## NDA (MATHS) MOCK TEST - 41 (SOLUTION)

1. (A) We have
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\(\mathrm{B}=\mathrm{B} \cup(\mathrm{A} \cap \mathrm{B})\)
\(=B \cup(A \cap C)\)
\((\because \mathrm{A} \cap \mathrm{B}=\mathrm{A} \cap \mathrm{C})\)
\(=(B \cup A) \cap(B \cup C)(\) by Statement II)
\(=(A \cup C) \cap(B \cup C)\)
\(=(A \cap B) \cup C \quad(\because A \cap B=A \cap C)\)
\(=(A \cap C) \cup C=C\)
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Hence, Statement II is the correct explanation of Statement I.
2. $(\mathrm{C}) \because B=U-A=A^{\prime}$

$$
\therefore \quad \mathrm{n}(\mathrm{~B})=\mathrm{n}\left(\mathrm{~A}^{\prime}\right)=\mathrm{n}(\mathrm{U})-\mathrm{n}(\mathrm{~A})
$$

Hence, Statement I is true
but for any three arbitary sets A, B, C we can not always have

$$
\mathrm{n}(\mathrm{c})=\mathrm{n}(\mathrm{~A})-\mathrm{n}(\mathrm{~B}) \text { if } \mathrm{C}=\mathrm{A}-\mathrm{B}
$$

as it is not specified $A$ is universal set or not. In case not conclude $n(C)=n(A)-n(B)$. Hence, Statement II is false.
3. (A) Let S be the set of all even prime numbers. $\therefore \mathrm{S}=[2]=$ non empty set
4. (A)
5. (B)
6. (B) $\because \omega^{13}+\omega^{20}=\omega+\omega^{2}=-1$
$\therefore \quad \mathrm{E}=\sin \left(-\pi+\frac{\pi}{4}\right)$
$=-\sin \frac{3 \pi}{4}=-\frac{1}{\sqrt{2}}$
7. (D) $\mathrm{E}=\left(\frac{1-i}{1+i}\right)^{n-2}(1-i)^{2}=\left(-\frac{2 i}{2}\right)^{n-2}(-2 i)$ $=2(-i)^{\mathrm{n}-1}=2\left[(-i)^{2}\right]^{(\mathrm{n}-1 / 2)}=2(-1)^{(\mathrm{n}-1) / 2}$ Since, E is real and positive.

Therefore, $\frac{n-1}{2}=2 \lambda$
$\therefore \mathrm{n}=4 \lambda+1$
i.e., odd of this type but not any odd.
8. (B) Now, $\frac{i+\sqrt{3}}{-i+\sqrt{3}}=\frac{(i+\sqrt{3})^{2}}{(\sqrt{3}-i)(\sqrt{3}+i)}$

$$
\begin{aligned}
& =\frac{i^{2}+3+2 \sqrt{3 i}}{3+1}=\frac{-1+3+2 \sqrt{3 i}}{4} \\
& =\frac{1+\sqrt{3 i}}{2}=-\omega^{2}
\end{aligned}
$$

and $\frac{i-\sqrt{3}}{i+\sqrt{3}}=\frac{(i-\sqrt{3})^{2}}{i^{2}-(\sqrt{3})^{2}}$

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16. (D) Sum of $n$ terms of an AP is $S_{n}=\frac{n}{2}$
$[2 \mathrm{~A}+(\mathrm{n}-1) \mathrm{D}]$
where, $A$ and $D$ are first term and common difference.
Hence, sum is always of the form $a^{2}+b n$
Hence, Statement I is false, and Statement II is true.
17. (C) Given, $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$
$\Rightarrow \frac{1}{b-a}-\frac{1}{c}=\frac{1}{a}-\frac{1}{b-c} \Rightarrow \frac{(c-b+a)}{c(b-a)}=\frac{(b-c-a)}{a(b-c)}$
$\Rightarrow \frac{1}{c(b-a)}=-\frac{1}{a(b-c)} \Rightarrow \mathrm{ba}-\mathrm{ca}=-\mathrm{cb}+\mathrm{ac}$
$\Rightarrow \mathrm{ab}+\mathrm{bc}=2 \mathrm{ac}$
$\therefore \quad \mathrm{b}=\frac{2 a c}{a+c}$
Hence, $a, b, c$ are in HP.
18. (C) Replacing $x$ by $\frac{1}{x}$ in the first equation, we get the second equation and hence, its roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
19. (A) Dividing the equation $a^{3} x^{2}+a b c x+c^{3}=0$ by $\mathrm{c}^{2}$, we get $\mathrm{a}\left(\frac{a x}{c}\right)^{2}+\mathrm{b}\left(\frac{a x}{c}\right)+\mathrm{c}=0$
$\Rightarrow \frac{a x}{c}=\alpha, \beta$
$\Rightarrow x=\frac{c}{a} \alpha, \frac{c}{a} \beta$
$\Rightarrow x=\alpha^{2} \beta, \alpha \beta^{2} \quad\left(\because \frac{c}{a}=\alpha \beta=\right.$ product of roots $)$ Hence, $\alpha^{2} \beta$ and $\alpha \beta^{2}$ are the roots of the equation $\mathrm{a}^{3} \times x^{2}+\mathrm{abc} x+\mathrm{c}^{3}=0$
20. (D) $\sin \theta+\cos \theta=-\frac{b}{a}$

$$
\sin \theta \cos \theta=\frac{c}{a}
$$

Now, $(\sin \theta+\cos \theta)^{2}=1+2 \sin \theta \cos \theta$
$\Rightarrow \frac{b^{2}}{a^{2}}=1+\frac{2 c}{a}=\frac{a+2 c}{a}$
$\Rightarrow \mathrm{b}^{2}=\mathrm{a}^{2}+2 \mathrm{ac}$
$\Rightarrow \mathrm{b}^{2}+\mathrm{c}^{2}=\mathrm{a}^{2}+2 \mathrm{ac}+\mathrm{c}^{2}=(\mathrm{a}+\mathrm{c})^{2}$
$\therefore \mathrm{b}^{2}+\mathrm{c}^{2}=(\mathrm{a}+\mathrm{c})^{2}$
21. (B) Use $\alpha^{3}+\beta^{3}=(\alpha+\beta)^{3}-3 \alpha \beta(\alpha+\beta)$
22. (A)
23. (B) Let there be $n$ persons in a room.
$\therefore$ Total number of shankhands $={ }^{n} C_{2}=66$

$$
\begin{aligned}
& \Rightarrow \frac{1}{2} \mathrm{n}(\mathrm{n}-1)=66 \Rightarrow \mathrm{n}^{2}-\mathrm{n}-132=0 \\
& \Rightarrow(\mathrm{n}+11)(\mathrm{n}-12)=0 \Rightarrow \mathrm{n}=12
\end{aligned}
$$

$$
(\because \mathrm{n} \neq-11)
$$

24. (C)
25. (B) Total number of lines in $n$-sided regular polygon $={ }^{n} \mathrm{C}_{2}$
and total number of sides in n -sided regular polygon $=n$
$\therefore$ Number of diagonals in n -sided regular polygon
$={ }^{\mathrm{n}} \mathrm{C}_{2}-\mathrm{n}=\frac{n(n-1)}{2}-\mathrm{n}=\mathrm{n}\left\{\frac{n-1}{2}-1\right\}$
$=\frac{n(n-3)}{2}$
26. (C) Total number of arrangements $=6!=720$ Total number of arrangements while all the Hindi books are together $=4!\times 3!=24 \times 6$ $=144$
$\therefore$ Number of ways, in which books are arranged, while all Hindi books are not together $=720-144=576$
27. (C) $\left(1+x+x^{2}+x^{3}\right)^{11}=\left[(1+x)\left(1+x^{2}\right)\right]^{11}$ $=(1+x)^{11} \cdot\left(1+x^{2}\right)^{11}$
$=\left({ }^{11} \mathrm{C}_{0}+{ }^{11} \mathrm{C}_{1} x+{ }^{11} \mathrm{C}_{2} x^{2}+{ }^{11} \mathrm{C}_{3} x^{3}+{ }^{11} \mathrm{C}_{4} x^{4}+\ldots.\right)$ $\left({ }^{11} \mathrm{C}_{0}+{ }^{11} \mathrm{C}_{1} x^{2}+{ }^{11} \mathrm{C}_{2} x^{4}+\ldots ..\right)$
$\therefore$ Coefficient of $x^{4}$ in $\left(1+x+x^{2}+x^{3}\right)^{11}$
$={ }^{11} \mathrm{C}_{0} \cdot{ }^{11} \mathrm{C}_{2}+{ }^{11} \mathrm{C}_{2} \cdot{ }^{11} \mathrm{C}_{1}+{ }^{11} \mathrm{C}_{4} \cdot{ }^{11} \mathrm{C}_{0}=990$
28. (C) Given, $\left(1+2 x+x^{2}\right)^{10}=\left\{\left(1+x^{2}\right\}^{10}=(1+x)^{20}\right.$ $\therefore$ Total terms $=20+1=21$
29. (A) Given $(1+i)^{5}+(1-i)^{5}$
$=\left({ }^{5} \mathrm{C}_{0}+{ }^{5} \mathrm{C}_{1} \mathrm{i}+{ }^{5} \mathrm{C}_{2} \mathrm{i}^{2}+{ }^{5} \mathrm{C}_{3} \mathrm{i}^{3}+{ }^{5} \mathrm{C}_{4} \mathrm{i}^{4}+{ }^{5} \mathrm{C}_{5} \mathrm{i}^{5}\right)$
$+\left({ }^{5} \mathrm{C}_{0}-{ }^{5} \mathrm{C}_{1} \mathrm{i}+{ }^{5} \mathrm{C}_{2} \mathrm{i}^{2}-{ }^{5} \mathrm{C}_{3} \mathrm{i}^{3}+{ }^{5} \mathrm{C}_{4} \mathrm{i}^{4}-{ }^{5} \mathrm{C}_{5} \mathrm{i}^{5}\right)$ (by Bionomial theorem)
$2\left({ }^{5} \mathrm{C}_{0}+{ }^{5} \mathrm{C}_{2} \mathrm{i}^{2}+{ }^{5} \mathrm{C}_{4} \mathrm{i}^{4}\right)=2[1-10+5]=-8$
30. (A) $\mathrm{T}_{\mathrm{r}+1}={ }^{9} \mathrm{C}_{\mathrm{r}}(3 x)^{9-\mathrm{r}}\left(-\frac{x^{3}}{6}\right)^{r}={ }^{9} \mathrm{C}_{\mathrm{r}} x^{9+2 \mathrm{r}}(3)^{9-\mathrm{r}}\left(-\frac{1}{6}\right)^{r}$

For coefficient of $x^{17}, 9+2 r=17 \Rightarrow r=4$
$\therefore \quad \mathrm{T}_{5}={ }^{9} \mathrm{C}_{4}(3)^{9-4}\left(-\frac{1}{6}\right)^{4}=126 \times 3^{5} \times \frac{1}{6^{4}}=\frac{189}{8}$
31. (C) Given expression can be rewritten as
$\log _{x y z} x y+\log _{x y z} y z+\log _{x y z} z x$
$\quad=\log _{x y z}(x y \cdot y z \cdot z x)=\log _{x y z}\left(x^{2} y^{2} z^{2}\right)$
$\quad=\log _{x y z}(x y z)^{2}=2 \times 1=2$
32. (B) $\left(\log _{3} x\right)\left(\log _{x} 2 x\right)\left(\log _{2 x} y\right)=\log _{x} x^{2}$

$$
\Rightarrow \frac{\log x}{\log 3} \times \frac{\log 2 x}{\log x} \times \frac{\log y}{\log 2 x}=\frac{\log x^{2}}{\log x}
$$

$$
\left(\because \log _{b} a=\frac{\log a}{\log b}\right)
$$

$\Rightarrow \frac{\log y}{\log 3}=\frac{2 \log x}{\log x} \quad\left(\because \log \mathrm{a}^{\mathrm{b}}=\mathrm{b} \log \mathrm{a}\right)$
$\Rightarrow \quad \log y=2 \log 3$
$\Rightarrow \quad \log \mathrm{y}=\log 3^{2}(\because \log \mathrm{~m}=\log \mathrm{n} \Rightarrow \mathrm{m}=\mathrm{n})$
$\Rightarrow \quad \log y=\log 9$
$\therefore \quad \mathrm{y}=9$
33. (D) $\mathrm{A}^{2}=\left[\begin{array}{ll}3 & -4 \\ 1 & -1\end{array}\right]\left[\begin{array}{ll}3 & -4 \\ 1 & -1\end{array}\right]=\left[\begin{array}{ll}5 & -8 \\ 2 & -3\end{array}\right]$
$\mathrm{A}^{2}=\mathrm{A}^{\mathrm{n}}$ for $\mathrm{n}=2$, putting $\mathrm{n}=2$ in the matrices given in $(\mathrm{A}),(\mathrm{B})$ and $(\mathrm{C})$, we do not get $\left[\begin{array}{ll}5 & -8 \\ 2 & -3\end{array}\right]$

## Solution(Q nos. 34-36)

Since, BA is defined.
$\therefore$ Number of columns in $B=$ Number of rows in A
$\Rightarrow 11-\mathrm{y}=\mathrm{x} \Rightarrow \mathrm{x}+\mathrm{y}=11$
Also AB is defined
$\therefore$ Number of columns in $A=$ Number of rows in B
$\therefore \mathrm{x}+5=\mathrm{y}$
$\Rightarrow \quad x-y=-5$
34. (B) On adding Eqs. (i) and (ii), we get

$$
2 x=6 \Rightarrow x=3
$$

35. (A) On subtracting Eq. (ii) from Eq. (i), we get $2 y=16 \Rightarrow y=8$
36. (A) Order of $A B=$ (Number of rows in $A$ ) $\times$ (Number of columns in B)
$=x \times 11-\mathrm{y}=3 \times 3$
37. (C) If $A B=0$, then it may be concluded that either $|A|=0$ or $|B|=0$.
38. (B) $\because A=\left[\begin{array}{ll}3 & 2 \\ 1 & 4\end{array}\right]$
$\therefore \quad \mathrm{A}[\operatorname{adj}(\mathrm{A})]=\mathrm{I}_{2}|\mathrm{~A}|$

$$
=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]\left|\begin{array}{ll}
3 & 2 \\
1 & 4
\end{array}\right|=\left[\begin{array}{cc}
10 & 0 \\
0 & 10
\end{array}\right]
$$

39. (D) Put $x=0$ in given equation $\mathrm{c}=\left|\begin{array}{ccc}0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0\end{array}\right|=0$
(since, skew symmetric determinant of odd order is zero)
40. (C) $\Delta=-\left(a^{3}+b^{3}+c^{3}-3 a b c\right)$
$=-(a+b+c) \frac{1}{2}\left[(a-b)^{2}+(b-c)^{2}+(c-a)^{2}\right]$
Since, $\mathrm{a}, \mathrm{b}$ and c are distinct negative real numbers, hence $\Delta \geq 0$.
41. (C) In a triangle, $A+B+C=\pi$
$\therefore \quad \cos (A+B)=\cos (\pi-C)=-\cos C$
$\Rightarrow \cos A \cos B+\cos C=\sin A \sin B$
and $\sin (\mathrm{A}+\mathrm{B})=\sin \mathrm{C}$
Expanding the given determinant, we get $\Delta=\left(1-\cos ^{2} \mathrm{~A}\right)+\cos \mathrm{C}[\cos \mathrm{C}+\cos \mathrm{A} \cos \mathrm{B}]$ $+\cos B[\cos B+\cos A \cos C]$
$=-\sin ^{2} A+\cos C(\sin A \sin B)+\cos B(\sin A$ $\sin \mathrm{C})$
$=-\sin ^{2} A+\sin A \sin (B+C)=-\sin ^{2} A+\sin ^{2} A$ $=0$
42. (A) We know that, a square matrix $A^{\prime}$ is an orthogonal matrix, if $A A^{T}=I$
$\Rightarrow\left|\mathrm{AA}^{\mathrm{T}}\right|=|\mathrm{I}| \Rightarrow|\mathrm{A}|\left|\mathrm{A}^{\mathrm{T}}\right|=\mathrm{I}$
$\Rightarrow|\mathrm{A}||\mathrm{A}|=1 \quad\left(\because|\mathrm{~A}|=\left|\mathrm{A}^{\mathrm{T}}\right|\right)$
$\Rightarrow|A|^{2}=1$
$\Rightarrow|\mathrm{A}|= \pm 1$
43. (D)
$\left|\begin{array}{ccc}6 a & 3 b & 15 c \\ 2 l & m & 5 n \\ 2 p & q & 5 r\end{array}\right|=30\left|\begin{array}{ccc}a & b & c \\ l & m & n \\ p & q & r\end{array}\right|=30 \times 2=60$
44. (A) $\tan ^{4} \mathrm{~A}-\sec ^{4} \mathrm{~A}+\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}$
$=\left(\tan ^{2} \mathrm{~A}\right)^{2}-\left(\sec ^{2} \mathrm{~A}\right)^{2}+\left(\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}\right)$
$=\left(\tan ^{2} \mathrm{~A}-\sec ^{2} \mathrm{~A}\right)\left(\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}\right)+\left(\tan ^{2} \mathrm{~A}\right.$
$\left.+\sec ^{2} \mathrm{~A}\right)$
$=(-1)\left(\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}\right)+\left(\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}\right)$
$\left(\because \sec ^{2} A-\tan ^{2} A=1\right)$
$=-\left(\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}\right)+\left(\tan ^{2} \mathrm{~A}+\sec ^{2} \mathrm{~A}\right)=0$
45. (A) Given that, $\frac{\sin \theta}{\operatorname{cosec} \theta}+\frac{\cos \theta}{\sec \theta}$

$$
\begin{array}{r}
=\frac{\sin \theta}{(1 / \sin \theta)}+\frac{\cos \theta}{(1 / \cos \theta)} \\
=\sin ^{2} \theta+\cos ^{2} \theta=1
\end{array}
$$

46. (D) $\sin 50^{\circ}-\sin 70^{\circ}+\sin 10^{\circ}$
$=2 \cos \frac{70^{\circ}+50^{\circ}}{2} \cdot \sin \frac{50^{\circ}-70^{\circ}}{2}+\sin 10^{\circ}$
$=-2 \cos 60^{\circ} \sin 10^{\circ}+\sin 10^{\circ}$
$=-\sin 10^{\circ}+\sin 10^{\circ}=0$
47. (C) I. $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)=\sin -1\left(-\frac{\sqrt{3}}{2}\right)=\sin ^{-1}$

$$
[-\sin (\pi / 3)]=-\frac{\pi}{3}
$$

II. $\sec ^{-1}\left(\frac{2}{\sqrt{3}}\right)=\cos ^{-1}\left(\frac{\sqrt{3}}{2}\right)=\cos ^{-1}\left(\cos \frac{\pi}{6}\right)=\frac{\pi}{6}$

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48. (D) Given, $\sin ^{-1} x+\sin ^{-1} y=\frac{\pi}{2}$
and $\cos ^{-1} x-\cos ^{-1} y=0$
$\Rightarrow\left(\frac{\pi}{2}-\sin ^{-1} x\right)-\left(\frac{\pi}{2}-\sin ^{-1} y\right)=0$
$\Rightarrow \sin ^{-1} y-\sin ^{-1} x=0$
$\Rightarrow \sin ^{-1} y=\sin ^{-1} x$
From Eqs. (i) and (ii),

$$
2 \sin ^{-1} x=\frac{\pi}{2} \Rightarrow \sin ^{-1} x=\frac{\pi}{4} \Rightarrow x=\frac{1}{\sqrt{2}}
$$

From Eq. (ii), $\mathrm{y}=\frac{1}{\sqrt{2}}$
49. (C


Now, In $\triangle \mathrm{ABC}$,
$\tan 15^{\circ}=\frac{120}{x} \Rightarrow \tan \left(60^{\circ}-45^{\circ}\right)=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{120}{x}$
$\Rightarrow x=120 \times \frac{\sqrt{3}+1}{\sqrt{3}-1}=120(2+\sqrt{3})$
$=120 \times 3.7=444 \mathrm{~m}$
50. (A) We know that,
$\sin \mathrm{C}=\sin [\pi-(\mathrm{A}+\mathrm{B})]=\sin (\mathrm{A}+\mathrm{B})$
and $\sin \mathrm{A}=\sin (\mathrm{B}+\mathrm{C})$
$\therefore \quad \frac{\sin A}{\sin C}=\frac{\sin (B+C)}{\sin (A+B)}$
But $\frac{\sin A}{\sin C}=\frac{\sin (A-B)}{\sin (B-C)}$
$\therefore \quad \frac{\sin (B+C)}{\sin (A+B)}=\frac{\sin (A-B)}{\sin (B-C)}$
$\left[\because \sin (A+B) \cdot \sin (A-B)=\sin ^{2} A-\sin ^{2} B\right]$
$\Rightarrow \sin ^{2} B-\sin ^{2} C=\sin ^{2} A-\sin ^{2} B$
$\Rightarrow \mathrm{b}^{2}-\mathrm{c}^{2}=\mathrm{a}^{2}-\mathrm{b}^{2}$
$\Rightarrow 2 b^{2}=a^{2}+c^{2}$
so, $a, b$ and $c$ are in AP.
51. (A) $\cos \mathrm{B}=\left(\frac{c^{2}+a^{2}-b^{2}}{2 c a}\right)$
$\Rightarrow \cos ^{2} \mathrm{~B}=\frac{a^{4}+b^{4}+c^{4}-2 b^{2} c^{2}-2 a^{2} b^{2}+2 a^{2} c^{2}}{4 a^{2} c^{2}}$
$\Rightarrow \cos ^{2} \mathrm{~B}=\frac{1}{2} \Rightarrow \cos \mathrm{~B}= \pm \frac{1}{\sqrt{2}}$
$\therefore B=45^{\circ}$ or $135^{\circ}$
52. (B) We know that,

$$
\begin{aligned}
& \mathrm{r}=4 \mathrm{R} \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \\
& =4 \mathrm{R}\left(\frac{1}{2}\right)^{3}=\frac{R}{2} \quad\left(\because \mathrm{~A}=\mathrm{B}=\mathrm{C}=60^{\circ}\right)
\end{aligned}
$$

53. (D) Equation of line prallel to $2 x+3 y+5=0$ is $2 x+3 y+\lambda=0$
But it passes throuht (1, 1).
$\therefore 2+3+\lambda=0 \Rightarrow \lambda=-5$
So, the required equations is $2 x+3 y-5=0$.
54. (D) Perpendicular distance of the line $3 x+$ $4 y-1=0$ from the point $(1,1)=$ Perpendicular distance of the line $4 x+3 y+2 k=0$ from the point $(1,1)$
$\Rightarrow \frac{|3 \times 1+4 \times 1-1|}{\sqrt{9+16}}=\frac{|4 \times 1+3 \times 1+2 k|}{\sqrt{16+9}}$
$\Rightarrow \frac{|3+4-1|}{5}=\frac{|4+3+2 k|}{5}$
$\Rightarrow 6=7+2 \mathrm{k} \Rightarrow 2 \mathrm{k}=-1$
$\Rightarrow \mathrm{k}=-\frac{1}{2}$
55. (B) A line which passes through the points $(5,0)$ and $(0,3)$ is
$(y-0)=\frac{3-0}{0-5}(x-5)$
$\Rightarrow-5 y=3 x-15$
$\Rightarrow 3 x+5 y-15=0$
Now, length of the perpendicular from the point $(4,4)$ on the line (i) is
$=\frac{|3(4)+5(4)-15|}{\sqrt{(3)^{2}+(5)^{2}}}=\frac{|12+20-15|}{\sqrt{9+25}}$
$=\frac{17}{\sqrt{34}}=\sqrt{\frac{17}{2}}$
56. $(\mathrm{C})$ Let $\mathrm{A}(\mathrm{a}, 0)$ and $\mathrm{B}(0, \mathrm{~b})$ are two points on respective coordinate axes and (-5, 4) divides $A B$ in the ratio $1: 2$
$\therefore-5=\frac{1 \times 0+2 \times a}{3} \Rightarrow \mathrm{a}=\frac{-15}{2}$

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and $4=\frac{1 \times b+2 \times 0}{3} \Rightarrow b=12$
Hence, equation of line joining $\left(-\frac{15}{2}, 0\right)$ and $(0,12)$ is
$(y-0)=\frac{12-0}{0+\frac{15}{2}} \cdot\left(x+\frac{15}{2}\right)$
$\Rightarrow \mathrm{y}=\frac{4}{5}(2 x+15)$
$\Rightarrow 5 y=(8 x+60) \Rightarrow 8 x-5 y+60=0$
57. (C) $\mathrm{A}=\pi \mathrm{r}^{2}$, where r is the distance between $(1,2)$ and $(4,6)$.
$\Rightarrow \sqrt{\left(x_{2}-x_{1}\right)^{2}\left(y_{2}-y_{1}\right)^{2}}=\sqrt{3^{2}+4^{2}}=\sqrt{9+16}=5$
$\therefore \mathrm{A}=\pi \mathrm{r}^{2}=\pi, 25=25 \pi$
58. (B) If $(x, y)$ be the point, then by ratio formula $x=4 \cos \theta+3, y=4 \sin \theta$
$\therefore(x-3)^{2}+y^{2}=16$
59. (A) In $\triangle \mathrm{AOD}$,

$\sin \frac{\theta}{2}=\frac{A D}{O A} \Rightarrow \sin \frac{\theta}{2}=\frac{A D}{R}$
$\Rightarrow \mathrm{AD}=\mathrm{R} \sin \frac{\theta}{2}$
$\therefore$ Length of the chord $\mathrm{AB}=2 \mathrm{AD}=2 \mathrm{R} \sin \frac{\theta}{2}$
60. (B) $x=2+\mathrm{t}^{2}, \mathrm{y}=2 \mathrm{t}+1$

Eliminating $t$, we get

$$
(y-1)^{2}=2(x-2)
$$

which is a parabola with vertex at $(2,1)$.
61. (A) Clearly, the race course will be an ellipse with the flag posts as its foci. If $a$ and $b$ are the semi major and semi minor axes of the ellipse, then $2 \mathrm{a}=10$ and $2 \mathrm{ae}=8$
$\therefore \quad \mathrm{a}=5, \mathrm{c}=\frac{4}{5}$
and $b^{2}=a^{2}\left(1-c^{2}\right)=9$
$\therefore$ Area of the ellipse $=\pi \mathrm{ab}=\pi \cdot 5 \cdot 3=15 \pi \mathrm{sq} \mathrm{m}$
62. (B) Equation of any tangent to the parabola $P: y^{2}=8 x$ is

$$
\mathrm{y}=\mathrm{m} x+\frac{2}{m}
$$

where, $m$ is the slope, of tangent,
Since, it touches $\mathrm{E}: \frac{x^{2}}{4}-\frac{y^{2}}{16}=1$
$\left(\frac{2}{m}\right)^{2}=4 \mathrm{~m}^{2}+15 \Rightarrow \mathrm{~m}= \pm \frac{1}{2}$
Equations of the tangents are $x \pm 2 \mathrm{y}+8=0$
63. (A) When $m=\frac{1}{2}$, the slope of the normal is -2 and equations of the normal to the parabola is
$y=2 x-2(2)(-2)-2(-2)^{3} \Rightarrow 2 x+y=24$
64. (C) We know that, the direction cosines of X -axis is $(1,0,0)$.
$\therefore$ Som of squares of direction cosine

$$
\begin{aligned}
& =(1)^{2}+(0)^{2}+(0)^{2} \\
& =1+0+0=1
\end{aligned}
$$

65. (B) The equation of line passing through $\left(x_{1} \cdot \mathrm{y}_{1}, z_{1}\right)$ and $\left(x_{2}, \mathrm{y}_{2}, z_{2}\right)$ is

$$
\begin{equation*}
\frac{x-x_{1}}{x_{2}-x_{1}}=\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{z-z_{1}}{z_{2}-z_{1}} \tag{i}
\end{equation*}
$$

Here, $\left(x_{2}-x_{1}\right),\left(y_{2}-y_{1}\right)$ and $\left(z_{2}-z_{1}\right)$ direction ratio's of that line.
Then, its direction cosines are

$$
\begin{aligned}
& l=\frac{(-2-1)}{\sqrt{(-3)^{2}+(1)^{2}+(4)^{2}}} \\
& \mathrm{~m}=\frac{(3-2)}{\sqrt{(-3)^{2}+(1)^{2}+(4)^{2}}}
\end{aligned}
$$

and $\quad \mathrm{n}=\frac{(1+3)}{\sqrt{(-3)^{2}+(1)^{2}+(4)^{2}}}$
$\Rightarrow l=\frac{-3}{\sqrt{26}}, \mathrm{~m}=\frac{1}{\sqrt{26}}, \mathrm{n}=\frac{4}{\sqrt{26}}$
$\therefore l^{2}+\mathrm{m}^{2}+\mathrm{n}^{2}=\left(\frac{-3}{\sqrt{26}}\right)^{2}+\left(\frac{1}{\sqrt{26}}\right)^{2}+\left(\frac{4}{\sqrt{26}}\right)^{2}$

$$
=\frac{9}{26}+\frac{1}{26}+\frac{16}{26}=\frac{26}{26}=1
$$

66. (A)
67. (A) $\mathrm{a}^{\mathrm{y}}=x+\sqrt{x^{2}+1} \Rightarrow \mathrm{a}^{-\mathrm{y}}=\frac{1}{x+\sqrt{x^{2}+1}}$

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=\frac{x-\sqrt{x^{2}+1}}{-1}
$$

$$
\therefore \quad \mathrm{a}^{\mathrm{y}}-\mathrm{a}^{-\mathrm{y}}=2 x \Rightarrow x=\frac{1}{2}\left(\mathrm{a}^{\mathrm{y}}-\mathrm{a}^{-\mathrm{y}}\right)=f^{-1}(\mathrm{y})
$$

68. (D)
69. (B) $\lim _{\theta \rightarrow \pi / 4} \frac{\sqrt{2}-\sqrt{2} \cos (\theta-\pi / 4)}{16(\theta-\pi / 4)^{2}}$

$$
=\lim _{y \rightarrow 0} \frac{\sqrt{2}}{16} \cdot \frac{(1-\cos y)}{y^{2}}
$$

where, $\mathrm{y}=\theta-\frac{\pi}{4} \rightarrow 0$ as $\theta \rightarrow \frac{\pi}{4}$

$$
\begin{aligned}
& \quad=\frac{1}{8 \sqrt{2}} \cdot \lim _{y \rightarrow 0} \frac{2 \sin ^{2}(y / 2)}{y^{2}} \\
& =\frac{1}{8 \sqrt{2}} \cdot \frac{1}{2}=\frac{1}{16 \sqrt{2}} \quad\left(\because \lim _{\theta \rightarrow 0} \sin \theta=0\right)
\end{aligned}
$$

70. (A) $\underset{x \rightarrow 0}{\operatorname{Lt}} \frac{a^{x}-b^{x}}{x}$

$$
\begin{aligned}
& =\lim _{x \rightarrow 0} \frac{a^{x}-b^{x}-1+1}{x} \\
& =\lim _{x \rightarrow 0} \frac{\left(a^{x}-1\right)-\left(b^{x}-1\right)}{x} \\
& =\lim _{x \rightarrow 0} \frac{a^{x}-1}{x}-\lim _{x \rightarrow 0} \frac{b^{x}-1}{x} \\
& =\log a-\log b=\log \frac{a}{b}
\end{aligned}
$$

71. (A) I. $\lim _{x \rightarrow 0} \frac{x^{2}}{x}=\lim _{x \rightarrow 0}(x)=0$
II. It is true that $\frac{x^{2}}{x}$ is not continuous at $x=0$
III. LHL $=\lim _{h \rightarrow 0} \frac{|0-h|}{(0-h)}$

$$
=\lim _{h \rightarrow 0} \frac{h}{-h}=-1
$$

RHL $=\lim _{h \rightarrow 0} \frac{|0+h|}{(0+h)}$

$$
=\lim _{h \rightarrow 0} \frac{h}{h}=1
$$

$\therefore$ LHL $\neq$ RHL
So, it does not exist.
72. (A) $f(x)=\log _{\mathrm{a}}\left(\log _{\mathrm{a}} x\right)=\log _{\mathrm{a}}\left(\frac{\log _{e} x}{\log _{e} a}\right)$

$$
=\log _{a}\left(\log _{e} x\right)-\log _{a}\left(\log _{e} a\right)
$$

$\Rightarrow f(x)=\frac{\log _{e}\left(\log _{e} x\right)}{\log _{e} a}-\log _{\mathrm{a}}\left(\log _{e} \mathrm{a}\right)$
$\Rightarrow f^{\prime}(x)=\frac{1}{\log _{e} a}\left(\frac{1}{\log _{e} x} \cdot \frac{1}{x}\right) \Rightarrow f^{\prime}(x)=\frac{\log _{a} e}{x \log _{e} x}$
73. (C) $\left(\mathrm{y}=\left(x+\sqrt{1+x^{2}}\right)^{\mathrm{n}}\right.$

On differentiating w.r.t.x, we get

$$
\begin{aligned}
& \frac{d y}{d x}=\mathrm{n}\left(x+\sqrt{1+x^{2}}\right)^{\mathrm{n}-1}\left(1+\frac{x}{\sqrt{x^{2}+1}}\right) \\
& =\frac{n\left[x+\sqrt{1+x^{2}}\right]^{n}}{\sqrt{1+x^{2}}}
\end{aligned}
$$

$$
\Rightarrow \frac{d y}{d x}=\frac{n y}{\sqrt{1+x^{2}}}
$$

74. (C) From solution 40,

$$
\begin{aligned}
& \frac{d y}{d x}=\frac{n y}{\sqrt{1+x^{2}}} \\
\Rightarrow & \left(\frac{d y}{d x}\right)^{2}\left(1+x^{2}\right)=\mathrm{n}^{2} \mathrm{y}^{2}
\end{aligned}
$$

Again, differentiating w.r.t. $x$ we get
$2 \frac{d y}{d x} \frac{d^{2} y}{d x^{2}}\left(1+x^{2}\right)+2 x\left(\frac{d y}{d x}\right)^{2}=2 \mathrm{n}^{2} \mathrm{y} \frac{d y}{d x}$

$$
\Rightarrow \frac{d^{2} y}{d x^{2}}\left(1+x^{2}\right)+x \frac{d y}{d x}=n^{2} y
$$

75. (D) Given that, $y=\cos t$ and $x=\sin t$

$$
\text { Then, } \frac{d y}{d t}=-\sin \mathrm{t} \text { and } \frac{d x}{d t} \cos \mathrm{t}
$$

Now, $\frac{d y}{d x}=\frac{d y / d t}{d x / d t}=-\frac{-\sin t}{\cos t}=\frac{-x}{y}$
76. (A) $\because x=\mathrm{k}(\theta+\sin \theta)$ and $\mathrm{y}=\mathrm{k}(1+\cos \theta)$
$\Rightarrow \frac{d x}{d \theta}=\mathrm{k}(1+\cos \theta)$ and $\frac{d y}{d \theta}=-\mathrm{k} \sin \theta$
$\therefore \frac{d y}{d x}=\frac{-k \sin \theta}{k(1+\cos \theta)}=\frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos ^{2} \frac{\theta}{2}}=-\tan \frac{\theta}{2}$
$\Rightarrow\left(\frac{d y}{d x}\right)_{\theta=\frac{\pi}{2}}=-\tan \frac{\pi}{4}=-1$

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77. (B)
78. (B) Given equation is $\mathrm{y}=2 x^{2}-x+1$.

On differentiating w.r.t. $x$, we get $\frac{d y}{d x}=4 x-1$
Since, tangent is parallel to the given line $y=3 x+9$
Slope of second line $=\frac{d y}{d x}=3$
Therefore, these slopes are equal.
$\Rightarrow 4 x-1=3 \Rightarrow x=1$
At $x=1, \mathrm{y}=2(1)^{2}-1+1 \Rightarrow \mathrm{y}=2$
Thus, the point is $(1,2)$
79. (C) Surface area of sphere, $\mathrm{S}=4 \pi \mathrm{r}^{2}$ and $\frac{d r}{d t}=2$
$\therefore \frac{d S}{d t}=4 \pi \times 2 \mathrm{r} \frac{d r}{d t}=8 \pi \mathrm{r} \times 2=16 \pi \mathrm{r}$
$\Rightarrow \frac{d S}{d t} \alpha \mathrm{r}$
80. (C) The given function is
$f(x)=x^{3}-1, \in[-1,1]$
$f^{\prime}(x)=3 \mathrm{x}^{2} \geq 0$
So, $f(x)$ is increasing function in $[-1,1]$
Also, $f(x)$ has no root between $(-1,1)$.
81. (A) Given curve is $\mathrm{y}=x^{2}-4 x+3$

Now, differential w.r.t. $x$, we get

$$
\begin{equation*}
\frac{d y}{d x}=2 x-4=2(x-2) \tag{i}
\end{equation*}
$$

Here, at $x=2, \frac{d y}{d x}=0$
i.e, for the given curve only one tangent is possible because slope of tangent parallel to $x$-axis is zero.
82. (B) Let $f(x)=2 x^{3}-3 x^{2}-12 x+5$

$$
f^{\prime}(x)=6 x^{2}-6 x-12
$$

For largest value, $f^{\prime}(x)=0$
$\Rightarrow 6 x^{2}-6 x-12=0$
$\Rightarrow x^{2}-x-2=0$
$\Rightarrow(x+1)(x-2)=0$
$\Rightarrow x=-1,2$
$f^{\prime \prime}(x)=12 x-6$
At $x=2, f^{\prime \prime}(2)=24-6=18>0$ (minimum)
At $x=-1, f^{\prime \prime}(-1)=12-6=-18<0$ (maximum)
So , the function is maximum (largest) at $x$ $=-1$ and its largest value is

$$
\begin{aligned}
& f(-1)=2(-1)^{3}-3(-1)^{2}-12(-1)+5 \\
& \quad=-2-3+12+5=12
\end{aligned}
$$

83. (C) $f(x)=x^{2}-2 x$

On differentiating w.r.t. $x$, we get
$f^{\prime}(x)=2 x-2$
For function to be increasing,
$f^{\prime}(x)=>0$
$\therefore 2 x-2>0 \Rightarrow x>1$
84. (A)
85. (D) Let $\mathrm{I}=\int e^{\log x} \mathrm{~d} x$

By logarithm property, $\mathrm{e}^{\log \mathrm{a}}=\mathrm{a}$
$\therefore \mathrm{I}=\int x d x=\left[\frac{x^{2}}{2}\right]+\mathrm{C}$
86. (C) Put $x \mathrm{e}^{x}=\mathrm{t}$
87. (C) Put $\log x=\mathrm{t} \Rightarrow \frac{1}{x} \mathrm{~d} x=\mathrm{dt}$
$\therefore \mathrm{I}=\int \frac{t e^{t} d t}{(1+t)^{2}}=\int \frac{e^{t}}{1+t} \mathrm{dt}-\int \frac{e^{t}}{(1+t)^{2}} \mathrm{dt}$
$=\frac{e^{t}}{1+t}-\int-e^{t} \frac{1}{(1+t)^{2}} \mathrm{dt}-\int \frac{e^{t}}{(1+t)^{2}} \mathrm{dt}$
$=\frac{x}{1+\log x}+\mathrm{C}$
88. (D) Given, $f^{\prime}(x)=g^{\prime}(x)$

On integrating both sides, we get

$$
f(x)=g(x)+\mathrm{c} \Rightarrow f(x)=x^{3}-4 x+6+\mathrm{C}
$$

$\because f(1)=2$
$\therefore 2=1-4+6+C \Rightarrow C=-1$
$\therefore f(x)=x^{3}-4 x+5$
89. (D) $\cos ^{2}(\pi+x)=\cos ^{2} x$
$\therefore \mathrm{I}_{1}=\int_{0}^{3 \pi} f\left(\cos ^{2} x\right) \mathrm{d} x=3 \int_{0}^{\pi} f\left(\cos ^{2} x\right) \mathrm{d} x=3 \mathrm{I}_{2}$
$\therefore \mathrm{I}_{1}=3 \mathrm{I}_{2}$
90. (A) Put $x=a \sin \theta$
$\Rightarrow \mathrm{d} x=\mathrm{a} \cos \theta \mathrm{d} \theta$ and adjust the limits
$\therefore \mathrm{I}=\int_{0}^{\pi / 2} \frac{\cos \theta d \theta}{\sin \theta+\cos \theta}=\frac{\pi}{4}$
91. (A)
92. (B)
93. (B)
94. (C) Line and the curve meet at $\mathrm{P}(\sqrt{3}, 1)$ in Ist quadrant.
Draw perpendicular PM.
$\therefore$ Area $=\Delta$ OPM $+\int_{\sqrt{3}}^{2} y d x$
Now, $x=2 \cos \theta, y=2 \sin \theta$, then the limits changes

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$=\frac{1}{2} \sqrt{3} \cdot 1+\int_{\pi / 6}^{0}(2 \sin \theta)(-2 \sin \theta) d \theta$
$=\frac{\sqrt{3}}{2}+4 \int_{0}^{\pi / 6} \frac{(1-\cos 2 \theta)}{2} d \theta$
$=\frac{\sqrt{3}}{2}+2\left[\theta-\frac{\sin 2 \theta}{2}\right]^{\pi / 6}$
$=\frac{\sqrt{3}}{2}+2\left[\frac{\pi}{6}-\frac{1}{2} \cdot \frac{\sqrt{3}}{2}\right]=\frac{\pi}{3}$
95. (B) Given equation of curves

$\mathrm{y}=\tan x$
andy $=0$ and $x=\frac{\pi}{4}$
$\therefore$ Required area $=\int_{0}^{\pi / 4} y d x$
$=\int_{0}^{\pi / 4} \tan x \mathrm{~d} x=[\log |\sec x|]_{0}^{\pi / 4}$
$=\log \left|\sec \frac{\pi}{4}\right|-\log |\sec \theta|=\log |\sqrt{2}|-\log |1|$
$=\log \sqrt{2}-0=\frac{1}{2} \log 2$ sq units
96. (C)


Required area $\left(\mathrm{OBAB}^{\prime} \mathrm{C}\right)=\int_{0}^{\pi} \sin x d x+$
$\int_{\pi}^{2 \pi}-\sin x d x$
$=[-\cos x]_{0}^{\pi}+[\cos x]_{\pi}^{2 \pi}$
$=-(\cos \pi-\cos 0)+(\cos 2 \pi-\cos \pi)$
$=-(-1-1)+(1+1)=4$ sq units
97. (C) The equations of curves are

$$
\begin{equation*}
y=x^{2} \tag{i}
\end{equation*}
$$

and $y=16$
On solving Eqs. (i) and (ii), we get $x=4,-4$
So, the points of intersection are $(4,16)$ and $(-4,16)$.


Required area $=\int_{-4}^{4}\left(16-x^{2}\right) d x$

$$
=2 \int_{0}^{4}\left(16-x^{2}\right) \mathrm{d} x
$$

$=2\left[16 x-\frac{x^{3}}{3}\right]_{0}^{4}=2\left[64-\frac{64}{3}\right]$
$=\frac{128 \times 2}{3}=\frac{256}{3}$ sq units

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98. (A) Putting, $\mathrm{y}=\mathrm{v} x$,

$$
\begin{aligned}
& \mathrm{v}+x \frac{d v}{d x}=1+\mathrm{v}+\mathrm{v}^{2} \\
\Rightarrow & \frac{d v}{1+v^{2}}=\frac{d x}{x} \Rightarrow \tan ^{-1} \mathrm{v}=\log x+\mathrm{C}
\end{aligned}
$$

$\therefore \tan ^{-1}\left(\frac{y}{x}\right)=\log x+C$
99. (D) Put $x+\mathrm{y}=\mathrm{v} \Rightarrow \frac{d v}{d x}-1=\frac{d y}{d x}$
$\therefore \frac{d v}{d x}=1+\sin \mathrm{v}+\cos \mathrm{v}$
$\Rightarrow \frac{d v}{2 \cos ^{2} \frac{v}{2}+2 \sin \frac{v}{2} \cdot \cos \frac{v}{2}}=\mathrm{d} x$
$\Rightarrow \frac{\frac{1}{2} \sec ^{2} \frac{v}{2}}{1+\tan \frac{v}{2}} \mathrm{dv}=\mathrm{d} x \Rightarrow \log \left(1+\tan \frac{x+y}{2}\right)=x+\mathrm{C}$
100. (B) We have $x^{2} y d y-(x d y-y d x)=0$
or it can be rewritten as $y d y-x d y+x^{2} y d y=0$
On dividing Eq. (i) by $x^{2}$, we get

$$
\frac{y d x-x d y}{x^{2}}+y d y=0
$$

$\Rightarrow-d\left(\frac{y}{x}\right)+y d y=0$
On integrating both sides, we get
$-\frac{y}{x}+\frac{y^{2}}{2}=C$
$x y^{2}-2 y=2 \mathrm{C} x$
101. (A) Given differential equation is

$$
x \frac{d y}{d x}+\mathrm{y}=0 \Rightarrow x \frac{d y}{d x}-\mathrm{y}
$$

$$
\Rightarrow \quad-\frac{d y}{y}=\frac{d x}{x} \Rightarrow \int \frac{d x}{x}+\int \frac{d y}{y}=0
$$

On integrating both sides, we get
$\log x+\log y=\log C$
$\Rightarrow \log (x y)=\log C \Rightarrow x y=C$
Alternate Method

$$
\begin{array}{ll} 
& \frac{x d y}{d x}+\mathrm{y}=0 \quad \Rightarrow x \mathrm{dy}+\mathrm{y} \mathrm{~d} x=0 \\
\Rightarrow \quad & \mathrm{~d}(x \mathrm{y})=0 \\
\therefore \quad & x \mathrm{y}=\mathrm{C}
\end{array}
$$

102. (C) Given, $x^{2} \mathrm{dy}+\mathrm{y}^{2} \mathrm{~d} x=0 \Rightarrow \frac{d y}{y^{2}}+\frac{d x}{x^{2}}=0$

On integrating, we get

$$
\int y^{-2} \mathrm{dy}+\int x^{-2} \mathrm{~d} x=0
$$

$\Rightarrow \frac{y^{-2+1}}{-2+1}+\frac{x^{-2+1}}{-2+1}=-\mathrm{C}_{1}$
$\Rightarrow \frac{y^{-1}}{-1}+\frac{x^{-1}}{-1}=\mathrm{C}_{1} \Rightarrow \frac{-1}{y}-\frac{1}{x}=\mathrm{C}_{1}$
$\Rightarrow \frac{1}{x}+\frac{1}{y}=\mathrm{C}_{1} \Rightarrow \mathrm{x}+\mathrm{y}=\mathrm{C}_{1} x \mathrm{y}$
$\Rightarrow \frac{1}{C_{1}}(x+y)=x y$
$\therefore \mathrm{C}(x+y)=x y$, where $\frac{1}{C_{1}}=\mathrm{C}$
103. (D) Given equation can be rewritten as

$$
\begin{gathered}
\frac{x d y-y d x}{y^{2}}=x \mathrm{~d} x \\
\Rightarrow \quad-\mathrm{d}\left(\frac{x}{y}\right)=x \mathrm{~d} x
\end{gathered}
$$

On inegrating both sides, we get

$$
-\frac{x}{y}=\frac{x^{2}}{2}-\frac{C}{2}
$$

$\therefore \quad x^{2}+2 x y^{-1}=C$
104. (D) Given, A $(2,3), \mathrm{B}(5,6), \mathrm{C}(8, \lambda),, \mathrm{O}(1,1)$
$\therefore \quad \mathrm{AB}=\mathrm{OB}-\mathrm{OA}=(5,6)-(2,3)=(3,3)$
Similarly, BC $=3, \lambda-6$
Since, A, B and C are collinear,

$$
\mathrm{AB}=\mathrm{p} \mathrm{BC}
$$

$\therefore \quad(3 i+3 j)=p[3 i+(\lambda-6) j]$
On comparing both sides, we get

$$
3=3 \mathrm{p}
$$

$\therefore \mathrm{p}=1$ and $3=\mathrm{p}(\lambda-6)=\lambda-6$
$\therefore \lambda=9$
105. (A) a is perpendicular to both b and c and hence it is parallel to $\mathrm{b} \times \mathrm{c}$.
$\therefore \quad a=t(b \times c)$
On squaring both sides, we get all are unit vectors.
$1=\mathrm{t}^{2}\left(1 \cdot 1 \sin 30^{\circ}\right)^{2} \cdot 1=\mathrm{t}^{2} \cdot \frac{1}{4}$
$\therefore \mathrm{t}= \pm 2$
106. (C) Given that, $|\mathrm{a}|=|\mathrm{b}|$
(a) If $(a+b)$ is prallel to $(a-b)$

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Then, $(a+b) \times(a-b)$ should be equal to zero.
$\therefore(a+b) \times(a-b)=a \times a+b \times a-a \times b-b \times$ $b=0-a \times b-a \times b-0$
$=-2 a \times b \neq 0$
(b) $(\mathrm{a}+\mathrm{b}) \cdot(\mathrm{a}-\mathrm{b})=\mathrm{a} \cdot \mathrm{a}+\mathrm{b} \cdot \mathrm{a}-\mathrm{a} \cdot \mathrm{b}-\mathrm{b} \cdot \mathrm{b}$ $=1+\mathrm{a} \cdot \mathrm{b}-\mathrm{a} \cdot \mathrm{b}-1=0 \neq 1$
i.e., $(a+b)$ is perpendicular to $(a-b)$.
107. (C) Arithmetic mean of the squares of the first ' n ' natural numbers
$=\frac{1^{2}+2^{2}+3^{2}+\ldots .+n^{2}}{n}$
$=\frac{n(n+1)(2 n+1)}{6 \times n}=\frac{(n+1)(2 n+1)}{6}$
108. (D) According to question,

$$
\begin{aligned}
& \sum_{i=1}^{20}\left(x_{i}-30\right)=2 \\
\Rightarrow & \sum_{i=1}^{20} x_{i}-600=2 \Rightarrow \sum_{i=1}^{20} x_{i}=602 \\
\therefore & \text { Mean }=\frac{\sum_{i=1}^{20} x_{i}}{20}=\frac{602}{20}=30.1
\end{aligned}
$$

109. (C) Let, A, B, C be the section of class having 30, 30 and 40 students respectively.
Also given, the students of each section securing the Arithmetic means of the marks $72.2,69.0$ and 64.1 respectively.
Now, the total marks secured by the students of section $A=30 \times 72.2=2166$
The total marks secured by the students of sections B
$=30 \times 69=2070$
and the total marks secured by the students of sections $C=40 \times 64.1=2564$
So, the arithmetic mean of marks of all the students of three sections
$=\frac{2166+2070+2564}{100}=\frac{6800}{100}=68$
110. (D) Since, lines of regression passes though $(\bar{x}, \bar{y})$.
$\therefore 3 \bar{x}+\bar{y}-12=0$
and $\bar{x}+2 \bar{y}-14=0$
On solving Eqs. (i) and (ii), we get

$$
\bar{x}=2 \text { and } \bar{y}=6
$$

111. (B) $\because \mathrm{P}(\mathrm{A})=0.6, \mathrm{P}(\mathrm{B})=0.7$

Here, A and B are independent events.
$\therefore \mathrm{P}(\mathrm{A} \cap \mathrm{B})=\mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B})$
$=0.6 \times 0.7=0.42$
$\mathrm{P}(\bar{A} \cap \bar{B})=\mathrm{P}(\bar{A}) \times \mathrm{P}(\bar{B})$

$$
=0.4 \times 0.3=0.12
$$

Since, probability that A and B describe single event.
Probability that both speak truth or false

$$
\begin{aligned}
& =\mathrm{P}(\mathrm{~A} \cap \mathrm{~B})+\mathrm{P}(\bar{A} \cap \bar{B}) \\
& =0.42+0.12=0.54
\end{aligned}
$$

112. (B) Given that, in a binomial distribution, the occurrence and the non-occurrence of an event are equally likely.
i.e, $p=q=\frac{1}{2}$
and mean of Binomial distribution $=n p=6$
$\Rightarrow \mathrm{n} \times \frac{1}{2}=6 \Rightarrow \mathrm{n}=12$
So, the required number of trials is 12 .
113. (D) Probability of getting head in a single toss,
$\mathrm{P}(\mathrm{H})=\frac{1}{2}$
Probability of getting tail in a single toss,
$\mathrm{P}(\mathrm{T})=\frac{1}{2}$
$\therefore$ Required probability $=\mathrm{P}(\mathrm{H} H H H T$ or TTTTH $)$
$=\mathrm{P}($ HHHHT $)+\mathrm{P}($ TTTTH $)$
$=P(H) \cdot P(H) \cdot P(H) \cdot P(H) \cdot P(T)$
$+\cdot \mathrm{P}(\mathrm{T}) \cdot \mathrm{P}(\mathrm{T}) \cdot \mathrm{P}(\mathrm{T}) \cdot \mathrm{P}(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H})$
$=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
$=2 \times \frac{1}{32}=\frac{1}{16}$
114. (B) Favourable numbers $=[222,444,666$, 888]
Total digit numbers $=4 \times 5 \times 5$
$\therefore$ Required probability $=\frac{4}{4 \times 25}=\frac{1}{25}$
115. (D) Given $\mathrm{p}, \mathrm{q}, \mathrm{r} \in \mathrm{z}^{+}$
and $\omega$ is the cube root of unity.
Then, $f(x)=x^{3 \mathrm{p}}+x^{3 \mathrm{q}+1}+x^{3 \mathrm{r}+2}$

$$
\begin{gathered}
\Rightarrow f(\omega)=\omega^{3 \mathrm{p}}+\omega^{3 \mathrm{q}+1}+\omega^{3 \mathrm{r}+2}\left\{\begin{array}{c}
\omega^{3}=1 \\
\text { and } 1+\omega+w^{2}=0
\end{array}\right\} \\
=(1)^{\mathrm{p}}+(1)^{\mathrm{q}} \cdot \omega+(1)^{\mathrm{r}} \cdot \omega^{2}=1+\omega+\omega^{2}=0
\end{gathered}
$$

116. (C) A.

$$
\begin{aligned}
& {\left[\frac{-1+\sqrt{-3}}{29}\right]^{29} \therefore+\left[\frac{-1-\sqrt{-3}}{29}\right]^{29}} \\
& =\left[\frac{-1+\sqrt{3} i}{2}\right]^{29}+\left[\frac{-1-\sqrt{3} i}{2}\right]^{29}
\end{aligned}
$$

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$$
=\omega^{2}+\omega=-1
$$

R. $\omega^{2} \neq-1$

Therefore, A is true but R is false.
117. (D) $A . M=\left[\begin{array}{cc}5 & 10 \\ 4 & 8\end{array}\right]$
$|\mathrm{M}|=\left[\begin{array}{cc}5 & 10 \\ 4 & 8\end{array}\right]=40-40=0$
So, M is not invertible.
R. $M$ is singular matrix.

Therefore, is $A$ is false and $R$ is true.
118. (A) A. $\int \frac{e^{x}}{x}(1+x \log x) \mathrm{d} x=$

$$
\int \frac{e^{x}}{x} d x \int e^{x} \log x d x
$$

$=\mathrm{e}^{x} \log x-\int e^{x} \log x d x+\int e^{x} \log x d x$
$=\mathrm{e}^{x} \log x+\mathrm{C}$
$\int e^{x}\left[f(x)+f^{\prime}(x)\right] \mathrm{d} x=\int e^{x} f(x) d x+\int e^{x} f(x) d x$ $=\mathrm{e}^{x} f(x)+\mathrm{C}$
Therefore, both $A$ and $R$ are true but $R$ is the correct explanation of $A$.
119. (D) $\frac{d y}{d x}=5 x^{2}(x-1)(x-3)=0$
$\Rightarrow x=0,1,3$

$$
\frac{d^{2} y}{d x^{2}}=10 x\left(2 x^{2}-6 x+3\right)
$$

At $x=1, \frac{d^{2} y}{d x^{2}}=<0$, maxima
At $x=3, \frac{d^{2} y}{d x^{2}}=>0$, minima
At $x=0, \frac{d^{2} y}{d x^{2}}=0$ and $\frac{d^{3} y}{d x^{3}} \neq 0$
Neither maxima nor minima. 120. (D) In DACD,

$C D+D A=C A$
Now, in $\triangle \mathrm{ABC}$,

$$
\begin{equation*}
\mathrm{CA}+\mathrm{AB}=\mathrm{CB} \tag{i}
\end{equation*}
$$

From Eqs. (i) and (ii),
$C D+D A+A B=C B$
$\Rightarrow \mathrm{CB}+\mathrm{CD}+\mathrm{DA}+\mathrm{AB}=2 \mathrm{CB}$

## NDA (MATHS) MOCK TEST - 41 (Answer Key)

| 1. $(\mathrm{A})$ | 21. (B) | 41. (C) | 61. (A) | 81. (A) | 101. (A) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2. (C) | 22. (A) | 42. (A) | 62. (B) | 82. (B) | 102. (C) |
| 3. (A) | 23. (B) | 43. (D) | 63. (A) | 83. (C) | 103. (D) |
| 4. (A) | 24. (C) | 44. (A) | 64. (C) | 84. (A) | 104. (D) |
| 5. (B) | 25. (B) | 45. (A) | 65. (B) | 85. (D) | 105. (A) |
| 6. (B) | 26. (C) | 46. (D) | 66. (A) | 86. (C) | 106. (C) |
| 7. (D) | 27. (C) | 47. (C) | 67. (A) | 87. (C) | 107. (C) |
| 8. (B) | 28. (C) | 48. (D) | 68. (D) | 88. (D) | 108. (D) |
| 9. (A) | 29. (A) | 49. (C) | 69. (B) | 89. (D) | 109. (C) |
| 10. (A) | 30. (A) | 50. (A) | 70. (A) | 90. (A) | 110. (D) |
| 11. (C) | 31. (C) | 51. (A) | 71. (A) | 91. (A) | 111. (B) |
| 12. (A) | 32. (B) | 52. (B) | 72. (A) | 92. (B) | 112. (B) |
| 13. (B) | 33. (D) | 53. (D) | 73. (C) | 93. (B) | 113. (D) |
| 14. (A) | 34. (B) | 54. (D) | 74. (C) | 94. (C) | 114. (B) |
| 15. (A) | 35. (A) | 55. (B) | 75. (D) | 95. (B) | 115. (D) |
| 16. (D) | 36. (A) | 56. (C) | 76. (A) | 96.(C) | 116. (C) |
| 17. (C) | 37. (C) | 57. (C) | 77. (B) | 97. (C) | 117. (D) |
| 18. (C) | 38. (B) | 58. (B) | 78. (B) | 98. (A) | 118. (A) |
| 19. (A) | 39. (D) | 59. (A) | 79. (C) | 99. (D) | 119. (D) |
| 20. (D) | 40. (C) | 60. (B) | 80. (C) | 100. (B) | 120. (D) |

Note:- If you face any problem regarding result or marks scored, please contact 9313111777

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

