(100^0) + \cos(140^0)$

$= \cos 20^0 + \frac{1}{2} + 2\cos 120^0 \cos 20^0$

$= \cos 20^0 + \frac{1}{2} - 2\sin 30^0 \cos 20^0$

$= \cos 20^0 + \frac{1}{2} - \cos 20^0 = \frac{1}{2}$

41. (B) Given, $(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2$

$= k + \tan^2 x + \cot^2 x$

$\Rightarrow \sin^2 x + \operatorname{cosec}^2 x + 2 + \cos^2 x + \sec^2 x + 2$

$= k + \tan^2 x + \cot^2 x$

$\Rightarrow 1 + \operatorname{cosec}^2 x - \cot^2 x + \sec^2 x - \tan^2 x + 4 = k$

$\Rightarrow 1 + 1 + 1 + 4 = k \Rightarrow k = 7$

42. (C) $\cos 2\phi - 1 = \frac{1-\tan^2\phi}{1+\tan^2\phi} - 1 = \frac{-2\tan^2\phi}{1+\tan^2\phi}$

$= \frac{-(\tan^2\theta - 1)}{1+\frac{\tan^2\theta-1}{2}}$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta} \times 2$

$= \cos 2\theta.2$

Thus, $\cos 2\theta = \frac{\cos 2\phi - 1}{2}$

Solutions (Q. Nos. 43 - 44)

$\alpha = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left[\frac{\left(\frac{1}{2}+\frac{1}{3}\right)}{1-\frac{1}{2}\times\frac{1}{3}}\right]$

$= \tan^{-1}\left[\frac{\frac{5}{6}}{\frac{5}{6}}\right] = \tan^{-1}(1) = \frac{\pi}{4}$

$\beta = \cos^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$= \cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{2}{3}\right) = \frac{\pi}{2}$

$$\gamma = \sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right] + \frac{1}{2}\cos^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$$

$$= \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right] + \frac{1}{2}\cos^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$$

$$= \sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right] + \frac{1}{2}\cos^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$$

$$= \frac{\pi}{3} + \frac{1}{2} \times \frac{2\pi}{3} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

43.(B) Now, $\cos(\alpha + \beta + \gamma)$

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{2} + \frac{2\pi}{3}\right)$$

$$= \cos\left(\frac{3\pi + 6\pi + 8\pi}{12}\right) = \cos\left(\frac{17\pi}{12}\right)$$

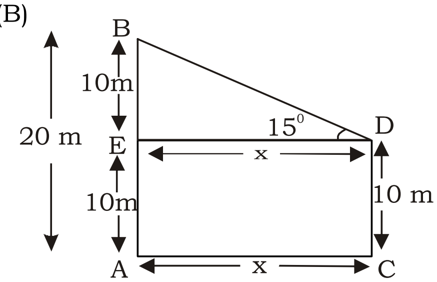
44. (D) $\tan\alpha - \tan\frac{\beta}{2} + \sqrt{3}\tan\frac{\gamma}{4} = \tan\frac{\pi}{4} - \tan\frac{\pi}{4}$

$$+\sqrt{3}\tan\frac{\pi}{6} = \sqrt{3} \times \frac{1}{\sqrt{3}} = 1$$

45. (C) $\text{cosec}^{-1}(-\sqrt{2})$

$$= \text{cosec}^{-1}\,\text{cosec}\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

46. (B)



Now, in ΔEDB,

$$\tan 15^0 = \frac{10}{x} \Rightarrow \tan(60^0 - 45^0) = \frac{10}{x}$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{10}{x}$$

$$\Rightarrow x = 10(2 + \sqrt{3}) = 37.3 \text{ m}$$

47. (A) Let $\angle A = 30^0$, $\angle B = 45^0$ and AB = $\sqrt{3} + 1$

Then, $\angle C = 180^0 - (\angle A + \angle B)$

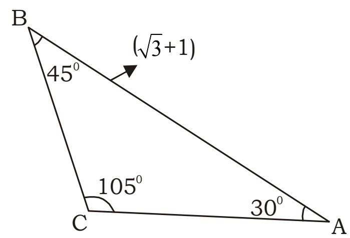
(since, the sum of inrernal angles of a triangle is 180^0).

$= 180^o - (30^o + 45^o) = 105^o$

By Sine Formula,

$$\frac{\sin 30^o}{Bc} = \frac{\sin 105^o}{\sqrt{3}+1}$$

$$\Rightarrow BC = (\sqrt{3} + 1) \times \left(\frac{2\sqrt{2}}{\sqrt{3}+1}\right) \times \frac{1}{2} = \sqrt{2}$$



Again, now by sine rule $\frac{\sin 45^0}{AC} = \frac{\sin 105^0}{\sqrt{3}+1}$

$$\Rightarrow AC = \frac{(\sqrt{3}+1)}{\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{3}+1} = 2$$

$\therefore$ Area of ΔABC $= \frac{1}{2} \times BC \times AC \times \sin 105^0$

$$= \frac{1}{2} \times 2 \times \sqrt{2} \times \frac{(\sqrt{3}+1)}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2} \text{ cm}^2$$

48. (B) P : $x^2 - y^2 + 2x - 1 = 0$

$\Rightarrow x^2 = (y-1)^2$

$\Rightarrow (x - y + 1)(x + y - 1) = 0$

$\therefore$ equation of angle bisector is

$$\frac{(x+y-1)}{\sqrt{2}} = \pm \frac{(x-y+1)}{\sqrt{2}}$$

$\Rightarrow x = 0$ or, $y - 1 = 0$

combined equation is

$x(y-1) = 0$

$= xy - x = 0$

49. (B) Given, $v = -x^2 \log x$

On differentiating w.r.t. x, we get

$$\frac{dv}{dx} = -2x\log x - \frac{x^2}{x} = -2x\log x - x$$

For maximum or minimum value of velocity,

put $\frac{dv}{dx} = 0 \Rightarrow -2x\log x - x = 0$

$$\Rightarrow \log x = -\frac{1}{2} \Rightarrow x = e^{-1/2}$$

Now, $\frac{d^2v}{dx^2} = -\frac{2x}{x} - 2\log x - 1$

$= -3 - 2\log x$

At $x = e^{-1/2}$

$\frac{d^2v}{dx^2} = -3 - 2\left(-\frac{1}{2}\right) = -2$ maxima.

$\therefore$ At $x = e^{-1/2}$, the velocity is maximum

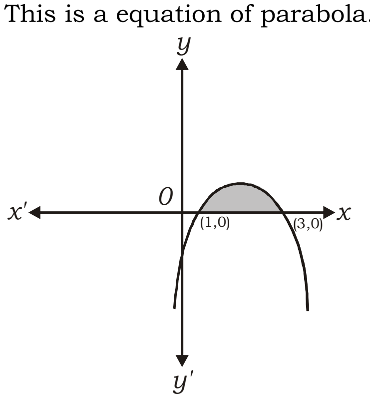
50. (B) Given, $4x - x^2 - 3 = y$

$\Rightarrow -(x^2 - 4x) = y + 3$

$\Rightarrow -(x^2 - 4x + 4) = y + 3 - 4$

$\Rightarrow (x - 2)^2 = -(y - 1)$

This is a equation of parabola.



$\therefore$ Required area $= \int_1^3 y\,dx$

$= \int_1^3 (4x - x^2 - 3)dx$

$= \left[2x^2 - \frac{x^3}{3} - 3x\right]_1^3$

$= 18 - 9 - 9 - \left(2 - \frac{1}{3} - 3\right) = \frac{4}{3}$ sq. units

51. (D) Given, $f^1(x) = 6 - 4\sin 2x$

On integrating both the sides, we get

$f(x) = 6x + \frac{4\cos 2x}{2} + C$

As $f(0) = 3$

As $f(0) = 3 = 0 + 2(1) + C$

$\Rightarrow C = 1$

$\therefore f(x) = 6x + 2\cos 2x + 1$

52. (B) $(gof)x = g(f(x))$

$= g(e^x) = \log e^x = x$

On differentiating w.r.t.x, we get

$(gof)'(x) = 1$

53. (D) Given, $f'(x) = g'(x)$

On integrating both sides, we get

$f(x) = g(x) + C$

$\Rightarrow f(x) = x^3 - 4x + 6 + C$ .... (i)

$\because$ $f(1) = 2$ (Given)

$\therefore$ $2 = 1 - 4 + 6 + c \Rightarrow C = -1$ [From Eq. (i)]

$f(x) = x^3 - 4x + 5$

54. (C) Given, $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

Now, redefine the given function

$f(x) = \begin{cases} 1, & x > 0 \\ 2, & x = 0 \\ -1, & x < 0 \end{cases}$

$\therefore$ Range of $f(x)$ is $\{-1, 1, 2\}$

55. (B) We know that the equation of circle, which touches both the axes, is

$x^2 + y^2 - 2r\,x - 2r\,y + r^2 = 0$

The centre (r, r) of this circle lies on the line

$x + y = 4.$

$\therefore r + r = 4$

$r = 2$

On putting the value of r in Eq. (i), we get

$r^2 + y^2 - 4x - 4y + 4 = 0$

which is required equation of circle.

56. (D) The equation of first circle is $x^2 + y^2 - 2x - 2y = 0$

Redius of this circle $= \sqrt{(1)^2 + (1)^2}$

$= \sqrt{2}$ $\left(by \sqrt{g^2 + f^2 - c}\right)$

and equation of second circle is $x^2 + y^2 = 1$

Radius of this circle = 1

From above it is clear that the radius of first circle is not twice that of second circle.

57. (B) $\because$ Foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are (ae, 0) and (- ae, 0) equation of circle with centre (0,0) and radius ae is

$x + y^2 = (ae)^2$ [where, $(ae)^2 = a^2 - b^2$]

$\therefore x^2 + y^2 = a^2 - b^2$

58. (B) $e_1 = \sqrt{1 - \frac{25}{169}} = \frac{12}{13} \Rightarrow e_2 = \sqrt{1 - \frac{b^2}{a^2}}$

$\because e_1 = e_2$ (given)

$\therefore \frac{12}{13} = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow \frac{a}{b} = \frac{13}{5}$

59. (B)

60. (D) Given that, equation of straight line is

$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ ...... (i)

and equation of plane is

ax + by + cz + d = 0 ...... (ii)

Since, the straight line is parallel to plane i.e, normal to plane is perpendicular to the straight line.

By perpendicularity condition,

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \Rightarrow al + bm + cn = 0$

61. (B) Direction ratios of the diagonal OP

= 2 - 0, 2 - 0. 2 - 0 and

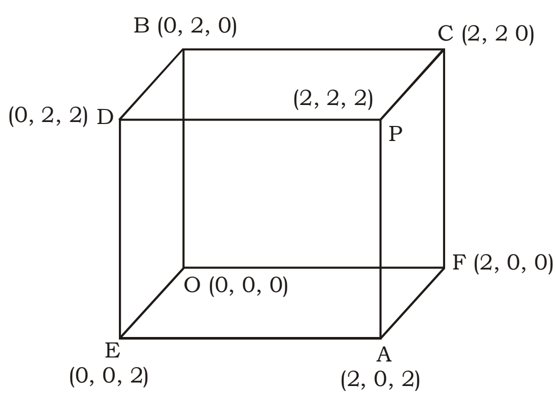
direction cosine = $\frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Direction ratios of diagonal AB

= 2 - 0, 0 - 2, 2 - 0 = 2, - 2, 2

and direction cosine = $\frac{2}{2\sqrt{3}}, \frac{-2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}$

$= \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$



Let θ be the angle between them,

then $\cos\theta = \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{-1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{1}{3} - \frac{1}{3} + \frac{1}{3} = \frac{1}{3} \Rightarrow \theta = \cos^{-1}(1/3)$

62. (C) direction ratios of side OB

= 0- 0, 2 - 0, 0 - 0

and direction cosine = $\frac{0}{2}, \frac{2}{2}, \frac{0}{2}$ = 0, 1, 0

Let the angle between diagonal OP and the side OB be θ_1 then,

$\cos\theta_1 = 0.\frac{1}{\sqrt{3}} + 1.\frac{1}{\sqrt{3}} + 0.\frac{1}{\sqrt{3}}$

$\Rightarrow \theta_1 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

63. (C)

64. (A) The intersection of given plane is

$x - y + 2z - 1 + \lambda(x + y - z - 3) = 0$

$\Rightarrow x(1+\lambda) + y(\lambda - 1) + z(2 - \lambda) - 3\lambda - 1 = 0$

Dr's of normal to the above plane is

$(1 + \lambda, \lambda - 1, 2 - \lambda)$

Taking option (A),

$-1(1+\lambda) + 3(\lambda - 1) + 2(2 - \lambda) = 0$

$\Rightarrow -1 - \lambda + 3\lambda - 3 + 4 - 2\lambda = 0 \Rightarrow 0 = 0$

65. (C) Given centre of sphere is (6, - 1, 2)

$\therefore$ Radius = $\frac{2(6)-1(-1)+2(2)-2}{\sqrt{4+1+4}} = \frac{15}{3} = 5$

$\therefore$ Equation of sphere is

$(x-6)^2 + (y+1)^2 + (z-z)^2 = 5^2$

$\Rightarrow x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

66. (A) The relation given in (A),

i, e, $f(x) = g(\sin^2 x)$ and $g(x) = \sqrt{x}$

Satisfy the given relations,

$g[f(x)] = g(\sin^2 x) = |\sin x|$

$f[g(x)] = f(\sqrt{x}) = \sin^2\sqrt{x} = (\sin\sqrt{x})^2$

67. (D) For (x) to be defined

$x + 3 > 0 \Rightarrow x > -3$

$\therefore x \in (-3, \infty)$

Also, $x^2 + 3x + 2 \neq 0 \Rightarrow (x+2)(x+1) \neq 0$,

i. e., $x \neq 1, x \neq -2$

So, the domain is $\frac{(-3,\infty)}{\{-1,-2\}}$

68.(D) $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+4}{5}\frac{5}{x+1}(x+1)}$

$$= \lim_{x\to\infty}\left[\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{5\frac{x+4}{x+1}}$$

$$e^{5\lim_{x\to\infty}\frac{1+\frac{4}{x}}{1+\frac{1}{x}}} = e^5$$

69.(B) Put $x = \cos^2\theta \Rightarrow \theta = \cos^{-1}\sqrt{x}$

$\therefore$ $y = \sin^{-1}\sqrt{1-x} + \cos^{-1}\sqrt{x}$

$= \sin^{-1}\sqrt{\sin^2\theta} + \cos^{-1}\sqrt{\cos^2\theta}$

$\Rightarrow y = \sin^{-1}\sin\theta + \cos^{-1}\cos\theta$

$\Rightarrow y = \theta + \theta = 2\theta \Rightarrow y = 2\cos^{-1}\sqrt{x}$

$\therefore \frac{dy}{dx} = -\frac{2}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} \Rightarrow \frac{dy}{dx}$

$= \frac{-1}{\sqrt{x(1-x)}}$

70. (B) On putting $x = \tan\theta$, we have

$y = \tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+x^2} = \frac{1}{2}$ at $x = 0$

Again, putting $x = \sin\phi$, we get

$z = \tan^{-1}\left(\frac{2\sin\phi\cos\phi}{1-2\sin^2\phi}\right) = \tan^{-1}\frac{\sin 2\phi}{\cos 2\phi}$

$= \tan^{-1}\tan 2\phi = 2\phi = 2\sin^{-1}x$

$\Rightarrow \frac{dz}{dx} = \frac{2}{\sqrt{1-x^2}} = 2$ at $x = 0$

$\therefore \frac{dy}{dz} = \frac{dy}{dx}\frac{dx}{dz} = \frac{1/2}{2} = \frac{1}{4}$

71. (C) Statement I Given, y = In (secx + tan x)

On differentiating it w.r.t.x, we get

$\frac{dy}{dx} = \frac{1}{(\sec x+\tan x)}\frac{d}{dx}(\sec x + \tan x)$

$= \frac{1}{(\sec x+\tan x)}(\sec x.\tan x + \sec^2 x)$

$= \frac{1}{(\sec x+\tan x)}\sec x(\tan x + \sec x)$

$= \sec x$

Statement II Given, y

$= \log(\text{cosec } x - \cot x)$

$\frac{dy}{dx} = \frac{1}{(\text{cosec } x-\cot x)}\frac{d}{dx}(\text{cosec } x - \cot x)$

$= \frac{1}{(\text{cosec } x-\cot x)}\times(-\text{cosec}x.\cot x + \text{cosec}^2 x)$

$= \text{cosec } x.\frac{(\text{cosec } x-\cot x)}{(\text{cosec } x-\cot x)} = \text{cosec}x$

So, Statements I and II both are true.

72. (B) $3^x + 3^y = 3^{x+y}$

On differenting w. r. t. x, we get

$3^x\log 3 + 3^y\log 3\frac{dy}{dx} = + 3^{(x+y)}\log 3\left(1+\frac{dy}{dx}\right)$

$\Rightarrow 3^x + 3^y\frac{dy}{dx} = 3^{x+y}\,3^{(x+y)}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(-3^{x+y} + 3^y) = 3^{x+y} - 3^x$

$\Rightarrow \frac{dy}{dx} = \frac{3^x(3^y-1)}{3^y(1-3^x)} = \frac{3^{x-y}(3^y-1)}{(1-3^x)}$

73. (C) Let a, b and c be in HP.

$\therefore \quad b = \frac{2ac}{a+c}$

Now,

$\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{\frac{2ac}{a+c}-a} + \frac{1}{\frac{2ac}{a+c}-c}$

$= \frac{1}{a\left(\frac{2c-a-c}{a+c}\right)} + \frac{1}{c\left(\frac{2a-a-c}{a+c}\right)}$

$= \frac{a+c}{a(c-a)} + \frac{a+c}{c(a-c)}$

$= \left(\frac{a+c}{c-a}\right)\left(\frac{1}{a}-\frac{1}{c}\right)$

$= \frac{a+c}{c-a}\times\frac{c-a}{ca}$

$= \frac{a+c}{ca} = \frac{1}{a}+\frac{1}{c}$

Hence, a, b, c are in H.P.

74. (A) Total number of terms = $\left(1-\frac{x}{2}\right)^8$ = 9

$$\left[\begin{array}{r}\because n = 8\,(\text{even})\\ \text{Middle term} = \left(\frac{n}{2}+1\right)\text{th term}\end{array}\right]$$

$\therefore$ Middle term is 5th term.

Hence, $T_5 = {}^8C_4(1)^4\left(-\frac{x}{2}\right)^4 = \frac{70x^4}{16} = \frac{35x^4}{8}$

75. (A) The given equation is

$$(2-\sqrt{3})x^2 - (7-4\sqrt{3})x + (2+\sqrt{3}) = 0$$

$\therefore$ Sum of roots $= \frac{(7-4\sqrt{3})}{2-\sqrt{3}}$

$= \frac{(2-\sqrt{3})^2}{(2-\sqrt{3})}$

$= 2-\sqrt{3}$

76. (D) $\because$ Combinations formed after taking 1, 2, 3, ..., n things at a time are ${}^nC_1, {}^nC_2, \ldots, {}^nC_n$.
$\therefore$ Total number of combinations
$= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$
$= 1 + {}^nC_n + {}^nC_2 + \ldots + {}^nC_n - 1$
$= 2^n - 1$
$[\because 2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n]$

77. (B) Since, one root of $ax^2 + bx + c = 0$, $a \neq 0$ is positive and another root is negative which is possible only if $a > 0$, $b < 0$, $c > 0$.

78. (C) $\because \frac{dy}{dx} = \frac{ax+3}{2y+f}$ [Given]

$\Rightarrow \int (2y+f)dy = \int (ax+3)dx$

$\Rightarrow c + y^2 + fy = \frac{ax^2}{2} + 3x$

$\Rightarrow \frac{-a}{2}x^2 + y^2 - 3x + fy + C = 0$

This equation represents a circle, if the coefficient of x^2 = the coefficient of y^2

$-1 = \frac{a}{2} \Rightarrow a = -2$

79. (D) $\because$ A, B and C are in AP.
$\therefore$ 2B = A + C
$\because$ A + B + C = 180°
$\Rightarrow$ 3B = 180° $\Rightarrow$ B = 60°
Now, by sine rule,

$$\frac{b}{c} = \frac{\sin B}{\sin C} \Rightarrow \frac{\sqrt{3}}{\sqrt{2}} \quad \left(\because \frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}}\right)$$

$\Rightarrow \sin C = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$

80. (C) Since, the points with position vectors $10i + 3j$, $12i - 5j$ and $ai + 11j$ are collinear, *i.e.*, area of triangle formed by these positions vectors should be zero.

Therefore, $\frac{1}{2}\begin{vmatrix}10 & 3 & 1\\ 12 & -5 & 1\\ a & 11 & 1\end{vmatrix} = 0$

$\Rightarrow a(3 + 5) - 11(10 - 12) + 1(-50 - 36) = 0$
$\Rightarrow 8a + 22 - 86 = 0$
$\Rightarrow 8a = 64$
$\Rightarrow a = 8$

81. (B) We know that the angle between the vectors $a_1i + b_1j + c_1k$ and ad $a_2i + b_2j + c_2k$ is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$\therefore$ Angle between the vector $i + 2j + 3k$ and $-i + 2j + 3k$ is given by

$$\cos\theta = \left|\frac{1\times(-1)+2\times2+3\times3}{\sqrt{1+4+9}\sqrt{1+4+9}}\right|$$

$= \frac{-1+4+9}{14} = \frac{12}{14} = \frac{6}{7}$

Now, $\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\frac{36}{49}}$

$= \sqrt{\frac{49-36}{49}}$

$= \sqrt{\frac{13}{49}}$

$= \frac{\sqrt{13}}{7}$

82. (D) $\sin\left[\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x\right] = 1$

$\Rightarrow \sin\left[\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x\right] = \sin\frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{1}{5} = \cos^{-1}\frac{1}{5}$

$\left(\because \sin^{-1}x+\cos^{-1}x = \frac{\pi}{2}\right)$

$\Rightarrow \quad x = \frac{1}{5}$

83. (B) $\log(a+\sqrt{a^2+1})+\log\left(\frac{1}{a+\sqrt{a^2+1}}\right)$

$= \log(a+\sqrt{a^2+1})+\log(a+\sqrt{a^2+1})^{-1}$

$= \log(a+\sqrt{a^2+1})-\log(a+\sqrt{a^2+1})$

$= 0$

84. (B) Number of ways when one specified book is included $= {}^9C_4 = m$

$\Rightarrow \quad m = 126,$

and number of ways when one specific book is excluded

$= {}^9C_5 = n$

$\Rightarrow \quad n = 126$

$\Rightarrow \quad m = n$

85. (C) $\because \quad f(x) = |x| + x^2$

$\Rightarrow f(x) = \begin{cases} x^2+x & x \ge 0 \\ x^2-x & x<0 \end{cases}$

LHL $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0}(-h)^2 + h = 0$

and,

RHL $= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0}(+h^2)+h = 0$

$\Rightarrow$ LHL $=$ RHL $= f(0)$

$\Rightarrow f(x)$ is continuous at $x = 0$.

Now,

$Lf'(0) = \text{LHD} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0}\frac{h^2+h}{-h} = \lim_{h\to 0} h+1 = 1$

$\because f(0) = 0$

$= -1$

$Rf'(0) = \text{RHD} = \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h^2+h}{h}$

$= \lim_{h\to 0}(h+1) = 1$

$\Rightarrow$ LHD $\neq$ RHD

$\Rightarrow f(x)$ is not differentiable at $x = 0$.

86. (B) Let the roots of the equation $ax^2 + bx + c = 0$ be α and 2α.

$\therefore \quad \alpha + 2\alpha = \frac{-b}{a}$, and $\alpha.2\alpha = \frac{c}{a}$

$\Rightarrow \quad \alpha = \frac{-b}{3a}$, and $\alpha^2 = \frac{c}{2a}$

$\Rightarrow \left(\frac{-b}{3a}\right)^2 = \frac{c}{2a} \Rightarrow \frac{b^2}{9a^2} = \frac{c}{2a}$

$\Rightarrow 2b^2 = 9ac$

87. (C) Since, on the set of real numbers, R is a relation defined by xRy if and only if $3x + 4y = 5$ for which $1R\frac{1}{2}$ and $\frac{2}{3}R\frac{3}{4}$.

i.e., $1R\frac{1}{2} \Rightarrow 3.1 + 4.\frac{1}{2} = 5,$

and $\frac{2}{3}R\frac{3}{4} \Rightarrow 3 \times \frac{2}{3} + \frac{3}{4} \times 4 = 5$

Hence, both the statements II and III are correct.

88. (C) $f(x) = k\sin x + \frac{1}{3}\sin 3x$ (given)

$\Rightarrow \quad f'(x) = k\cos x + \frac{3}{3}\cos 3x$

Put $f'(x) = 0$, for maxima

$k\cos x + \cos 3x = 0$

At $\quad x = \frac{\pi}{3}$, $k\cos\frac{\pi}{3} + \cos\pi = 0$

$\Rightarrow \quad k\left(\frac{1}{2}\right) = 1 \Rightarrow k = 2$

89. (A)Let I= $\int \sin^{-1}(\cos x)\,dx$

$$= \int \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right] dx$$

$$= \int \left(\frac{\pi}{2} - x\right) dx$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C$$

where C is a constant of integration.

90. (C) $\because$ α and β are the roots of the equation.
$4x^2 + 3x + 7 = 0$

$$\therefore \quad \alpha + \beta = -\frac{3}{4} \text{ and } \alpha\beta = \frac{7}{4}$$

Now, $\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\frac{9}{16} - \frac{7}{2}}{\frac{49}{16}}$$

$$= \frac{\frac{9-56}{16}}{\frac{49}{16}}$$

$$= \frac{-47}{16} \times \frac{16}{49}$$

$$= \frac{-47}{49}$$

91. (C) The equation of line passing through (2, –3) and parallel to Y-axis is $(y + 3) = \tan 90(x - 2)$
$\Rightarrow x - 2 = 0 \Rightarrow x = 2$.

92. (C) The given equation are
$x^2 + y^2 = 4$,
and $x + y = 2$
These equations are satisfied by only (2, 0) and (0, 2).
Hence, the required set is {(0, 2), (2, 0)}.

93. (A) The inverse of a square matrix, if it exists, is unique but if A and B are singular matrices of order n, then AB is not a singular matrix of order n.
Hence, only statement I is correct.

94. (A) $\because 2 \times 1 + 3 \times (-2) + 4 \times 1 = 0$
$(\because \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2)$
$\Rightarrow \cos\theta = 0 = \cos 90° \Rightarrow \theta = 90°$
$\therefore$ Angle between the lines is 90°.

95. (A) $\because f(x) = \begin{cases} \frac{x^3 - 3x + 2}{(x-1)^2}, & \forall x \neq 1 \\ k & \forall x = 1 \end{cases}$

and $f(x)$ is continuous.

$$\therefore \lim_{x\to 1} \frac{x^3 - 3x + 2}{(x-1)^2} = k \qquad \left(\because \frac{0}{0} \text{form}\right)$$

By L Hospital rule

$$\Rightarrow \quad k = \lim_{x\to 1} \frac{3x^2 - 3}{2(x-1)} \qquad \left(\because \frac{0}{0} \text{form}\right)$$

By L Hospital rule

$$= \lim_{x\to 1} \frac{6x}{2}$$

$= 3$

96. (D) The given equation is
$x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$
Now, $B^2 - 4AC = (-2p)^2 - 4(1)(p^2 - q^2 + 2pr - r^2)$
$= 4p^2 - 4p^2 + 4(q - r)^2$
$= 4(p - r)^2$
which is always greater than zero.
Therefoere, the roots of the given equation are rational.

97. (B) Let I $= \int_0^1 \frac{\tan^{-1}}{1+x^2}.dx$

Put $\begin{cases} \tan^{-1} x = dt \\ \frac{dx}{1+x^2} = dt \end{cases}$

when $x = 0$, then $t = 0$

$x = 1$, then $t = \frac{\pi}{4}$

$$\therefore \int_0^{\pi/2} t\,dx = \left[\frac{t^2}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right)^2 = \frac{\pi^2}{32}$$

98. (A) Let $I = \int_0^{\pi/2} \sin 2x\, In(\cot x) dx$

$\because \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$= \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right) \text{In } \cot\left(\frac{\pi}{2} - x\right)$

$I = \int_0^{\pi/2} \sin 2x\, In(\tan x) dx$ ... (ii)

On adding Eqs. (i) and (ii), we get,

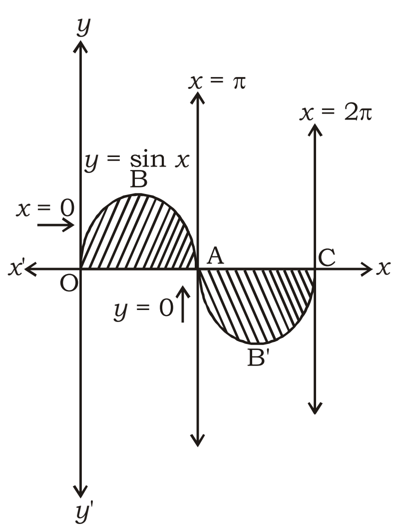
$2I = \int_0^{\pi/2} \sin 2x\, [\text{In } \cot x + \text{in}(\tan x)\}.dx$

$= \int_0^{\pi/2} \sin 2x\, [\text{In } \cot x + \tan x)]dx$

$= \int_0^{\pi/2} \sin 2x\, .\text{In}1.dx = 0$

$I = 0$

99. (C)



Required area (OBAB′C)

$= \int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} -\sin x\, dx$

$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$

$= -(\cos \pi - \cos 0) + (\cos 2\pi - \cos \pi)$

$= -(-1 - 1) + (1 + 1)$

= 4 sq. units

100. (A) Let $I = \int \frac{In\, x}{x} dx$

$I = \int t dt$

Put $\begin{cases} In\, x = t \\ \frac{1}{x} dx = dt \end{cases}$

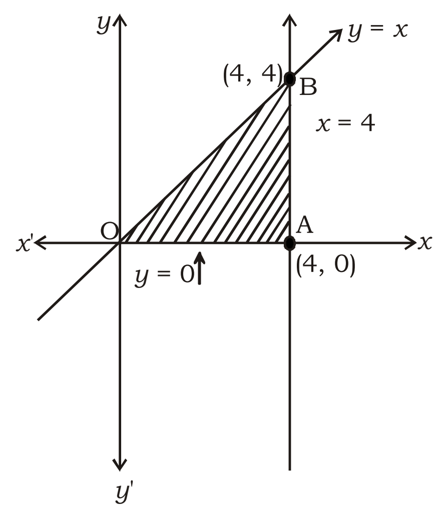
$I = \frac{t^2}{2} + C$

$= \frac{(In\, x)^2}{2} + C$

101. (B) $\therefore$ Required Area = area (Δ OAB)

$= \frac{1}{2} \times 4 \times 4$

= 8 sq. units



102. (A) $\int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}\right) dx$

$= \int (\sec^2 x - \text{cosec}^2 x) dx$

$= \tan x + \cot x + C$

$= \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) + C$

$= \frac{1}{\sin x.\cos x} + C$

$= \frac{2}{\sin 2x} + C$

$= 2\,\text{cosec}\, 2x + C$

103. (D) The power of highest derivative is 1.
So, degree = 1.

104. (B) The pairs $\left(2, \frac{3}{2}\right)$ is not feasible. Because, the degree of any differential equation cannot be rational type. If so, then we use rationization and convert it into integer.

105. (A) Given, $y = a\sin(\lambda x + \alpha)$ ... (i)

On differentiating it *wrt* x, we get

$$\frac{dy}{dx} = \frac{d}{dx} a\sin(\lambda x + \alpha)$$

$$= a\cos(\lambda x + \alpha)\lambda$$

$$\frac{dy}{dx} = a\lambda\cos(\lambda x + \alpha)$$

Again differentiating it *wrt* x, we get

$$\frac{d^2y}{dx^2} = a\lambda \frac{d}{dx}\cos(\lambda x + \alpha)$$

$$= a\lambda[-\sin(\lambda x + \alpha)] \times \lambda$$

$$= -a\lambda^2 \sin(\lambda x + \alpha)$$

$$\frac{d^2y}{dx^2} = -\lambda^2 y \quad \text{[from Eq. (i)]}$$

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0$$

106. (C) $y\frac{dy}{dx} + x = a$, $y\,dy + x dx = a dx$

On integrating both sides, we get

$$\int y\,dy + \int x\,dx = \int a\,dx, \frac{y^2}{2} + \frac{x^2}{2} = \text{ax}$$

$\Rightarrow x^2 + y^2 - 2ax = 0$

Which represents a set of circles.

107. (D) The given differential equation is

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0 \quad \text{... (i)}$$

(a) $y = x - 1 \Rightarrow \frac{dy}{dx} = 1$

From equation (i),

$(1)^1 - x(1) + (x - 1)$

$= 1 - x + x - 1 = 0$

So, $y = x - 1$ is a solution of Eq. (i).

(b) $4y = x^2 \Rightarrow y = \frac{x^2}{4} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$

From Equation (i),

$$\left(\frac{x}{2}\right)^2 - x\left(\frac{x}{2}\right) + \left(\frac{x^2}{4}\right)$$

$$= \frac{x^2}{4} - \frac{x^2}{2} + \frac{x^2}{4} = \frac{x^2}{2} - \frac{x^2}{2} = 0$$

So, $4y = x^2$ is a solutions of Equation (i).

(c) $y = x \Rightarrow \frac{dy}{dx} = 1$

From equation (i),

$(1)^2 - x(1) + x = 1 \neq 0$

$\therefore$ $y = -x - 1$ is a solution of Eq. (i).

108. (C) Given,

$$x^2dy + y^2dx = 0, \frac{dy}{y^2} + \frac{dx}{x^2} = 0$$

On integrating, we get

$$\int y^{-2}dy + \int x^{-2}dx = 0$$

$$\frac{y^{-2+1}}{-2+1} + \frac{x^{-2+1}}{-2+1} = C_1$$

$$\frac{y^{-1}}{-1} + \frac{x^{-1}}{-1} = C_1, \frac{-1}{y} - \frac{1}{x} = C_1$$

$$\frac{1}{x} + \frac{1}{y} = -C_1, x + y = C_1xy \ \frac{1}{C_1}(x + y) = xy$$

$C(x + y) = xy$, where $\frac{1}{C_1} = C$

109. (D) Given, $e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$

$$\Rightarrow \frac{e^x}{1 - e^x}.dx + \frac{\sec^2 y}{\tan y}.dy = 0$$

On integrating, we get

$$\Rightarrow \int \frac{e^x dx}{1 - e^x} + \int \frac{\sec^2 y}{\tan y} = 0$$

$-\log(1 - e^x) + \log\tan y = \log C$

$\log \tan y = \log C + \log(1 - e^x) = \log C(1 - e^x)$

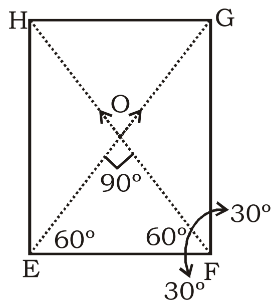
$\tan y = C(1 - e^x)$

110. (D) Let the one side of rhombus be a.

Then, in ΔOEF,

$$\sin 60° = \frac{OF}{a} \Rightarrow OF = a \times \frac{\sqrt{3}}{2}$$

We know that the diagonal of rhombus bisect each other perpendicularly.



$\because$ FH = 2FO = $2a\frac{\sqrt{3}}{2}$ ... (i)

Again, in ΔOEF,

$$\sin 30° = \frac{OE}{a} \Rightarrow OE = a \times \frac{1}{2}$$

$\therefore$ EG = 2EO = $2.\frac{a}{2} = a$

Given magnitude of FH = magnitude of $\{mEG\}$.

$\therefore$ $a\sqrt{3} = ma$

On comparing, we get $m = \sqrt{3}$

111. (C) Given that;

$a \circ b = 0$

i.e. a and b are perpendicular to each other and $a \times b = 0$.

i.e. a and b are parallel to each other.

So, both conditions are possible if

$a = 0$ and $b = 0$

112. (C) Given that,

$a \times (b \times a)$

which is the vector triple product

$= (a \circ a)b - (a \circ b)a$

$= \lambda b - ua$

where λ and μ are scalar quantity.

$\Rightarrow a \times (b \times a)$ is coplanar with both a and b.

113. (B) Both statements are true.

Statements 1

$4i \times 3i$

$= 12(i \times i)$

$= 12 \times 0$ $[\because i \times i = 0]$

Statements 2

$$\frac{4i}{3i} = \frac{4}{3}$$

Divisibility in vectors are not possible.

114. (A) Given,

$(\lambda i + j - k) \times (3i - 2j + 4k)$

$= (2i - 11j - 7k)$

$$\Rightarrow \begin{vmatrix} i & j & k \\ \lambda & 1 & -1 \\ 3 & -2 & 4 \end{vmatrix} = (2i - 11j - 7k)$$

$\Rightarrow 2i - (4\lambda + 3)j + (-2\lambda - 3)k$

$= 2i - 11j - 7k$

On comparing the coefficient of 'j'

$(4\lambda + 3) = 11 \Rightarrow 4\lambda = 8 \Rightarrow \lambda = 2$

115. (D) $|p(-3i - 2j + 13k)| = 1$

$$\Rightarrow \sqrt{(-3p)^2 + (-2p)^2 + (13p)^2} = 1$$

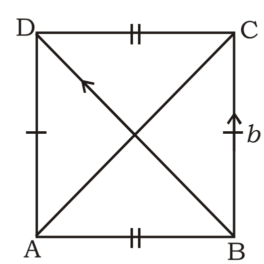
$$\Rightarrow \sqrt{9p^2 + 4p^2 + 169p^2} = 1$$

$$\Rightarrow \sqrt{182p^2} = 1$$

$$\Rightarrow p = \frac{1}{\sqrt{182}}$$

116. (B) The vector $2j - k$ lies in the plane of YZ. Because its x-coordinates is zero.

117. (D)



Since, opposite sides of parallelogram are same.

AB = $a \Rightarrow$ CD = $-a$

and BC= $b \Rightarrow$ DA = – b

Applying addition formula in ΔBCD.

BD = BC + CD

= $b - a = -a + b$

118. (A) The geometric mean of 1, 2, 4, 8 ...... 2^n

$= (1.2.4.8......2^n)^{\frac{1}{n+1}}$

$= (2.2^2.2^3......2^n)^{\frac{1}{n+1}}$

$= (2^{1+2+3+}.....2^n)^{\frac{1}{n+1}} = \left(2^{\Sigma n}\right)^{\frac{1}{n+1}}$

$2^{\frac{n(n+1)}{2}\times\frac{1}{n+1}} = 2^{\frac{n}{2}}$

119. (D)Let observations are $x_1, x_2 ... x_{10}$

Given,

$\frac{x_1 + x_2 + x_3 + ...x_{10}}{10} = 5$

$\Rightarrow x_1 + x_2 + x_3 + ... + x_{10} = 50$

Again, according to question

New mean

$= \frac{\left[(x_1+2)+(x_2+2)+(x_3+2)+......+(x_{10}+2)\right]\times 3}{10}$

$= \frac{(50+20)\times 3}{10} = \frac{70\times 3}{10} = 21$

120. (A) 1 + 3 + 5 + 7 + 9 .... n term

$= \frac{n}{2}[(2\times 1) + (n-1)\, 2] = \frac{n}{2}\times 2n = n^2$

$\therefore$ Mean = $\frac{\text{Sum of } n \text{ odd natural numbers}}{\text{Total numbers}}$

$= \frac{n^2}{n} = n$

NDA MATHS MOCK TEST- 66 (ANSWER KEY)

1.	(B)	26.	(A)	51.	(D)	76.	(D)	101.	(B)
2.	(A)	27.	(A)	52.	(B)	77.	(B)	102.	(A)
3.	(B)	28.	(A)	53.	(D)	78.	(C)	103.	(D)
4.	(B)	29.	(D)	54.	(C)	79.	(D)	104.	(B)
5.	(B)	30.	(C)	55.	(B)	80.	(C)	105.	(A)
6.	(D)	31.	(B)	56.	(D)	81.	(B)	106.	(C)
7.	(D)	32.	(D)	57.	(B)	82.	(D)	107.	(D)
8.	(B)	33.	(B)	58.	(B)	83.	(B)	108.	(C)
9.	(C)	34.	(B)	59.	(B)	84.	(B)	109.	(D)
10.	(D)	35.	(B)	60.	(D)	85.	(C)	110.	(D)
11.	(B)	36.	(C)	61.	(B)	86.	(B)	111.	(C)
12.	(B)	37.	(B)	62.	(C)	87.	(C)	112.	(D)
13.	(D)	38.	(C)	63.	(C)	88.	(C)	113.	(B)
14.	(D)	39.	(B)	64.	(A)	89.	(A)	114.	(A)
15.	(C)	40.	(D)	65.	(C)	90.	(C)	115.	(D)
16.	(B)	41.	(B)	66.	(A)	91.	(C)	116.	(B)
17.	(D)	42.	(C)	67.	(D)	92.	(C)	117.	(D)
18.	(D)	43.	(B)	68.	(D)	93.	(A)	118.	(A)
19.	(B)	44.	(D)	69.	(B)	94.	(A)	119.	(D)
20.	(B)	45.	(C)	70.	(B)	95.	(A)	120.	(A)
21.	(C)	46.	(B)	71.	(C)	96.	(D)		
22.	(C)	47.	(A)	72.	(B)	97.	(B)		
23.	(A)	48.	(B)	73.	(C)	98.	(A)		
24.	(A)	49.	(B)	74.	(A)	99.	(C)		
25.	(C)	50.	(B)	75.	(A)	100.	(A)		

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*