2007, OUTRAM LINES, 1ST FLOOR, OPPOSITE MUKHERJEE NAGAR POLICE STATION, DELHI-110009

## NDA (MATHS) MOCK TEST - 64 (SOLUTION)

1. (D)Let $x=\sqrt{8+2 \sqrt{8+2 \sqrt{8+\ldots+\infty}}}$

Squaring both the sides of the equation, we have,
$x^{2}=[\sqrt{8+2 \sqrt{8+2 \sqrt{8+\ldots+\infty}}}]^{2}$
$\Rightarrow x^{2}=8+2 \sqrt{8+2 \sqrt{8+2 \sqrt{8+\ldots+\infty}}}$
$\Rightarrow x^{2}=8+2 \mathrm{x}$
$\Rightarrow x^{2}-2 \mathrm{x}-8=0$
$\Rightarrow x(x-4)+2(x-4)=0$
$\Rightarrow x+2=0$ or $x-4=0$
$\Rightarrow x+2=0$ or $x-4=0$
Neglecting the negative sign, we have, $x=4$
2. (C) Given that $\mathrm{A}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$

There are 3 elements in the set,
If the number of elements in the set is ' $n$ ', then the
number if subsets is $2^{n}$.
But the set $A$ is also a subset of $A$.
Since we required the number of proper subsets.
The total number of proper subset is $2^{n}-1$
Therefore, there are $2^{3}-1=8-1=7$ proper subsets of $A$.
3. (A) Total number of arrangements $=\frac{10!}{2}$

Consider a single unit which comprises of two 'I's.
Thus, there are 9! ways in which two 'I's are together.
So, the number of arrangements in
which two 'I's are not together $=\frac{10!}{2}-9!$
Thus, required probability $\mathrm{SP}=$
Number of favourable events
Total number of events
$=\frac{\frac{10!}{2}-9!}{\frac{10!}{2}}$
$=\frac{4}{5}$
4. $(\mathrm{C}) \Rightarrow \mathrm{A}=\tan ^{-1} \frac{1}{2}$ and $\mathrm{B}=\tan ^{-1} \frac{1}{3}$

$$
\begin{aligned}
& \Rightarrow 4 A=4 \tan ^{-1} \frac{1}{2} \text { and } 4 B=4 \tan ^{-1} \frac{1}{3} \\
& \Rightarrow 4 A+4 B=4 \tan ^{-1} \frac{1}{2}+4 \tan ^{-1} \frac{1}{3} \\
&=4\left(\tan ^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right)\right) \\
&=4\left(\tan ^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)\right) \\
&=4\left(\tan ^{-1}\left(\frac{6}{5}\right)\right. \\
&=4\left(\frac{\pi}{4}\right) \\
&=\pi
\end{aligned}
$$

5. (C)


In $\triangle \mathrm{APB}$,
$\tan 45^{\circ}=\frac{\mathrm{BP}}{\mathrm{AB}}$
$\Rightarrow 1=\frac{h}{A B}$
$\Rightarrow \mathrm{h}=\mathrm{AB} \ldots$... 1 )
In triangle AQB.
$\tan 60^{\circ}=\frac{\mathrm{BQ}}{\mathrm{AB}}$
$\Rightarrow h=\frac{300}{\sqrt{3}}$ [Form equation(1)]

$$
h=100 \sqrt{3} m
$$

6. (C) If the position vectors in the plane are collinear,
$\mathrm{AB}=\lambda \mathrm{BC}$
$\Rightarrow \overrightarrow{O B}-\overrightarrow{O A}=\lambda \overrightarrow{O C}-\overrightarrow{O B}$
$\Rightarrow 2 \hat{\mathrm{i}}-8 \hat{\mathrm{j}}=\lambda(\mathrm{m} \hat{\mathrm{i}}-12 \hat{\mathrm{i}}+16 \hat{\mathrm{j}})$
$\Rightarrow 2 \hat{i}-8 \hat{j}=\lambda(\mathrm{m}-12) \hat{i}+16 \hat{j}$
Compaing the coefficients of $i$ and $j$, we have,
$\lambda(m-12)=2$ and $16 \lambda=-8$
$\Rightarrow \lambda(m-12)=2$ and $\lambda=-\frac{1}{2}$
Substituting the value of $\lambda$ in $\lambda(\mathrm{m}-12)=2$, we have,
$-\frac{1}{2}(m-12)=2$
$\Rightarrow-(\mathrm{m}-12)=4$
$\Rightarrow \mathrm{m}=8$
7. (C) Let $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}, X_{6}, X_{7}$, be the set of 7 observations.
Since mean of 7 observations is 10 , we have,
$10=\frac{X_{1}+X_{2}+X_{3}+X_{4}+X_{5}+X_{6}+X_{7}}{7}$
$\Rightarrow 70=X_{1}+X_{2}+X_{3}+X_{4}+X_{5}+X_{6}+X_{7}$
Let $Y_{1}, Y_{2}, Y_{3}$ be the set of 3 observations. Since mean of 3 Observations is 5, we have,
$\Rightarrow 5=\frac{Y_{1}+Y_{2}+Y_{3}}{3}$
$\Rightarrow 15=Y_{1}+Y_{2}+Y_{3} \ldots$ (2)
Now adding equations (1) and (2), We have,
$X_{1}+X_{2}+X_{3}+X_{4}+X_{5}+X_{6}+X_{7}+Y_{1}+Y_{2}+Y_{3}$
$=70+15$
$\Rightarrow \frac{X_{1}+X_{2}+X_{3}+X_{4}+X_{5}+X_{6}+X_{7}+Y_{1}+Y_{2}+Y_{3}}{10}$
$=\frac{70+15}{10}$
$=8.5$
8. (B)Thus, the $n^{\text {th }}$ term of binomial expansion $\left(X^{2}+\frac{2}{X}\right)^{15}$ is given as
$\mathrm{T}_{\mathrm{r}+1}={ }^{15} \mathrm{C}_{\mathrm{r}} \mathrm{X}^{2(15-\eta)}\left(\frac{2}{X}\right)^{r}$
$={ }^{15} \mathrm{C}_{\mathrm{r}} \mathrm{X}^{30-3 \mathrm{r}} 2^{\mathrm{r}} \ldots$ (1)
To get the /Coeffuecient of $\mathrm{x}^{15}, 30-3 \mathrm{r}=15$ $\Rightarrow 3 r=30-15$
$\Rightarrow r=5$
Thus, from equation(1), the coefficient of $x^{15}$ $={ }^{15} \mathrm{C}_{5} \times 2^{5} \ldots$ (2)
To get the independent of the term $x$, we have, $30-3 \mathrm{r}=0$
$\Rightarrow r=10$
Thus, from equation(1), the independent term $\mathrm{x}={ }^{15} \mathrm{C}_{10} \times 2^{10} \ldots$ (3)
From equations (2) and (3), required ratio
$=\frac{{ }^{15} \mathrm{C}_{5} \times 2^{5}}{{ }^{15} C_{10} \times 2^{10}}=\frac{1}{2^{5}}=\frac{1}{32}$
9. (B) $\left|\begin{array}{ccc}1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c\end{array}\right|=(1+a)[(1+b)(1+c)-1]-$
$1[1(1+c)-1]+1[(1-1)+b]$
$\Rightarrow \lambda=(1+\mathrm{a})(\mathrm{c}+\mathrm{b})+\mathrm{bc}-\mathrm{c}-\mathrm{b}$
$\Rightarrow \lambda=\mathrm{bc}+\mathrm{ac}+\mathrm{ab}+\mathrm{abc} \ldots$ (1)
Given that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$
$\Rightarrow \frac{b c+a c+a b}{a b c}=0$
$\Rightarrow \mathrm{bc}+\mathrm{ac}+\mathrm{ab}=0$
Therefore, equation (1) becomes, $\lambda=\mathrm{abc}$
10. (A)

$\therefore$ Required probability
$=\frac{\text { Favourable number of events }}{\text { Total number of events }}$
$=\frac{65}{125}$
$=\frac{13}{25}$
11. (C) Given that, $\log _{10} 2, \log _{10}\left(2^{x}-1\right)$ and $\log _{10}$
$\left(2^{x}+3\right)$ are in AP.
$\therefore 2 \log _{10}\left(2^{x}-1\right)=\log _{10} 2+\log _{10}\left(2^{x}+3\right)$
$\Rightarrow \log _{10}\left(2^{x}-1\right)^{2}=\log _{10}\left[2 \times\left(2^{x}+3\right)\right]$
$\Rightarrow\left(2^{x}-1\right)^{2}=\left[2 \times\left(2^{x}+3\right)\right]$
$\Rightarrow 2^{2 x}+1-2 \times 2^{x}=2 \times 2^{x}+6$
$\Rightarrow 2^{2 x}-2 \times 2^{x}-2 \times 2^{x}-5=0$
$\Rightarrow\left(2^{x}\right)^{2}-4\left(2^{x}\right)-5=0$
Let $2^{x}=y$, then above equation becomes
$y^{2}-4 y-5=0$
$\Rightarrow y(y-5)+1(y-5)=0$
$\Rightarrow(y+1)=0$ or $(y-5)=0$
$\Rightarrow y=-1$ or $y=5$
$\Rightarrow 2^{x}=-1$ or $2^{x}=5$
$\Rightarrow x=\log _{2}(-1)$ or $x=\log _{2} 5$
Logarithm of negative numbers does not exist.
$\therefore x=\log _{2} 5$
12. (A) The locus of points of intersection of a sphere and a plane is the circle.

13. (D) Imaginary roots always occur in conjugate pairs.
Thus, conjugate pair of $2+5 i$ is $2-5 i$.
Therefore, the other root of the equation is 2 $-5 i$.
14. (A) Let $x+i y=\sqrt{-2 i}$

Squaring both the sides, we have,
$(x+i y)^{2}=(\sqrt{-2 i})^{2}$
$\Rightarrow(x+i y)^{2}=-2 i$
$\Rightarrow x^{2}+i^{2} y^{2}+2 x y i=-2 i$
$\Rightarrow x^{2}-y^{2}+2 x y i=-2 i$
Comparing the real and imaginary parts, we have,
$x^{2}-y^{2}=0$ and $2 x y=-2$
Consider
$\left(x^{2}+y^{2}\right)^{2}=\left(x^{2}-y^{2}\right)+4 x^{2} y^{2}$

$$
=0+(-2)^{2}
$$

$$
=4
$$

$\Rightarrow\left(x^{2}+y^{2}\right)^{2}=2^{2}$
$\Rightarrow x^{2}+y^{2}=2$
We have,
$x^{2}-y^{2}=0$
$x^{2}+y^{2}=2$
Adding the above equations we have,
$2 x^{2}=2 \Rightarrow x^{2}=1 \Rightarrow x= \pm 1$
Substituting the value, $x^{2}=1$ in the equation $x^{2}+y^{2}=2$, we have, $y^{2}=1 \Rightarrow y= \pm 1$
$\therefore( \pm 1)+i( \pm 1)=\sqrt{-2 i}$
$\Rightarrow \sqrt{-2 i}= \pm(1+i)$
15. (C) Consider the equation of the curve

$$
\begin{aligned}
& \begin{aligned}
& y=\cos 3 x, 0 \leq x \leq \frac{\theta}{6} \\
& \text { Area }=\int_{0}^{\frac{\theta}{6}} \cos 3 x d x \\
&=\left.\left|\frac{\sin 3 x}{3}\right|\right|_{0} ^{\frac{\theta}{6}} \\
&=\frac{\sin 3 \propto \frac{\theta}{6}}{3}=\frac{\sin \frac{\theta}{2}}{3}=\frac{1}{3} \text { square unit }
\end{aligned}
\end{aligned}
$$

16. (A) At extreme point of a function, $f(x)$, the slope of the curve $\frac{d y}{d x}=0$

Since $\frac{d y}{d x}=0$, the tangent is parallel to $x$ axis.
17. (B) Consider the given function,

$$
y=\left|1, x^{\frac{1}{4}}\right|\left|k^{1}, x^{\frac{1}{2}}\right|\left|-x^{\frac{1}{4}}\right|
$$

Rewriting the above function, we have,
$\left.y=\left|1, x^{\frac{1}{4}}\right|\left|k^{1-x^{\frac{1}{4}}}\right| \right\rvert\, k^{1}, x^{\frac{1}{2}} \downarrow$
Using the identity, $(a+b)(a-b)=a^{2}-b^{2}$, we have,
$y=\left|1^{2}-\left|x^{\frac{1}{4}}\right|^{2}\right|_{\lambda}^{2}\left|k^{1, x^{\frac{1}{2}}}\right|$
$\Rightarrow y=\left|1-x^{\frac{1}{2}}\right|\left|k^{1}, x^{\frac{1}{2}}\right|$
Againg using the identity, $(a+b)(a-b)=a^{2}$
$-b^{2}$, we have, $\left.y=\left|1^{2}-\left|x^{\frac{1}{2}}\right|^{2}\right| x \right\rvert\,$
$\Rightarrow y=(1-x)$
$\Rightarrow \frac{d y}{d x}=-1$
18. (A) Consider the decimal number 0.3
$0.3 \times 2=0.60$
$0.6 \times 2=1.2$ 1
$0.2 \times 2=0.40$
$0.4 \times 2=0.80$
$0.8 \times 2=1.61$
Thus, binary equivalent of 0.3 is :
$(0.3)_{10}=(0.01001 \ldots)_{2}$
19. (A)It is a four letter word out of which two, O and E, are vowels.
Number of ways of selecting 2 letters from 4 $={ }^{4} \mathrm{C}_{2}$
Thus, the total number of events $={ }^{4} \mathrm{C}_{2}$
Number of ways of selecting 2 vowels from 2 letters $={ }^{2} \mathrm{C}_{2}$
Thus, the favourable number of events $={ }^{2} \mathrm{C}_{2}$ The probability of selecting two vowels
$=\frac{\text { Favourable number of events }}{\text { Total number of events }}$
$=\frac{{ }^{2} \mathrm{C}_{2}}{{ }^{4} \mathrm{C}_{2}}$
$=\frac{1}{6}$
20. (B) Given that $z=1+\operatorname{itan} \alpha$

$$
\begin{aligned}
& \pi<\alpha<\frac{3 \theta}{2} \\
& z=1+i \tan \alpha \\
& \begin{aligned}
|z| & =\sqrt{1, \tan ^{2} \beta} \\
& =\sqrt{\sec ^{2} \beta}
\end{aligned}
\end{aligned}
$$

Given that $\alpha$ lies in the third quadrant, and in third quadrant, tangent and cotagent are positive, and all other ratios are negative. Hence, $|z|-\sec \alpha$
21. (D) We know that if the vectors are coplanar, then thier scalar triple product is zero.
Hence,

$$
\left|\begin{array}{ccc}
2 & -3 & 4 \\
1 & 2 & -1 \\
m & -1 & 2
\end{array}\right|=0
$$

$2[4-1]-(-3)[2-(-m)]+4-1-2 m=0$
$\Rightarrow 8-5 \mathrm{~m}=0$
$\Rightarrow 5 \mathrm{~m}=8$
$\Rightarrow \mathrm{m}=\frac{8}{5}$
22. (B) $\operatorname{Cos} \mathrm{C}=\frac{a^{2}+b^{2}-c^{2}}{2 a b}$

$$
\begin{aligned}
& =\frac{8^{2}+10^{2}-12^{2}}{2 \times 8 \times 10} \\
& =\frac{64+100-144}{160}
\end{aligned}
$$

$$
\begin{equation*}
=\frac{1}{8} . \tag{1}
\end{equation*}
$$

Now, $\cos \mathrm{A}=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$

$$
=\frac{10^{2}+12^{2}-8^{2}}{2 \times 10 \times 12}
$$

$$
\begin{equation*}
=\frac{3}{4} \ldots \tag{2}
\end{equation*}
$$

$$
\cos 2 \mathrm{~A}=2 \cos ^{2} \mathrm{~A}-1
$$

$$
=2 \times\left(\frac{3}{4}\right)^{2}-1
$$

$$
=\frac{1}{8} \ldots \text { (3 }
$$

Consider $\cos ^{2}\left(\frac{A}{2}\right)=\frac{1+\cos A}{2}$
$\Rightarrow \operatorname{Cos}^{2}\left(\frac{\mathrm{~A}}{2}\right)=\frac{1+\frac{3}{4}}{2}=\frac{7}{8}$ from equation (2)
$\Rightarrow \operatorname{Cos} \frac{A}{2}=\frac{\sqrt{7}}{2 \sqrt{2}}$
$\therefore \mathrm{C} \neq \frac{\mathrm{A}}{2}$.
We know that, $\cos 3 \mathrm{~A}=4 \cos ^{3} \mathrm{~A}-3 \cos \mathrm{~A}$
$=\frac{27-36}{16}$

$$
\begin{equation*}
=\frac{-9}{16} \ldots \tag{5}
\end{equation*}
$$

$\cos ^{2}\left(\frac{3 A}{2}\right)=\frac{1+\operatorname{Cos} 3 A}{2}$
$\Rightarrow \cos ^{2}\left(\frac{3 A}{2}\right)=\frac{1+\left(\frac{-9}{16}\right)}{2}[$ from equation(5)]
$\therefore C \neq \frac{3 A}{2} \cdots(6)$
From equations (1), (2), (3), (4), (5), (6), we have,
$\mathrm{C}=2 \mathrm{~A}$
23. (C) $\operatorname{Sin}_{\theta}=\mathrm{x}^{+} \frac{a}{x}, \mathrm{x} \in \mathrm{R}-0$

We know that, $-1 \leq \sin \theta \leq 1$

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$\Rightarrow-1 \leq \mathrm{x}+\frac{a}{x} \leq 1$
$\Rightarrow-1 \leq \frac{x^{2}+a}{x} \leq 1$
$\Rightarrow-x \leq x^{2}+a \leq x$
Thus, equations are:
$\Rightarrow x^{2}+a+x \geq 0$ or $x^{2}+a-x \leq 0$
$\Rightarrow x^{2}+x+a \geq 0$ or $x^{2}-x+a \leq 0$
The above equations will have real roots, if the discriminant is greater than or equal to zero.
That is $(1)^{2}-4 \times a \times 1 \geq 0$ or $1^{2}-4 \times a \times 1 \geq 0$
$\Rightarrow 1-4 \mathrm{a} \geq 0$
$\Rightarrow a \leq \frac{1}{4}$
24. (C) $\tan ^{4} X-2 \sec ^{2} X+a^{2}=0$
$\tan ^{4} X-2\left(1+\tan ^{2} X\right)+a^{2}=0$
$\Rightarrow \tan ^{4} \mathrm{X}-2-2 \tan ^{2} \mathrm{X}+\mathrm{a}^{2}=0$
$\Rightarrow \tan ^{4} \mathrm{X}-2 \tan ^{2} \mathrm{X}+\mathrm{a}^{2}-2=0$
25. (C) ATQ,
$2 b=a+c$
$\Rightarrow \mathrm{a}=2 \mathrm{~b}-\mathrm{c}$
$\operatorname{Cos} \mathrm{A}=\frac{b^{2}+c^{2}-2 b-c^{2}}{2 b c}$

$$
=\frac{-3 b^{2}+4 b c}{2 b c}
$$

$$
=\frac{4 c-3 b}{2 c}
$$

26. (A) Let $\mathrm{I}=\int_{1}^{2}\left[\mathrm{k}^{2}+(4-4 \mathrm{k}) \mathrm{x}+4 \mathrm{x}^{3}\right] \mathrm{dx}$

$$
\begin{aligned}
& \Rightarrow 12 \geq\left[k^{2} X+(4-4) k \frac{X^{2}}{2}+\frac{4}{4} X^{4}\right] \\
& \Rightarrow 12 \geq\left[k^{2}(2-1)+\frac{4-4 k}{2}\left(2^{2}-1^{2}\right)+2^{4}-1^{4}\right] \\
& \Rightarrow 12 \geq\left[k^{2}+\frac{3(4-4 k)}{2}+15\right] \\
& \Rightarrow k-3^{2} \leq 0 \\
& \Rightarrow k=3
\end{aligned}
$$

27. (D) Given that $p, q$ and $r$ are positive integers and is the cube root of unity.
Also $f(x)=\mathrm{x}^{3 \mathrm{p}}+\mathrm{X}^{3 \mathrm{q}+1}+\mathrm{X}^{3 \mathrm{r}+2}$
So, $f(\omega)=\omega^{3 \mathrm{p}}+\omega^{3 \mathrm{q}+1}+\omega^{3 \mathrm{r}+2}$
$\Rightarrow \mathrm{f}(\omega)=\omega^{3^{p}}+\omega^{3 q} \omega^{+} \omega^{3^{r}} \omega^{2}$
$\Rightarrow f(\omega)=1+\omega+\omega^{2}$
$\Rightarrow f(\omega)=0 \quad\left[\therefore 1+\omega+\omega^{2}=0\right]$
28. (C) We know that the equation
$\operatorname{acos} \theta+\mathrm{b} \sin \theta=\mathrm{C}$ is solvable for $|\mathrm{c}| \leq \sqrt{a^{2}+b^{2}}$

Comparing the general equation with the given equation, we have, $a=3$ and $b=4$

$$
\begin{aligned}
& -\sqrt{3^{2}+4^{2}} / 3 \cos \theta+4 \sin \theta / \sqrt{3^{2}+4^{2}} \\
& \Rightarrow-5 \leq 3 \cos \theta+4 \sin \theta \leq 5 \\
& \Rightarrow-5+5 / 3 \cos \theta+4 \sin \theta+5 / 5+5
\end{aligned}
$$

Thus, the maximum value of the fuction is 10 .
29. (A) $\cos ^{2} \theta\left(1+\cos ^{2} \theta\right)=\sin \theta(1+\sin \theta)$

$$
\begin{aligned}
& =\sin \theta+\sin ^{2} \theta \\
& =\sin \theta+1-\cos ^{2} \theta \\
& =\sin \theta+1-\sin \theta \\
& =1
\end{aligned}
$$

30. (D) Let (he equation of the required plane be $\mathrm{a}\left(\mathrm{x}-\mathrm{x}_{1}\right)+\mathrm{b}\left(\mathrm{y}-\mathrm{y}_{1}\right)+\mathrm{c}\left(\mathrm{z}-\mathrm{z}_{1}\right)=0$, where $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ is a point on the plane.
Givene point $(1,-1,-1)$ lies on the plane.
Thus, the equation of the required plane is $\mathrm{a}(x-1)+\mathrm{b}(y+1)+c(z+1)=0 \ldots(1)$
Also, given that the given plane is perpen dicular to
$x-2 y-8 z=0$
and
$2 x+5 y-z=0$
Thus, we have,
$a-2 b-8 c=0$
and
$2 a+5 b-c=0$
Cross multiplying, we have,

$$
\begin{aligned}
& \frac{a}{(2+40)}=\frac{b}{(-16+1)}=\frac{c}{(5+4)} \\
& \Rightarrow \frac{a}{14}=\frac{b}{-5}=\frac{c}{3}=R \\
& \Rightarrow \mathrm{a}=14 \mathrm{R}, \mathrm{~b}=-5 \mathrm{R} \text { and } \mathrm{c}=3 \mathrm{R}
\end{aligned}
$$

Substituting the values of $a, b$ and $c$ in equa tion (1), we have,
$14 \mathrm{R}(\mathrm{X}-1)-5 \mathrm{R}(\mathrm{Y}+1)-3 \mathrm{R}(\mathrm{Z}+1)=0$
$\Rightarrow 14 \mathrm{x}-14-5 \mathrm{y}-5+3 \mathrm{z}+3=0$
$\Rightarrow 14 \mathrm{x}-5 \mathrm{y}+3 \mathrm{z}-16$
31. (C) One man can vote in ${ }^{4} C_{1}$ ways $=4$ ways.
$\therefore 5$ men can vote in $=4 \times 4 \times 4 \times 4 \times 4$ ways

$$
=1024 \text { ways }
$$

32. (A)
A) $\int_{r>1}^{n} \frac{P) n, r^{*}}{r!}=\frac{P) n, 1^{*}}{1!}+\frac{P) n, 2^{*}}{2!}+\frac{P) n, 3^{*}}{3!}+$

$$
\ldots+\frac{P) n, n^{*}}{n!}
$$

$=\frac{n!}{\sqrt{n-1 *!1!}}+\frac{n!}{n-2 *!2!}+\frac{n!}{\sqrt{n-3 *!3!}}+\ldots+$
$\frac{n!}{n-n *!n!}$
$=1+{ }^{n} C_{1}+{ }^{n} C_{2}+{ }^{n} C_{3}+\ldots+{ }^{n} C_{n-1}+{ }^{n} C_{n}-1$
$=\left({ }^{n} C_{0}+{ }^{n} C_{1}+{ }^{n} C_{2}+{ }^{n} C_{3}+\ldots+{ }^{n} C_{n-1}+{ }^{n} C_{n}\right)-1$
$=(1+1)^{n}-1$
$=2^{n}-1$
33. (B) Let $\mathrm{M}(a, b)$ be the coordinates of the foot of perpendicular on the given line.
Equation joining two points is
$\frac{y-y_{1}}{y_{1}-y_{2}}=\frac{x-x_{1}}{x_{1}-x_{2}}$
$\therefore$ The equation of the line joining the point $\mathrm{P}(2,3)$ and $\mathrm{M}(\mathrm{a}, \mathrm{b})$ is
$\frac{y-3}{3-b}=\frac{x-2}{2-a}$
$\Rightarrow(y-3)(2-a)=(x-2)(3-\mathrm{b})$
$\Rightarrow(y-3)=\frac{) x-2^{*}}{) 2-a^{*}}(3-b)$
$\Rightarrow y=\frac{x) 3-b^{*}}{\sqrt{2-a^{*}}}-\frac{2) 3-b^{*}}{) 2-a^{*}}+3$
Thus, the slope of the line is $\frac{) 3-b *}{2-a^{*}}$
Slope of the given line is -1 Since the product of slopes of
two perpendicular lines is -1 , we have,
$\frac{) 3-b^{*}}{2-a^{*}}=1$
$\Rightarrow 3-b=2-a$
$\Rightarrow a-b=-1$
Since $\mathrm{M}(a, b)$ lies on the line $x+y-11=0$, we have, $a+b-11=0$
$\Rightarrow a+\mathrm{b}=11$
and
$\Rightarrow a-\mathrm{b}=-1$
From equations (i) and (ii) $a=5, b=6$
Thus, the foot of perpendicular is $\mathrm{M}(5,6)$.
34. (B) Degree of the differential equation is the power of the highest order derivative, when differential coefficients are made free from radicals and fractions, in the given equation. Consider the given differential equation
$\left.\left\lvert\,-\frac{d^{3} y}{d x^{3}}\right.\right)^{\frac{2}{3}}+4-3\left|-\frac{d^{2} y}{d x^{2}}\right|+5\left|-\frac{d y}{d x}\right|=0$
$\left.\Rightarrow \left\lvert\,-\frac{d^{3} y}{d x^{3}}\right.\right)^{\frac{2}{3}}=3\left|-\frac{d^{2} y}{d x^{2}}\right|-5\left(\left.-\frac{d y}{d x} \right\rvert\,=0\right.$
Cubing both the sides of the equation, we
have,
$\left|-\frac{d^{3} y}{d x^{3}}\right|^{2}=\left\{3\left|-\frac{d^{2} y}{d x^{2}}\right|-\left\lvert\, 5 \frac{d y}{d x}-4\right.\right.$
$\therefore$ Degree of the above differential equation
$=$ Power of the highest order $=2$
35. (B) We need to find area enclosed by the equation
$x^{2}+y^{2}=2$
$\Rightarrow x^{2}+y^{2}=(\sqrt{2})^{2}$
$\Rightarrow y^{2}=2-x^{2}$
$\Rightarrow y= \pm \sqrt{2-x^{2}}$
Thus, Area $=4 \int_{0}^{\sqrt{2}} \sqrt{2-x^{2} d x}$
$=4\left|\frac{x}{4} \sqrt{2-x^{2}}, \frac{2}{2} \sin ^{-1} \frac{x}{\sqrt{2}}\right|_{0}^{\sqrt{2}}$
$=4\left|\frac{x}{42} \sqrt{2-x^{2}}, \frac{2}{2} \sin ^{-1} \frac{x}{\sqrt{2}}\right|_{0}^{\sqrt{2}}$
$=4\left|\frac{\theta}{4}-0\right|$
$=2 \pi$ sq. unit
36. (A) $I=\int \frac{d x}{\sin ^{2} x \cos ^{2} x}$

$$
=\int \frac{\lambda \sin ^{2} x, \cos ^{2} x * d x}{\sin ^{2} x \cos ^{2} x}
$$

$=\int \frac{d x}{\cos ^{2} x}+\int \frac{d x}{\sin ^{2} x}$
$=\int \frac{d x}{\cos ^{2} x}+\int \frac{d x}{\sin ^{2} x}$
$=\int \sec ^{2} x d x+\int \operatorname{cosec}^{2} \mathrm{xdx}$
$=\tan x+\cot x+C$
37. (A)
A) $\left|\begin{array}{lll}x & \beta & 1 \\ \chi & x & 1 \\ \chi & \eta & 1\end{array}\right|=0$

Expanding the determinant, we have,
$x(x-\gamma)-\alpha(\beta-\beta)+1(\beta \gamma-\beta x)=0$
$\Rightarrow x^{2}-x \gamma-0+\beta \gamma-\beta x=0$
$\Rightarrow x^{2}-x(\beta-\gamma)+\beta \gamma=0$
Thus the roots of the above equation are $\alpha$ and $\gamma$.
38. (C) $\bar{x}=\frac{\int_{i>1}^{n} x_{1} f_{1}}{\int_{i>1}^{n} f_{1}}=\frac{760}{40}=19$

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39. (C) If $\vec{a} \cdot \vec{b}=0$, then $\vec{a}$ and $\vec{b}$ are perpendicular vectors.
If $\vec{a} \times \vec{b}=0$, then $\vec{a}$ and $\vec{b}$ are parallel vectors.
Since both of the above conditions cannot be satisfied simultaneously, either one of the vectors $\vec{a}$ or $\vec{b}$ should be a null vector.
40. (D) $y=\log \sqrt{\tan x}$
$\Rightarrow y=\frac{1}{2} \log (\tan x)$
Differentiating the above function with respect to $x$, we have,
$\frac{d y}{d x}=\frac{1}{2} \times \frac{1}{\tan x} \times \sec ^{2} x$
$\left.\frac{d y}{d x}\right|_{x>\frac{\theta}{4}}=\frac{1}{2} \times \frac{1}{\tan \left|-\frac{\theta}{4}\right|} \times \sec ^{2}\left|-\frac{\theta}{4}\right|$
$\left.=\frac{1}{2} \times \frac{1}{1} \times\right) \sqrt{2} *^{2}$
= 1
41. (A) $\tan 15^{\circ} \tan 195^{\circ}=\tan 15^{\circ} \tan 180^{\circ}+15^{\circ}$

$$
\begin{aligned}
& =\tan 15^{\circ} \tan 15^{\circ}\left[\because \tan 180^{\circ}+\theta=\tan \theta\right] \\
& =\tan ^{2} 15^{\circ}
\end{aligned}
$$

$$
\tan ^{2} 15^{\circ}=\frac{1-\cos 2 \times 15^{\circ}}{1+\cos 2 \times 15^{\circ}}
$$

$$
=\frac{1-\cos 30^{\circ}}{1+\cos 30^{\circ}}
$$

$$
=\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}
$$

$$
=7-4 \sqrt{3}
$$

42. (C) Volume of the sphere is given as
$\mathrm{V}=\frac{4}{3} \pi r^{3}$
Here, $\mathrm{V}=f(r, t)$ and $\frac{\mathrm{dV}}{\mathrm{dt}}=\mathrm{K}$
Differentiating with respect to ' t ', we have,
$\frac{d V}{d t}=\frac{d V}{d r} \times \frac{d r}{d t}$

$$
=3 \times \frac{4}{3} \pi r^{2} \times \frac{\mathrm{dr}}{\mathrm{dt}}
$$

$K=4 \pi r^{2} \times \frac{\mathrm{dr}}{\mathrm{dt}}$
$\frac{\mathrm{dr}}{\mathrm{dt}}=\frac{K}{4 \pi r^{2}} \ldots$
Similarly, we have, $S=4 \pi r^{2}$
$\frac{\mathrm{ds}}{\mathrm{dt}}=\frac{\mathrm{ds}}{\mathrm{dr}} \times \frac{\mathrm{dr}}{\mathrm{dt}}$
$=\frac{2 K}{r}$
$\Rightarrow \frac{\mathrm{ds}}{\mathrm{dt}} \alpha \frac{1}{r}$
Thus the rate of change of surface area is inversely proportional to radius.
43. (B) $\frac{\sin \mathrm{x}}{1+\cos \mathrm{x}}+\frac{1+\cos \mathrm{x}}{\sin \mathrm{x}}=\frac{\sin ^{2} \mathrm{x}+1+\cos \mathrm{x}^{2}}{(1+\cos \mathrm{x}) \sin \mathrm{x}}$

$$
\begin{gathered}
=\frac{\sin ^{2} x+1+\cos ^{2} x+2 \cos x}{(1+\cos x) \sin x} \\
=\frac{2+2 \cos x}{(1+\cos x) \sin x} \\
=\frac{2(1+\cos x)}{(1+\cos x) \sin x} \\
=2 \operatorname{cosec} x
\end{gathered}
$$

44. (D) Let a and b be two observations.
1.Arithmetic mean, $\mathrm{AM}=\frac{a+b}{2}$

Multiplying by c to each and every observation, we have,
$\mathrm{AM}=\frac{a c+b c}{2}=c\left(\frac{a+b}{2}\right)$
2. Geomatric mean $\mathrm{GM}=\sqrt{a b}$

Multiplying by c to each and every observa tion, we have,
$\mathrm{GM}=\sqrt{a c b c}=c \sqrt{a b}$
3. Harmonic mean, $\mathrm{HM}=\frac{2 a b}{a+b}$

Multiplying by c to each and every observa tion, we have,
$\mathrm{HM}=\frac{2 a c b c}{a c+b c}=\frac{2 a c^{2} b}{c a+b}=c\left(\frac{2 a b}{a+b}\right)$
4. Median
(i)

Let A, B, C, D and E be five observations. Number of observations is $n=5$, odd, and hence

Median $=\frac{n+1}{2}=3$
Thus, median is 3rd term, c.
Now consider Ac, Bc, Cc, Dc and Ec.
Median $=\frac{n+1}{2}=3$
Thus, median is 3rd term, Cc.
(ii)

Let A, B, C ans D be four observations. Number of Observations is $n=4$, even, and hence

Median $=\frac{\left(\frac{n}{2}\right)^{\text {th }} \text { term }+\left(\frac{n}{2}+1\right)^{\text {th }} \text { term }}{2}$

$$
=\frac{2^{\text {nd }} \text { term }+3^{\text {rd }} \text { term }}{2}
$$

Thus, median is $=\left(\frac{B+C}{2}\right)$
Now consider Ac, Bc, Cc and Dc, Number of observations is $n=4$, even, and hence

Median $=\frac{\left|-\frac{n}{2}\right|^{\text {th }} \operatorname{term}\left|\frac{n}{k},\right|^{\text {th }} \text { term }}{2}$
$=\frac{2^{\text {nd }} \text { term }+3^{\text {rd }} \text { term }}{2}$
Thus, median is $=c\left|-\frac{B, C}{2}\right|$
45. (C) For maximum of minimum, we have, $f^{\prime} x=0$

$$
\begin{aligned}
& \therefore x \times \frac{1}{x}+\log x=0 \\
& \Rightarrow \log x=-1 \\
& \Rightarrow e^{\log x}=e^{-1} \\
& \Rightarrow x=e^{-1}
\end{aligned}
$$

46. (A) The values of the variate are

$$
2,3,4,2,5,4,3,2,1
$$

In the above data values, the value ' 2 ' has been repeated thrice and hence the mode of the data is 2 .
47. (D) $z=\frac{1+2 i}{2-i}-\frac{2-i}{1+2 i}$

$$
=\frac{1-4+4 i-4+1+4 i}{2+4 i-i+2}
$$

$$
\begin{aligned}
& =\frac{8 \mathrm{i}-6}{4+3 \mathrm{i}} \\
& =\frac{8 \mathrm{i}-6}{4+3 \mathrm{i}} \times \frac{4-3 \mathrm{i}}{4-3 \mathrm{i}} \\
& =2 \mathrm{i} \\
& z \bar{z}=2 \mathrm{i}-2 \mathrm{i}=4 \\
& z^{2}=|z|^{2}=z z=4 \\
& \text { Thus, } z^{2}+z \bar{z}=4+4=8
\end{aligned}
$$

48. (D) Argument of $1-\sin \theta+i \cos \theta$
$=\tan ^{-1}\left(\frac{\cos \theta}{1-\sin \theta}\right)$
$=\tan ^{-1}\left(\frac{\cos ^{2} \frac{\theta}{2}-\sin ^{2} \frac{\theta}{2}}{\cos ^{2} \frac{\theta}{2}-\sin ^{2} \frac{\theta}{2}-2 \sin \frac{\theta}{2} \sin \frac{\theta}{2}}\right)$
$=\tan ^{-1} \frac{\left(\cos ^{2} \frac{\theta}{2}+\sin \frac{\theta}{2}\right)}{\left(\cos ^{2} \frac{\theta}{2}-\sin \frac{\theta}{2}\right)}$
$=\tan ^{-1} \frac{\left(1+\tan \frac{\theta}{2}\right)}{\left(1-\tan \frac{\theta}{2}\right)}$
$=\tan ^{-1} \tan \left(\frac{\pi}{4}+\frac{\theta}{2}\right)$
$=\frac{\pi}{4}+\frac{\theta}{2}$
49. (D) When the curve meets X axis, we have,
$y=0$
$\Rightarrow \sqrt{X}=\sqrt{a}$
$\Rightarrow X=a$
When the curve meets y axis, we have,
$\mathrm{x}=0$
$\Rightarrow \sqrt{Y}=\sqrt{a}$
$\Rightarrow \mathrm{Y}=\mathrm{a}$
Rewriting the equation (1), we have,
$\sqrt{Y}=\sqrt{a}-\sqrt{X}$
$\Rightarrow \sqrt{Y}^{2}=\sqrt{a}-\sqrt{X}^{2}$
$\Rightarrow \mathrm{y}=\mathrm{a}+\mathrm{X}-2 \sqrt{\mathrm{a}} \sqrt{X}$
Thus,
Area $=\int_{0}^{a} \mathrm{ydx}$

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$$
\begin{aligned}
& =\int_{0}^{a}[a+X-2 \sqrt{a} \sqrt{X}] \mathrm{dx} \\
& =\left[a X+\frac{X^{2}}{2}-\frac{4}{3} \sqrt{a X^{\frac{3}{2}}}\right]_{0}^{a} \\
& =\left[a^{2}+\frac{a^{2}}{2}-\frac{4}{3} \sqrt{a a} \frac{3}{2}\right] \\
& =\left[\frac{9 a^{2}-8 a^{2}}{6}\right] \\
& =\left[\frac{a^{2}}{6}\right]
\end{aligned}
$$

50. (A) Let A and B be two square matrices of same order. Given statement $\mathrm{AB}=0$ $\Rightarrow|\mathrm{A}|=0$ or $|\mathrm{B}|=0$
We Know that:
If the product of two non-null square matrices is a null matrix, then both of them must be singular matrices.
$\therefore|A|=0$ or $|B|=0$
Hence statement I is true.
We know that the product of two matrices can be null matrix, while neither of them is the null matrix.
Hence statement II is false.
Thus option (a) is correct.
51. (B) $y=\mathrm{e}^{x} \sin x$

Differentiating the above function
with respect to $x$, we have,
$\frac{d y}{d x}=e^{x} \cos x+\sin x e^{x}$
$=e^{x}(\cos x+\sin x)$
The slope of the function $y=e^{x} \sin x \sin e^{x}(\cos x+\sin x)$ Slope, $m=e^{x}(\cos x+\sin x)$
$\frac{d m}{d x}=e^{x}(-\sin x+\cos x)+(\cos x+\sin x) e^{x}$
$\Rightarrow \frac{d m}{d x}=2 e^{x} \cos x$
For the slope, the attain its maximum, we have,
$\frac{d m}{d x}$ should be zero.
$\Rightarrow 2 e^{x} \cos x=0$
$\Rightarrow e^{x} \cos x=0$
$\Rightarrow e^{x}=e^{-\infty}=0$ or $\cos x=\cos \frac{\theta}{2}$
$\Rightarrow x=-\infty$ or $x=\frac{\theta}{2}$ and $\frac{3 \theta}{2}$ in $(0,2 \pi)$
Differentiating once again, we have,
$\frac{d^{2} m}{d x^{2}}=2 \mathrm{e}^{x} \cos ^{x}-2 \mathrm{e}^{x} \sin x$
At $x=\frac{\theta}{2}$,
$\frac{d^{2} m}{d x^{2}}=2 e^{\frac{3 \theta}{2}} \cos \frac{3 \theta}{2}-2 e^{\frac{3 \theta}{2}} \sin \frac{3 \theta}{2}$
$=2 \mathrm{e}^{\frac{3 \theta}{2}}>0$
So, the function $y=\mathrm{e}^{x} \sin x$ has maximum
slope at $x=\frac{3 \theta}{2}$.
52. (D) $\tan \theta=\sqrt{m}$
$\Rightarrow \tan ^{2} \theta=\mathrm{m}$
Now consider $\sec 2 \theta$
$\sec 2 \theta=\frac{1}{\cos 2 \rho}$
$=\frac{1}{\frac{1-\tan ^{2} \rho}{1, \tan ^{2} \rho}}$
$=\frac{1, \tan ^{2} \rho}{1-\tan ^{2} \rho}$
$=\frac{1, m}{1-m}$
$=$ a rational number
53. (C) $\cos \mathrm{C}=\frac{a^{2}, b^{2}-c^{2}}{2 a b}$
$\Rightarrow \operatorname{cosC}=\frac{6^{2}, 10^{2}-14^{2}}{2 \propto 6 \cdot 10}$
$\Rightarrow \cos C=\frac{36,100-196}{120}$
$\Rightarrow \cos \mathrm{C}=\frac{136-196}{120}$
$\Rightarrow \cos \mathrm{C}=\frac{-60}{120}$
$\Rightarrow \cos \mathrm{C}=\frac{-1}{2}$
$\Rightarrow \cos \mathrm{C}=\frac{-1}{2}=\cos \frac{2 \theta}{3}$
$\Rightarrow \mathrm{C}=\frac{2 \theta}{3}$

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$\Rightarrow \mathrm{C}=120^{\circ}$
54. (B) $\tan ^{-1} 2+\tan ^{-1} 3=\tan ^{-1}\left|\frac{2,3}{1-2 \propto 3}\right|$

$$
\begin{aligned}
& =\tan ^{-1}\left|-\frac{5}{1-6}\right| \\
& =\tan ^{-1}\left|-\frac{5}{-5}\right| \\
& =\tan -1-1 \\
& =135^{\circ}
\end{aligned}
$$

$\Rightarrow \mathrm{A}+\mathrm{B}=135^{\circ}$
We have, $\mathrm{A}+\mathrm{B}+\mathrm{C}=180^{\circ}$
$\Rightarrow \mathrm{C}=180^{\circ}-\mathrm{A}+\mathrm{B}$
$\Rightarrow \mathrm{C}=180^{\circ}-135^{\circ}$
$\Rightarrow C=45^{\circ}=\frac{\theta}{4}$
55. (C) $(x-a)(x-b)=c$, where $c \neq 0$
$\Rightarrow x^{2}-a+b x+a b-c=0$
Given that $\alpha$ and $\beta$ are the root of the equation.
$\alpha+\beta=a+b$
$\alpha \beta=a b-c$
Now consider the equation,
$(x-\alpha)(x-\beta)+c=0$
$\Rightarrow x^{2}-\alpha+\beta x+\alpha \beta+c=0$
$\Rightarrow x^{2}-a+b x+a b-c+c=0$
Thus the root of the above equation are $a$ and $b$.
56. (D) Let $\alpha$ and $\beta$ are the roots of the equation $x^{2}$ $-p x+q=0$
$\Rightarrow \alpha+\beta=p$ and $\alpha \beta=q$
And let $\alpha$ be the common root of $x^{2}-p x+q=0$
$\Rightarrow 2 \alpha=a$ and $\alpha^{2}=b$
$\Rightarrow\left|-\frac{a}{2}\right|^{2}=b$
$\Rightarrow a^{2}=4 b \ldots$ (2)
Consider equation (1).
$\left.\Rightarrow \frac{a}{2}+\frac{\frac{q}{a}}{2}=p\left|\because \frac{a}{2}>\beta \sin \right\rangle \quad \frac{q}{\beta} \right\rvert\,$
$\Rightarrow a^{2}+4 q=2 a p$
$\Rightarrow 4 b+4 q=2 a p$
$\Rightarrow 2(b+q)=a p$
57. (C) Given that $n!, 3 \times(n!)$ and $(n+1)$ ! are in GP.
If $a, b$ and $c$ are in GP, then, $b^{2}=a c$
$\therefore[3 \times n!]^{2}=n!\times n \times 1!$
$\Rightarrow 9 \times \mathrm{n}!\times \mathrm{n}!\times \mathrm{n}!\times \mathrm{n}+1$
$\Rightarrow n+9-1$
$\Rightarrow n=8$
58. (A)
59. (D) $x^{2}=12 y$

Comparing the above equation with the standard equation, $x^{2}=4 a y$, we have
$4 a=12$
$\Rightarrow a=3$
Substituting $y=3$, in the equation, $x^{2}=12 y$,
$x^{2}=12 \times 3=36$
$\Rightarrow x= \pm 6$
Thus, the latus rectum passes through the points $(-6,0),(0,3)$ and $(6,0)$


We need to find the area of the triangle OAC We have, Area of the triangle $=\frac{1}{2} \times$ base $\times$ height

Thus, area of $\mathrm{OBC}=\frac{1}{2} \times \mathrm{AC} \times \mathrm{OB}$

$$
\begin{aligned}
& =\frac{1}{2} \times 12 \times 3 \\
& =18 \text { square units }
\end{aligned}
$$

60. (B) We need to find the angle between the planes $2 x-y+z=4$ and
$x+y+2 z=6$.
Let $\theta$ be the angle between the given planes. Thus,
$\cos \theta=\left|\frac{a_{1} a_{2}, b_{1} b_{2}, c_{1} c_{2}}{\sqrt{a_{1}^{2}, b_{1}^{2}, c_{1}^{2}} \sqrt{a_{2}^{2}, b_{2}^{2}, c_{2}^{2}}}\right|$
Here $a_{1}=2, b_{1}=-1, c_{1}=1$ and $a_{2}=1, b_{2}=1$ and $c_{2}=2$
$\cos \theta=\left|\frac{2) 1 *,)-1 * 1,) 1 * 2}{\sqrt{\left.2^{2},\right)-1 *^{2}, 1^{2}} \sqrt{1^{2}, 1^{2}, 2^{2}}}\right|$
$\Rightarrow \cos \theta=\left|\frac{3}{6}\right|$
$\Rightarrow \cos \theta=\cos \frac{\theta}{3}$
$\Rightarrow \theta=\frac{\theta}{3}$
61. (A) $x+2 y-9=0 \ldots$ (1)

$$
\Rightarrow 2 y=9-x
$$

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$\Rightarrow y=\frac{1}{2}(-x+9)$
Slope of the first line, $m=\frac{-1}{2}$
Consider the second line:
$\mathrm{kx}+4 \mathrm{y}+5=0$
$\Rightarrow 4 y=-K x-5$
$\Rightarrow \mathrm{y}=\frac{1}{4}(-\mathrm{Kx}-5)$
Slope of the Second line, $m=\frac{-k}{4}$
Given that the above lines are parallel and hence the
Slopes are equal.
$\frac{-1}{2}=\frac{-\mathrm{k}}{4}$
$\Rightarrow \mathrm{k}=2$
62. (A) $x^{2}+y^{2}-2 X-3=0$
$\Rightarrow y^{2}=-x^{2}+2 \mathrm{x}+3$
$\Rightarrow 2 y \frac{d y}{d x}=-2 x+2$
$\Rightarrow \frac{d y}{d x}=\frac{-X+1}{Y}$
Given that tangents to the curve are parallel to x -axis.
Thus slope $=0$
$\Rightarrow \frac{d y}{d x}=0$
$\Rightarrow-\mathrm{x}+1=0$
$\Rightarrow \mathrm{x}=1$
Substituting the value $\mathrm{x}=1$ in equation (1), we have,
$1^{2}+\mathrm{y}^{2}-2 \times 1-3=0$
$\Rightarrow \mathrm{y}^{2}-4=0$
$\Rightarrow y^{2}=4$
$\Rightarrow \mathrm{y}= \pm 2$
Thus, the points on the curve, where the tangents to the curve are parallet to x -axis are $(1,2)$ and $(1,-2)$
63. (B) Since, $3 \sin A-4 \sin ^{3} A=1$
$\Rightarrow 4 \sin ^{3} \mathrm{~A}-3 \sin \mathrm{~A}+1=0$
The above equation is a cubic polynomial in $\sin A$.
Therefore, there are 3 solutions for the above equation.
Substituting $\sin A=-1$, we have,
$4(1)^{3}-(-1)+1=0$
$\Rightarrow \sin A=-1$ is a solution of the equation.
Dividing $4 \sin ^{3} \mathrm{~A}-3 \sin \mathrm{~A}+1$ by $\sin \mathrm{A}+1$, we
have,
$\frac{4 \sin 3 A-3 \sin A+1}{\sin A+1}=4 \sin ^{2} A-4 \sin A+1$
Thus, $4 \sin ^{2} \mathrm{~A}-4 \sin \mathrm{~A}+1$ is a quadratic equation.
And $4 \sin ^{2} \mathrm{~A}-4 \sin \mathrm{~A}+1=(2 \sin \mathrm{~A}-1)^{2}$
Thus $\sin \mathrm{A}=\frac{1}{2}, \frac{1}{2}$
Therefore, $\sin A$ can assume two distinct values, -1 and $\frac{1}{2}$.
64. (A)


We know that the line joining the centre and the tangent is perpendicular to the tangent. Since x -axis is the tangent to the circle and OA is perpendicular to $x$-axis.
The length of the segment joining the centre and any point of on the circle is called the radius of the circle.
Thus, OA is the radius of the circle and (OA) $=3$ units.
Radius $=3$ units.
65. (B) $2^{4 n}-15 n-1$.

We have, $2^{4}=16$.
$\Rightarrow 2^{4 n}=16^{n}$
$\Rightarrow 2^{4 n}=(1+15)^{n}$
$\Rightarrow 2^{4 n}=1+{ }^{n} \mathrm{C}_{1} 15+{ }^{n} \mathrm{C}_{2} 15^{2}+\ldots{ }^{n} \mathrm{C}_{r} 15^{r}+\ldots+{ }^{n} \mathrm{C}_{n} 15^{n}$
$\Rightarrow 2^{4 n}-15 n-1=15^{2}\left({ }^{n} \mathrm{C}_{2}+{ }^{n} \mathrm{C}_{3} 15+\ldots+15^{n-2}\right) \ldots 1$
Let us consider ${ }^{n} \mathrm{C}_{2}+{ }^{n} \mathrm{C}_{3} 15+\ldots+15^{n-2}=k$
Thus equation (1) becomes,
$2^{4 n}-15 n-1=15^{2} k$
$\Rightarrow 2^{4 n}-15 n-1=225 k$
$\Rightarrow 2^{4 n}-15 n-1$ is divisible by 225 .
66. (D) $x^{2}-4 x-\log _{3} \mathrm{~N}=0$

Given that roots of the above equation are real.
Thus the discriminant, $b^{2}-4 a c \geq 0$
$\therefore-4^{2}-4 \times 1 \times-\log _{3} N \geq 0$
$\Rightarrow 16+4 \log _{3} N \geq 0$

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$\Rightarrow 16+4 \frac{\log _{10} \mathrm{~N}}{\log _{10} 3} \geq-4$
$\Rightarrow \log _{10} \mathrm{~N}>-4 \log _{10} 3$
$\Rightarrow \log _{10} \mathrm{~N} \geq \log _{10} 3^{-4}$
$\Rightarrow \log _{10} \mathrm{~N}>\log _{10}\left|-\frac{1}{81}\right|$
Thus the minimum value of N is $\frac{1}{81}$.
67. (B) $x^{2}+y^{2}+z^{2}+2 u x+2 v y+2 w z+d=0 \ldots$ (1) Since the sphere passing through the origin, the constant term, $d=0$
$\therefore$ equation of the sphere is $x^{2}+y^{2}+z^{2}+2 u x$
$+2 v y+2 w z=0 \ldots$ (2)
The sphere passing through origin and the point $(-1,0,0)(0,-2,0)$ and $(0,0,-3)$
$(-1)^{2}+0^{2}+0^{2}+2 u(-1)(-1)-2 v \times 0+2 w \times$ $0=0$
$0^{2}+(-2)^{2}+0^{2}+2 \mathrm{u} \times 0+2 \mathrm{v} \times(-2)-2 \mathrm{w} \times 0=0$
$0^{2}+0^{2}+(-3)^{2}+2 \mathrm{u} \times 0+2 \mathrm{v} \times 0+2 \mathrm{w} \times(-3)=0$
$\Rightarrow 1-2 u=0$
$\Rightarrow 4-4 v=0$
$9-6 w=0$
$\Rightarrow u=\frac{1}{2}, v=1, w=\frac{3}{2}$
Substituting, the above values in equation (2), the equation of the sphere is
$\Rightarrow x^{2}+y^{2}+z^{2}+x+2 y+3 z=0$
$\Rightarrow x^{2}+y^{2}+z^{2}+f(x, y, z)=0$
Therefore, $f(x, y, z)=x+2 y+3 z$
68. (A) We know that area of the triangle
$=\frac{1}{2}|\overrightarrow{\mathrm{AB}} \propto \overrightarrow{\mathrm{AC}}|$
$\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OA}}=(2-1) \hat{i}+(5-2) \hat{j}+-(1-3) \hat{k}$
$=\hat{i}+3 \hat{j}-4 \hat{k}$
Similarly,
$\overrightarrow{\mathrm{AC}}=\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OA}}=(-1-1) \hat{i}+(1-2) \hat{j}+(-1-3) \hat{k}$
$=-2 \hat{i}-\hat{j}-\hat{k}$
Thus, Area $=\frac{1}{2}\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -4 \\ -2 & -1 & -1\end{array}\right|$
$=\frac{1}{2}|-7 \hat{i}, 9 \hat{j}, 5 \hat{k}|$
$=\frac{1}{2} \sqrt{-7^{2}, 9^{2}, 5^{2}}$
$=\frac{\sqrt{155}}{2}$ square units.
69. (D) $\left[\begin{array}{lll}1 & 3 & 2\end{array}\right]$

$\Rightarrow(1 \times 1+3 \times 3+2)(1 \times 3+3 \times 0+2 \times 0)$
$(1 \times 0+3 \times 2+2 \times 1)\left[\left.\begin{array}{r}0 \\ -3 \\ 4 \times y\end{array} \right\rvert\,=0\right.$
$\Rightarrow 14 \times 0+3 \times 3+8 \times x=0$
$\Rightarrow x=\frac{-9}{8}$
70. (C) $(\mathrm{ab}-\mathrm{c}) x^{2}+(b c-a) x+c a-b=0$

Given that one of the roots of the above equation is 1 .
Let $\alpha$ be the other root.
Thus, $1+\alpha=\frac{-) b c-a^{*}}{a b-c}$
$\Rightarrow \alpha=\frac{\left.-) b c-a^{*}-\right) a b-c^{*}}{a b-c}$
$\Rightarrow \alpha=\frac{c a-b}{a b-c}$
71. (D) $\operatorname{Cos} x \sin y d y=\sin a \cos y d x$ $\sin x \cos y d x+\cos x \sin y d y=0$
$\Rightarrow \cos x \sin y d x+\cos x \sin y d y=0$
$\Rightarrow \frac{\sin y}{\cos y} d y=-\frac{\sin x}{\cos x} d x$
$\Rightarrow \tan y d y=-\tan x d x$
$\Rightarrow \tan y d y+\tan x d x=0$
$\Rightarrow \int \tan y d y+\int \tan x d x=0$
$\Rightarrow \log \cos \mathrm{y}+\log \cos x=\log \mathrm{C}$
$\Rightarrow \cos x \cos y=C$
When $x=0, y=\frac{\theta}{3}$
Thus, $\cos 0 \cos \frac{\theta}{3}=C$
$\Rightarrow \mathrm{C}=1 \times \frac{1}{2}=\frac{1}{2}$
Therefore, the equation of the curve is $\cos x$ $\cos x \cos y=1 \times \frac{1}{2}=\frac{1}{2}$
72. (B) Probability of selecting husband
$P(H)=\frac{1}{5}$
$\Rightarrow P(\overline{\mathrm{H}})=1-\frac{1}{5}=\frac{4}{5}$

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Probability of selecting wife $P(W)=\frac{1}{3}$
$\Rightarrow \mathrm{P}(\mathrm{W})=1-\frac{1}{3}=\frac{2}{3}$
$\therefore$ Probability of one of them is selected
$=\left|-\frac{1}{5}\right|\left|-\frac{2}{3}\right|+\left|-\frac{4}{5}\right|\left|-\frac{1}{3}\right|$
$=\frac{2}{15}+\frac{4}{15}$
$=\frac{2}{5}$
73. (C) Given $\alpha$ and $\beta$ are the complex cube roots of unity.
$1+\alpha+\beta=0$
$\Rightarrow \alpha+\beta=-1$
$\Rightarrow \alpha \beta=1$
Consider the expression

$$
\begin{align*}
& =(1+\alpha)(1+\beta)\left(1+\alpha^{2}\right)\left(1+\beta^{2}\right)  \tag{2}\\
& =(1+\alpha+\beta+\alpha \beta)\left(1+\alpha^{2}+\beta^{2}+\alpha^{2} \beta^{2}\right) \\
& =(1+(\alpha+\beta)+\alpha \beta)\left(1+(\alpha+\beta)^{2}-2 \alpha \beta+(\alpha \beta)^{2}\right) \\
& =(1-1+1)\left(1+(-1)^{2}-2(1)+(1)^{2}\right) \\
& =\left(1+1-2(1)+(1)^{2}\right) \\
& =1
\end{align*}
$$

74. (A)


In triangle ABC ,
$\tan 30^{\circ}=\frac{A B}{B C}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{15}{\mathrm{BC}}$
$\Rightarrow \mathrm{BC}=15 \sqrt{3} \mathrm{~m}$
The distance of the point from the foot of the tower is $15 \sqrt{3} \mathrm{~m}$.
75. (C) We know that
$\cot (\mathrm{A}-\mathrm{B})=\frac{\cot A \cot B, 1}{\cot B-\cot A}$
$\tan (\mathrm{A}-\mathrm{B})=\frac{\cot A-\tan B}{1, \tan A \tan B}$
Substituting the value, $x=\tan A-\tan B$, we have
$\tan (\mathrm{A}-\mathrm{B})=\frac{x}{1, \tan A \tan B}$

Also,

$$
\cot (\mathrm{A}-\mathrm{B})=\frac{1}{\tan ) \mathrm{A}-\mathrm{B}^{*}}
$$

$$
=\frac{1}{\frac{\tan A-\tan B}{1, \tan A \tan B}}
$$

$$
\begin{equation*}
=\frac{1, \tan A \tan B}{x} \tag{4}
\end{equation*}
$$

From equation (1), we have,

$$
\begin{align*}
\cot (\mathrm{A}-\mathrm{B}) & =\frac{1}{\frac{\tan A \tan B}{\cot B-\cot A}, 1} \\
& =\frac{1, \tan A \tan B}{\tan A \tan B \propto y} \tag{5}
\end{align*}
$$

Equating equations (4) and (5), we have
$\frac{1, \tan A \tan B}{\tan A \tan B \propto y}=\frac{1, \tan A \tan B}{x}$
$\Rightarrow \frac{1}{\tan A \tan B \propto y}=\frac{1}{x}$
$\Rightarrow \tan \mathrm{A} \tan \mathrm{B}=\frac{x}{y}$
Substituting the value $\tan \mathrm{A} \tan \mathrm{B}=\frac{x}{y}$ in equation (4), we have,
$\cot (\mathrm{A}-\mathrm{B})=\frac{1, \frac{x}{y}}{x}$

$$
=\frac{1}{x}+\frac{1}{y}
$$

76.(A) Let the radius of the given circle be ' $r$ '.

Now consider a inner circle of radius $\frac{r}{2}$


Area of the shaded region $=\pi r^{2}-\pi\left(\left.\frac{r}{2}\right|^{2}\right.$
$=\frac{3 \theta r^{2}}{4}$

Required Probability $=\frac{\frac{3 \theta r^{2}}{4}}{\theta r^{2}}=\frac{3}{4}$
77. (B) Let $n$ be the number of sides of a polygon. In an ' $n$ ' sided polygon, there are $n$ starting points.
A diagonal cannot connect to its own starting point and other two neighbouring points.
$\therefore$ Neglecting three points, each diagonal can connect to $(n-3)$ end points.
$\therefore$ There are total $n(1-3)$ ways to connect diagonally.
Neglecting the repetition of ways, the
total number of diagonal in an $n$-sided pentagon is $\frac{n) 1-3^{*}}{2}$.
78. (C) Mean of marks, $\bar{x}=\frac{\int_{i>1}^{3} f_{1} x_{1}}{\int_{i>1}^{3} f_{1}}=\frac{6800}{100}=68$
79. (D)


By the definition of parabola, we have,
$\frac{\mathrm{SP}}{\mathrm{PM}}=e$ and for parabola $e=1$
Thus, the focal distance of the point is $a+x_{1}$.
80. (D) $-\hat{i}-2 x \hat{j}-3 y k$ and $\hat{i}-3 x j-2 y k$

Given that they are orthogonal to each other.
Thus, $\vec{a} \cdot \vec{b}=0$
$(-\hat{i}-2 x \hat{j}-3 y k) \times(\hat{i}-3 x \hat{j}-2 y k)=0$
$\Rightarrow-1+6 x^{2}+6 y^{2}=0$
$\Rightarrow 6 x^{2}+6 y^{2}=1$
$\Rightarrow x^{2}+y^{2}=\frac{1}{6}$
which is the equation of the circle.
81. (A) $2(y+2)^{3}-5(y+2)=12$
$2(y+2)^{2}-5(y+2)-12=0$
Let $x=y+2$

Thus, above equation becomes,
$2 x^{2}-5 x-12=0$
$\Rightarrow 2 x+3=0$ or $x-4=0$
$\Rightarrow x=-\frac{3}{2}$ or $x=4$
$\therefore y=-\frac{7}{2}$ or $y=2$
82. (A) Let $x=\log _{3} 81$
$\Rightarrow x=4 \log _{3} 3$
$\Rightarrow x=4$
$\begin{aligned} \therefore \log _{2}\left(\log _{2} 81\right) & =\log _{2} 4 \\ & =2\end{aligned}$
83. (D) $3 x^{2}-5 x+q=0$

Given that the roots of the equation are equal.
Let $\alpha$ be the root.
Thus, sum of the roots
$\alpha+\alpha=\frac{--5}{3}$

$$
\alpha=\frac{5}{6}
$$

product of the roots, $\alpha^{2}=\frac{q}{3}$
$\Rightarrow\left|-\frac{5}{6}\right|^{2}=\frac{9}{3}$
$\Rightarrow q=\frac{25}{12}$
84. (D) We know that $\varphi$ is a set which contains nothing and it is called as the null set. Hence (d) is the correct option.
85. (D) Equation of the line parallel to $x$-axis is $y=c$, where $c$ is the distance of the line above or below the x -axis.
Since the line is 5 units below the $x$-axis, its equation is $y=-5$.
86. (A) The total population for the year $1997=$ 810
87. (B) The female urban population in the year $1995=410$
88. (C) The urban population in the year $1997=$ $310+180=490$
89. (D) The total population in the year $1998=$ $680+370=1050$
90. (A) Number of females in the year $1995=720$ Number of males in the year $1995=630$ Thus, the difference between number of females and the number of males in the year $1995=720-630=90$
91. (C) $\left|\begin{array}{lll}a-b & b, c & a \\ b-c & c, a & b \\ c-a & a, b & c\end{array}\right|$

Applying $C_{3} \rightarrow C_{3}+C_{2}$
$=\left|\begin{array}{llll}a-b & b, c & a, b, c \\ b-c & c, a & a, b, c \\ c-a & a, b & a, b, c\end{array}\right|$
$=(a+b+c)\left|\begin{array}{llll}a-b & b, c & 1 \\ b-c & c, & a & 1 \\ c-a & a, & b & 1\end{array}\right|$
Applying $R_{2} \rightarrow R_{2}-R_{1}, R_{3} \rightarrow R_{3}-R_{1}$

$$
\begin{aligned}
& =(a+b+c)\left|\begin{array}{ccc}
a-b & b, c & 1 \\
2) b-c^{*} & a-b & 0 \\
b-c & a-c & 0
\end{array}\right| \\
& =a+b+c[(a-b) 0-0-(b+c) 0-0+1 \\
& [b-(c-a)(a-c)-b+c-2 a a-b]] \\
& =a+b+c\left|\begin{array}{l}
2 a b-2 b c-a c, c^{2}-a^{2}, a c \\
-a b, a c-2 a^{2}-b^{2}-b c, 2 a b
\end{array}\right| \\
& =a+b+c\left|a^{2}+b^{2}+c^{2}-a b-b c-c a\right| \\
& =a^{3}+b^{3}+c^{3}-3 a b c
\end{aligned}
$$

92. (B) Given that, $a, b, c, d, e$ and $f$ are in arithmetic progression.
Thus, $b-a=c-b=d-c=e-d=f-e=k$
$\therefore e-d=d-c$
$\Rightarrow e=d+d-c$
$\Rightarrow e=2 d-\mathrm{c}$
$\Rightarrow e-c=2 d-c-c$
$\Rightarrow e-c=2 d-2 c$
$\Rightarrow e-c=2(d-c)$
93. (D) Let $\alpha$ and $\beta$ are the roots of the equation. Since $b^{2}-4 a c<0$, the roots of the equation are imaginary.
Thus $\alpha$ and $\beta$ are the cube roots of unity and hence,
Now consider $\alpha^{19} \alpha^{18} \times \alpha=\left(\alpha^{3}\right) 6 \times \alpha=1^{6} \times \alpha=\alpha$. And $\beta^{7}=\beta^{6} \times \beta=\left(\alpha^{3}\right)^{2} \times \beta=1^{2} \times \beta=\beta$
Thus the equation whose roots are $\alpha^{19}=\alpha$ and $\beta^{7}=\beta$ is $x^{2}+x+1=0$
94. (A)


In $\triangle B C P$
$\tan \beta=\frac{B C}{C P}$
$\mathrm{CP}=\frac{H}{\tan \chi}$
$=\frac{\mathrm{AC}}{\mathrm{CP}}=\frac{\mathrm{H}+\mathrm{h}}{\mathrm{CP}}=\frac{\mathrm{H}+\mathrm{h}}{\frac{\mathrm{H}}{\tan \chi}}$
$=\frac{\mathrm{H}+\mathrm{h} \tan \chi}{\mathrm{H}}$
$\Rightarrow \mathrm{H} \tan \alpha=\mathrm{H}+\mathrm{h} \tan \beta=h \tan \beta$
$\Rightarrow \mathrm{H}(\tan \alpha-\tan \beta)=h \tan \beta$
$\Rightarrow \mathrm{H}=\frac{h \tan \chi}{\tan \beta-\tan \chi *}$
95. (C) Consider the given determinant
$\left|\begin{array}{ccc}p & -q & 0 \\ 0 & p & q \\ q & 0 & p\end{array}\right|=0$
Expanding the determinant, we have,
$p\left[p^{2}\right]-(-q)\left[-q^{2}\right]=0$
$\Rightarrow p^{3}-q^{3}=0$
$\Rightarrow(p-q)\left(p^{2}+p q+q^{2}\right)=0$
$\Rightarrow p-q=0$ or $p^{2}+p q+q^{2}=0$
$\Rightarrow p=q$ or $\left|\frac{p}{q}\right|^{2}+\frac{p q}{q^{2}}+\frac{q^{2}}{q^{2}}=0$
$\Rightarrow p=q$ or $\left|\frac{p}{q}\right|^{2}+\frac{p}{q}+1=0$
$\Rightarrow p=q$ or $\frac{p}{q}$ is one of the cube roots of unity.
96. (A) Consider the given point $P(p, q)$

Given that, P is equidistant from the points
$\mathrm{A}(1,2)$ and $\mathrm{B}(2,3)$
$\Rightarrow \mathrm{PA}=\mathrm{PB}$
$\Rightarrow \sqrt{\left.p-1^{* 2},\right) q-2^{* 2}}=\sqrt{\left.p-2^{* 2},\right) q-3^{*^{2}}}$
$\Rightarrow p^{2}+1-2 p+q^{2}+4-4 q=p^{2}+4-4 p+q^{2}$
$+9-6 q$
$\Rightarrow 1-2 p+4-4 q=4-4 p+9-6 q$
$\Rightarrow 2 p+2 q-8=0$
$\Rightarrow p+q=4$
Given that the point P lies on the $x$-axis.
Thus, $p=4$ and $q=0$
97. (B) Given that the variance of the data $2,4,5$, 6,17 is v .
We know that var $\lambda x=\lambda^{2} \operatorname{var} \bar{x}$
Now consider the data, $4,8,10,12,34$.
We got the above data by multiplying the given data by 2 .
$\Rightarrow \operatorname{var} \overline{2 x}=4 v$
$\because$ variance of the first set is given as v .

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98. (C) We know that the vector perpendicular $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$.
Thus, the unit vector perpendicular to $\vec{a}$ and
$\vec{b}$ is $\frac{\vec{a} \propto \vec{b}}{|\vec{a} \propto \vec{b}|}$
Therefore, the other unit vector perpendicular to and $\vec{a}$ and $\vec{b}=-\frac{\vec{a} \propto \vec{b}}{|\vec{a} \propto \vec{b}|}$
99. (A) Given that $p$ is the length of the perpendicular drawn from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$.
Distance of any point P, $\left(x_{1}, y_{1}\right)$ to the line $A x+B y+C=0$
is $\mathrm{D}=\left|\frac{\left.\begin{array}{lll}\frac{1}{a} \propto 0 & \stackrel{1}{b} & 0-1 \\ \sqrt{\left|-\frac{1}{a}\right|^{2}} \frac{k}{\mathrm{k}} \frac{1 \frac{1}{b}}{}{ }^{2}\end{array} \right\rvert\,}{}\right|$
Here, $x_{1}=0$ and $y_{1}=0$
$\Rightarrow p=\left|\frac{-1}{\sqrt{\frac{1}{a^{2}}, \frac{1}{b^{2}}}}\right|$
$\Rightarrow \sqrt{\frac{1}{a^{2}}, \frac{1}{b^{2}}}=\frac{1}{p}$
$\Rightarrow \frac{1}{p^{2}}=\frac{1}{a^{2}}+\frac{1}{b^{2}}$
100. (C) The ratio of given sides of the triangle is $a: b: c=2: \sqrt{6}: 1+\sqrt{3}$
By applying sine rule, we have
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$
Thus, we have,
$2: \sqrt{6}: 1+\sqrt{3}$
$\frac{2}{\sin A}=\frac{\sqrt{6}}{\sin B}=\frac{1, \sqrt{3}}{\sin C}=k$
$\Rightarrow \sin \mathrm{A}=\frac{2}{k}, \sin \mathrm{~B}=\frac{\sqrt{6}}{k}, \sin \mathrm{C}=\frac{1, \sqrt{3}}{k}$
$\Rightarrow \sin \mathrm{A}: \sin \mathrm{B}: \sin \mathrm{C}=\frac{2}{\sqrt{6}}: 1: \frac{1, \sqrt{3}}{\sqrt{6}}$
$\Rightarrow \sin \mathrm{A}: \sin \mathrm{B}: \sin \mathrm{C}=\sqrt{\frac{2}{3}}: 1: \frac{1, \sqrt{3}}{\sqrt{6}}$
$\Rightarrow \sin \mathrm{A}: \sin \mathrm{B}: \sin \mathrm{C}=\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{3}: \frac{\sqrt{3}}{2}:$
$\frac{1, \sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{3}}{2}$
$\Rightarrow \sin \mathrm{A}: \sin \mathrm{B}: \sin \mathrm{C}=\frac{1}{\sqrt{2}}: \frac{\sqrt{3}}{2}: \frac{1, \sqrt{3}}{2 \sqrt{2}}$
101. (C) Number of males in the ascending order $=440,630,670,680$
Thus, the minimum number of males population $=440$
Therefore, the minimum number of males population is in the year 1997.
102.(A) Number of female in the ascending order $=370,370,450,720$
Thus, the maximum number of females population $=720$
Therefore, the maximum number of females population is in the year 1995.
103. (A) Number of male rural population for the year $1998=280$
The whole population for the year $1998=$ 1050
Therefore, percentage of male rural population over the whole population in the
year $1998=\frac{280}{1050} \times 100=\frac{80}{3} \%$
104. (C) Distribution of data in pie chart (in terms of angles) : $90^{\circ}, 45^{\circ}, 30^{\circ}, 120^{\circ}$ and $75^{\circ}$
Maximum angle is $120^{\circ}$.
Thus, Employment head is allocated maximum funds.
105. (A) Thus amount allocated for education.
$=\frac{30^{\circ}}{360^{\circ}} \times 36000=, 3,000$ crores.
106. (B) Thus amount allocated for Agriculture
$=\frac{90^{\circ}}{360^{\circ}} \times 36000=₹ 9000$ crores
Thus amount allocated for Employment
$=\frac{120^{\circ}}{360^{\circ}} \times 36000=₹ 12,000$ crores
Amount allocated for both
Agriculture and Employment $=₹ 9,000+$
₹ 12,000 crores
= ₹ 21,000 crores
107. (C) Thus, amount allocated for Miscellaneous

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$=\frac{75^{\circ}}{360^{\circ}} \times 36000=₹ 7,500$ crores
Thus amount allocated for education.
$=\frac{30^{\circ}}{360^{\circ}} \times 36000=₹ 3,000$ crores
Excess amount allocated to
Miscellaneous over Education $=$ ₹ 7,500 -
₹ 3,000 crores
= ₹ 4,500 crores.
108. (B) Geometric mean of data $10,20,40$

$$
\begin{aligned}
& =\sqrt[3]{10 \propto 20 c 40} \\
& =\sqrt[3]{8 \propto 1000} \\
& =10 \sqrt[3]{8} \\
& =20
\end{aligned}
$$

109. (A) $3,7,6,9,5,4,2$

Arranging the above data in ascending order, we have,
$2,3,4,5,6,7$ and 9
Number of terms $=7$
Thus median $=\left|-\frac{7,1}{2}\right|^{\text {th }}$ terms

$$
\begin{aligned}
& =4^{\text {th }} \text { term } \\
& =5
\end{aligned}
$$

110. (C) Let $a$ and $b$ be the two numbers.

Arithmetic mean of $a$ and $b$ is 10 and
the geometric mean of $a$ and $b$ is 8
$\Rightarrow \frac{a, b}{2}=10$ and $\sqrt{a b}=8$
$\Rightarrow a+b=20 \ldots(1)$
and $a b=64 \ldots$...2)
Consider $(a-b)^{2}=a+b^{2}-4 a b$

$$
\begin{aligned}
& =20^{2}-4 \times 64 \\
& =400-256 \\
& =144
\end{aligned}
$$

$\Rightarrow a-b=12 \ldots$ (3)
Adding equations (1) and (3), we have,
$a=16$
Substituting the value $\mathrm{a}=16$ in equation (2), we have
$b=\frac{64}{16}=4$
Thus, one number exceeds the other by 12 .
111. (A) Consider the given expression
$(1+i)^{5}+(1-i)^{5}$, where $i=\sqrt{-1}$
Applying binomial theorem, we have,
$(1+i)^{5}+(1-i)^{5}$
$=1+{ }^{5} \mathrm{C}_{1} i+{ }^{5} \mathrm{C}_{2} i^{2}+{ }^{5} \mathrm{C}_{3} i^{3}+{ }^{5} \mathrm{C}_{4} i^{4}-t^{5}$
$+1-{ }^{5} \mathrm{C}_{1} i+{ }^{5} \mathrm{C}_{2} i^{2}-{ }^{5} \mathrm{C}_{3} i^{3}+{ }^{5} \mathrm{C}_{4} i^{4}-i^{5}$
$=1+{ }^{5} \mathrm{C}_{2} i^{2}+{ }^{5} \mathrm{C}_{4} i^{4}+1+{ }^{5} \mathrm{C}_{2} i^{2}+{ }^{5} \mathrm{C}_{4} i^{4}$

Since $i=\sqrt{-1}$, we have, $i^{2}=-1, i^{3}=-i, i^{4}=1$, $i^{5}=i$
Thus equation (1) becomes,
$(1+i)^{5}+(1-i)^{5}=2-2 \times{ }^{5} C_{2} \times 2 \times{ }^{5} \mathrm{C}_{4}$
$=2-2 \times \frac{5 \propto 4}{1 \propto 2}+2 \times \frac{5 \propto 4 \propto 3 \quad 2}{1 \propto 2 \propto 3}$
$=2-10+10$
$=-8$
112. (D) $\tan 9^{\circ}-\tan 27^{\circ}-\tan 63^{\circ}+\tan 81^{\circ}$
$=\tan 9^{\circ}+\tan 81^{\circ}-\left(\tan 27^{\circ}+\tan 63^{\circ}\right)$
$=\tan 9^{\circ}+\tan \left(90^{\circ}-9^{\circ}\right)-\left(\tan 27^{\circ}+\tan \left(90^{\circ}-27^{\circ}\right)\right)$
$=\tan 9^{\circ}+\cot 9^{\circ}-\left(\tan 27^{\circ}+\tan 27^{\circ}\right)$
$=\frac{\sin 9^{\circ}}{\cos 9^{\circ}}+\frac{\cos 9^{\circ}}{\sin 9^{\circ}}-\left|-\frac{\sin 27^{\circ}}{\cos 27^{\circ}}, \frac{\cos 27^{\circ}}{\sin 27^{\circ}}\right|$
$=\frac{\sin ^{2} 9^{\circ}, \cos ^{2} 9^{\circ}}{\cos 9^{\circ} \sin 9^{\circ}}-\left|-\frac{\sin ^{2} 27^{\circ}, \cos ^{2} 27^{\circ}}{\cos 27^{\circ} \sin 27^{\circ}}\right|$
$=\frac{1}{\cos 9^{\circ} \sin 9^{\circ}}-\left|\frac{1}{\cos 27^{\circ} \sin 27^{\circ}}\right|$
$=\frac{2}{\sin ) 2 \propto 9^{\circ} *}-\left|\frac{2}{\sin ) 2 \propto 27^{\circ} \text { 平 }}\right|$
$=\frac{2}{\sin 18^{\circ}}-\left|-\frac{2}{\sin 54^{\circ}}\right|$
$=\frac{2}{\sin 18^{\circ}}-\frac{2}{\sin ) 90^{\circ}-36^{*}}$
$=\frac{2}{\sin 18^{\circ}}-\left|\frac{2}{\cos 36^{\circ}}\right|$
We know that $\sin 18^{\circ}=\frac{\sqrt{5}-1}{4}$ and $\cos 36^{\circ}$
$=\frac{\sqrt{5}, 1}{4}$
Thus, equation (1) becomes,
$=\frac{2}{\frac{\sqrt{5}-1}{4}}-\frac{2}{\frac{\sqrt{5}, 1}{4}}$
$=\frac{8}{\sqrt{5}-1}-\frac{8}{\sqrt{5}, 1}$
$=\frac{8) \sqrt{5}, 1-\sqrt{5}, 1^{*}}{\left.) \sqrt{5}-1^{*}\right) \sqrt{5}, 1^{*}}$

$$
=\frac{8) \sqrt{5}, 1-\sqrt{5}, 1^{*}}{) \sqrt{5}-1^{*}}
$$

$=\frac{8) 2^{*}}{4}=4$

113. (B) $x=y \cos \left|-\frac{2 \theta}{3}\right|=z \cos \left(\left.-\frac{4 \theta}{3} \right\rvert\,\right.$
$\Rightarrow x=y \cos \left|-\frac{\theta}{2}, \frac{\theta}{\sigma}\right|=-y \sin \left(\left.-\frac{\theta}{\sigma} \right\rvert\,\right.$
and
$x=z \cos \left|\begin{array}{ll}-\theta & \frac{\theta}{3}\end{array}\right|=-z \cos \left|-\frac{\theta}{3}\right|$
That is we have,
$x=-y\left(\left.-\frac{1}{2} \right\rvert\,\right.$
and
$x=-z\left(-\frac{1}{2}\right)$
Therefore, we have, $x=\frac{-y}{2}=\frac{-z}{2}$
$\Rightarrow 2 x=-y=-z$
$\Rightarrow \frac{x}{\left|-\frac{1}{2}\right|}=\frac{y}{-1}=\frac{z}{-1}=\mathrm{R}$
Thus,
$x y+y z+z x$
$=\left|-\frac{R}{2}\right|(-R)+(-R)(-R)+(-R)\left|-\frac{R}{2}\right|$
$=\frac{R^{2}}{2}+R^{2}-\frac{R^{2}}{2}$
$=R^{2}-R^{2}$
$=0$
114. (B) The maximum possible value of sine function is 1 .
Thus, $\sin A+\sin B+\sin C=3 \Rightarrow \sin A=1$, $\sin B=1$ and $\sin C=1$
Therefore, $\mathrm{A}=\mathrm{B}=\mathrm{C}=\frac{\theta}{2}$.
$\Rightarrow \cos \mathrm{A}+\cos \mathrm{B}+\cos \mathrm{C}=\cos \frac{\theta}{2}+\cos \frac{\theta}{2}+$
$\cos \frac{\theta}{2}=0$
115. (C)


We need to find the angle of elevation.
Let the angle of elevation be $\theta$.
In $\triangle \mathrm{ABC}$,
$\Rightarrow \frac{15}{15}=\tan \theta$
$\Rightarrow 1=\tan \theta$
$\Rightarrow \tan 45^{\circ}=\tan \theta$
$\Rightarrow \theta=45^{\circ}$
116. (A)

$3 x+5 y=7$
$6 x+10 y=18$
We have,
$3 x+5 y=7$
$\Rightarrow 5 y=-3 x+7$
$\Rightarrow \quad y=\frac{-3}{5} x+\frac{7}{5}$
Slope of first line is $\frac{-3}{5}$
Similarly,
$6 x+10 y=18$
$\Rightarrow \quad 10 y=-6 x+18$
$\Rightarrow y=\frac{-6}{10} x+\frac{18}{10}$
$\Rightarrow \quad y=\frac{-3}{5} x+\frac{9}{5}$
Slope of second line is $\frac{-3}{5}$
Slope first line $=$ Slope of second line.
The y -intercepts of both the lines are not uique.
Thus lines (1) and (2) are parallel to each other, As parallel lines do not intersect, the system of equations do not have a solution.
117. (D) Let $x=\sec ^{-1}\left|\frac{2}{\sqrt{3}}\right|$
$\Rightarrow \sec ^{-1}\left|-\frac{2}{\sqrt{3}}\right|=$ An angle in $[0, \pi]-\frac{\theta}{2}$
whose secant is $\left|-\frac{2}{\sqrt{3}}\right|$
$\Rightarrow \sec ^{-1}\left|-\frac{2}{\sqrt{3}}\right|=\frac{\theta}{6}$


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Thus, the principal value of $\sec ^{-1} \left\lvert\,\left(\frac{2}{\sqrt{3}}\right)\right.$ is $\frac{\theta}{6}$
118. (B) Consider the series $S_{1}=2+6+10+14$
$+18+22+26+30+34+38+42+46+$
Now, consider the second series
$\mathrm{S}_{2}=1+6+11+16+21+26+31+36$
$+41+46 \ldots$
In both the series, common terms are marked. The number sequence of common terms in $\mathrm{S}_{1}$,
$\mathrm{S}_{1}{ }^{\prime}=2^{\text {nd term }}, 7^{\text {nd term }}, 12^{\text {nd term }}, 17^{\text {nd term }}, 22^{\text {nd term }} \ldots$
The number sequence of common terms in $\mathrm{S}_{1}$, $S_{2}{ }^{\prime}=2^{\text {nd term }}, 6^{\text {nd term }}, 10^{\text {nd term }}, 14^{\text {nd term }}, 18^{\text {nd term }} \ldots$ Thus, 10th term in $\mathrm{S}_{1}{ }^{\prime}=2+(10-1) \times 5=$ $47^{\text {nd term }}$ of $\mathrm{S}_{1}$
Thus, 10th term in $\mathrm{S}_{2}{ }^{\prime}=2+(10-1) \times 4=$ $38^{\text {nd term }}$ of $\mathrm{S}_{2}$
So, 47th term in $\mathrm{S}_{1}$ and 38th term in $\mathrm{S}_{2}$ are:
For $S_{1}, t_{47}=2+(47-1) \times 4=186$
For $S_{2}, t_{38}=1+(38-1) \times 5=186$
119. (A) Given that $10^{\text {th }}$ term of a GP is 9 .
$\Rightarrow t_{10}=\mathrm{ar}^{10-1}=9$
$\Rightarrow a r^{9}=9 \ldots(1)$
And $4^{\text {th }}$ term is 4
$\Rightarrow t_{4}=a r^{4-1}=4$
$\Rightarrow a r^{3}=4$
Divide equation (1) by equation (2), we have,
$\frac{t_{10}}{t_{4}}=\frac{a r^{9}}{a r^{3}}=\frac{9}{4}$
$\Rightarrow \frac{r^{9}}{r^{6}}=\frac{9}{4}$
$\Rightarrow r^{6}=\frac{9}{4}$
Multiplying equations (1) and (2), we have,
$\left(a r^{9}\right)\left(a r^{3}\right)=9 \times 4$
$\Rightarrow a^{2} r^{12}=36$
$\Rightarrow a^{2}(r)^{2}=36$
$\Rightarrow a^{2}\left|-\frac{9}{4}\right|^{2}=6^{2}$
$\Rightarrow a=\frac{8}{3}$
Substituting the value of a in equation (4), we have,
$\Rightarrow t_{7}=6$
120. (D) Since the given equation $y=m x+c$ represents the equation of the straight line, there is neither maximum point nor minimum point.

## NDA MATHS MOCK TEST- 64 (ANSWER KEY)

1. (D)
2. (C)
3. (A)
4. (C)
5. (C)
6. (C)
7. (C)
8. (B)
9. (B)
10. (A)
11. (C)
12. (A)
13. (D)
14. (A)
15. (C)
16. (A)
17. (B)
18. (A)
19. (A)
20. (B)
21. (D)
22. (B)
23. (C)
24. (C)
25. (C)
26. (A)
27. (D)
28. (C)
29. (A)
30. (D)
31. (C)
32. (A)
33. (B)
34. (B)
35. (B)
36. (A)
37. (A)
38. (C)
39. (C)
40. (D)
41. (A)
42. (C)
43. (B)
44. (D)
45. (C)
46. (A)
47. (D)
48. (D)
49. (D)
50. (A)
51. (B)
52. (D)
53. (C)
54. (B)
55. (C)
56. (D)
57. (C)
58. (A)
59. (D)
60. (B)
61. (A)
62. (A)
63. (B)
64. (A)
65. (B)
66. (D)
67. (B)
68. (A)
69. (D)
70. (C)
71. (D)
72. (B)
73. (C)
74. (A)
75. (C)
76. (A)
77. (B)
78. (C)
79. (D)
80. (D)
81. (A)
82. (A)
83. (D)
84. (D)
85. (D)
86. (A)
87. (B)
88. (C)
89. (D)
90. (A)
91. (C)
92. (B)
93. (D)
94. (A)
95. (C)
96. (A)
97. (B)
98. (C)
99. (A)
100. (C)
101. (C)
102. (A)
103. (A)
104. (C)
105. (A)
106. (B)
107. (C)
108. (B)
109. (A)
110. (C)
111. (A)
112. (D)
113. (B)
114. (B)
115. (C)
116. (A)
117. (D)
118. (B)
119. (A)
120. (D)


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