NDA MATHS MOCK TEST - 198 (SOLUTION)

1. (B) tanA + 2tan2A + 4 tan4A + 8 cot8A
$\Rightarrow$ cotA – (cotA– tanA) + 2 tan2A + 4 tan4A + 8 cot8A
We know that
cotA – tanA = 2 cot2A
$\Rightarrow$ cotA – 2 cot2A + 2 tan2A + 4 tan4A + 8 cot8A
$\Rightarrow$ cotA – 2(cot2A – tan2A)+ 4tan4A + 8cot8A
$\Rightarrow$ cotA – 2×2 cot4A + 4 tan4A + 8 cot8A
$\Rightarrow$ cotA – 4(cot4A – tan4A) + 8 tan8A
$\Rightarrow$ cotA – 4×2cot8A + 8 cot8A = cotA

2. (B) When $\theta = 180°$
$M = \frac{60}{11}(H \pm 6)$ where $+H < 6$
$-H > 6$
H = 7(between 7 and 8 o'clock)
$M = \frac{60}{11}(7-6)$
$M = \frac{60}{11}\times 1 = 6\frac{5}{11}$
Hence time = $7 : 5\frac{5}{11}$

3. (C) $\sin\frac{\pi}{12} < \tan\frac{\pi}{12} < \cos\frac{\pi}{12}$

4. (B) 1 1 0 1 1 . 0 1
$1\times 2^0 = 1$
$1\times 2^1 = 2$
$0\times 2^2 = 0$
$1\times 2^3 = 8$
$1\times 2^4 = 16$
27
$0 = 0\times 2^{-1}$
$\frac{1}{4} = 1\times 2^{-2}$
$\frac{1}{4} = 0.25$
Hence $(11011.01)_2 = (27.25)_{10}$

5. (B) The required Probability $= \frac{^6C_3 \times {^4C_1}}{^{10}C_4}$
$= \frac{20\times 4}{10\times 3\times 7} = \frac{8}{21}$

6. (D) Let $\vec{a} = -\lambda\hat{i} + 2\hat{j} + (1-3\lambda)\hat{k}$ and
$\vec{b} = 6\hat{i} + -\lambda\hat{j} - 4\hat{k}$
$\vec{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}.\vec{b} = 0$
$\Rightarrow -\lambda \times 6 + 2(-\lambda) + (1-3\lambda)\times(-4) = 0$
$\Rightarrow -6\lambda - 2\lambda - 4 + 12\lambda = 0$
$\Rightarrow 4\lambda = 4 = \lambda = 1$

7. (B) Let $\vec{a} = \frac{1}{2}\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \frac{1}{2}\hat{j} + \hat{k}$
$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$
$\cos\theta = \frac{\frac{1}{2}\times 1 + 1\times\frac{1}{2} + 1\times 1}{\sqrt{\left(\frac{1}{2}\right)^2 + 1^2 + 1^2}\sqrt{1^2 + \left(\frac{1}{2}\right)^2 + 1^2}}$
$\cos\theta = \frac{2\times 4}{9} \Rightarrow \theta = \cos^{-1}\left(\frac{8}{9}\right)$

8. (D) Equation $ax^2 + bx + c = 0$
$\alpha + \beta = \frac{-b}{a}$ ...(i)
$px^2 + qx + r = 0$
Now, $\alpha - h + \beta - 'h = \frac{-q}{p}$
$\Rightarrow \frac{-b}{a} - 2h = \frac{-q}{p} \Rightarrow 2h = \frac{-b}{a} + \frac{q}{p}$
$\Rightarrow h = \frac{1}{2}\left[\frac{q}{p} - \frac{b}{a}\right]$

9. (B)
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       1001
101) 101111 (
      101
      ----
        111
        101
        ---
         10
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Quotient = $(1001)_2$ and Remainder = $(10)_2$

10. (B) $x = 1 + \left(\frac{y}{5}\right) + \left(\frac{y}{5}\right)^2 + \left(\frac{y}{5}\right)^3 +$,.... where $|y| < 5$
$\Rightarrow x = \frac{1}{1-\frac{y}{5}} \Rightarrow x = \frac{5}{5-y}$
$\Rightarrow 5x - xy = 5 \Rightarrow y = \frac{5x-5}{x}$

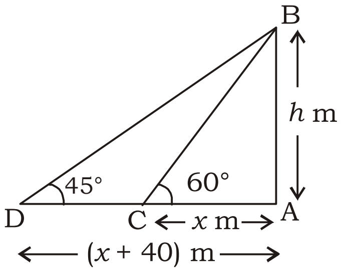
11. (B) sin(–1140) = – sin(1140)
$\Rightarrow$ sin(–1140) = – sin(3×360 + 60)
$\Rightarrow$ sin(–1140) = – sin60 = $\frac{-\sqrt{3}}{2}$

12. (A) A' = cofactor of A
$|A'| = |\text{co-factor of } A|$
$|A'| = (A)^{3-1}$ $[\because \text{order} = 3]$
$|A'| = A^2$

13. (C) $\lim_{x\to 0} \frac{\sin x + \tan x}{x}$ $\left[\frac{0}{0}\right]$ form
by L-Hospital's Rule
$\Rightarrow \lim_{x\to 0} \frac{\cos x + \sec^2 x}{1}$
$\Rightarrow \cos 0 + \sec^2 0$
$\Rightarrow 1 + 1 = 2$

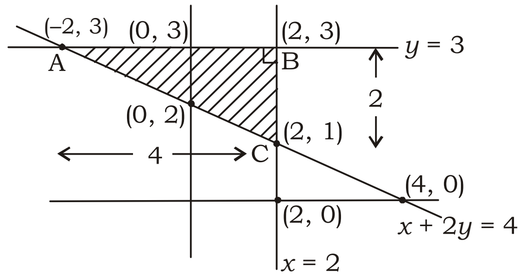
14. (C) $AA^T = I$
$\Rightarrow |AA^T| = 1$
$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

15. (B)



Let height of the house (AB) = h m
AC = x m
A.T.Q,
AD = $(x + 40)$ m
In ΔABC:-
$\tan 60° = \frac{AB}{AC}$
$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$
In ΔABD:-
$\tan 45° = \frac{AB}{AD}$
$\Rightarrow 1 = \frac{h}{x+40} \Rightarrow x + 40 = h$
$\Rightarrow \frac{h}{\sqrt{3}} + 40 = h \Rightarrow h\left(1 - \frac{1}{\sqrt{3}}\right) = 40$
$\Rightarrow h = \frac{40\sqrt{3}}{\sqrt{3}-1} \Rightarrow h = 20(3 + \sqrt{3})$ m
Height of the house = $20(3 + \sqrt{3})$ m

16. (A)

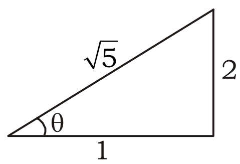


lines $x = 2$, $y = 3$ and $x + 2y = 4$
A.T.Q,
AB = 4, BC = 2
Area of ΔABC = $\frac{1}{2} \times AB \times BC$
$= \frac{1}{2} \times 4 \times 2 = 4$ sq.unit

17. (B) $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
$A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$
$A^2 = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$
Similarly $A^3 = \begin{bmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{bmatrix}$
and $A^4 = \begin{bmatrix} 2^4 & 2^4 \\ 2^4 & 2^4 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$

18. (C) $a^{1/3} - \frac{1}{a^{1/3}} = 4$
$\Rightarrow \left(a^{1/3} - \frac{1}{a^{1/3}}\right)^3 = 4^3$
$\Rightarrow a - \frac{1}{a} - 3 \times a = 64$
$\Rightarrow a - \frac{1}{a} = 64 + 12 = 76$

19. (D)



$\sin^{-1}\left(\frac{2}{\sqrt{5}}\right) = \theta$
$\Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$
Now, $\sec\theta = \sqrt{5} \Rightarrow \sec^{-1}(\sqrt{5}) = \theta$

20. (A) $I = \int_{-1}^{1} x^2 |x| dx$
$I = \int_{-1}^{0} x^2 |x| dx + \int_{0}^{1} x^2 |x| dx$
$I = -\int_{-1}^{0} x^3 dx + \int_{0}^{1} x^3 dx$
$I = -\left[\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$
$I = \frac{-1}{4}[0 - (-1)^4] + \frac{1}{4}[1^4 - 0]$
$I = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

21. (B) The no. of ways = $^{15-1}C_{11-1}$
$= {}^{14}C_{10} = 1001$

22. (C) $n = 25$
No. of diagonals = $\frac{n(n-3)}{2}$
$= \frac{25 \times 23}{2} = 275$

23. (B) zero

24. (C) Let $a - ib = \sqrt{5-12i}$
On squaring
$\Rightarrow (a^2 - b^2) - (2ab)i = 5 - 12i$
On comparing
$a^2 - b^2 = 5$ and $2ab = 12$ ...(i)
Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
$\Rightarrow (a^2 + b^2)^2 = 5^2 + (12)^2$
$\Rightarrow (a^2 + b^2)^2 = 13 \Rightarrow a^2 + b^2 = 13$ ...(iii)
On solving eq(i) and eq(ii)
$2a^2 = 18$ and $2b^2 = 8$
$\Rightarrow a = \pm 3 \quad b = \pm 2$
Hence $\sqrt{5-12i} = \pm(3 - 2i)$

25. (D) Given that P(A) = $\frac{2}{5}$, P(B) = $\frac{1}{2}$ and
$P\left(\frac{A}{B}\right) = \frac{3}{4}$
Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$
$\Rightarrow \frac{3}{4} = \frac{P(A \cap B)}{1/2}$
$\Rightarrow P(A \cap B) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$
$\Rightarrow P\left(\frac{B}{A}\right) = \frac{3/8}{2/5} = \frac{15}{16}$

26. (B) $(A \cap B) \cup (B \cap C)$

27. (C) $S_n = n^2 + n - 7$
$S_{n-1} = (n-1)^2 + (n-1) - 7$
$S_{n-1} = n^2 - n - 7$
Now, $T_n = S_n - S_{n-1}$
$\Rightarrow T_n = (n^2 + n - 7) - (n^2 - n - 7)$
$\Rightarrow T_n = 2n$
$\Rightarrow T_{21} = 2 \times 21 = 42$

28. (C) $\log_{10}\left(\frac{3}{4}\right) - \log_{10}\left(\frac{81}{32}\right) + \log_{10}\left(\frac{27}{8}\right)$
$\Rightarrow \log_{10}\left(\frac{\frac{3}{4} \times \frac{27}{8}}{\frac{81}{32}}\right)$
$\Rightarrow \log_{10}\left(\frac{81}{32} \times \frac{32}{81}\right) = \log_{10} 1 = 0$

29. (C)

30. (D) **Statement I**
We know that
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
Statement I is incorrect.
Statement II
We know that
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow \frac{\cos 2\alpha + 1}{2} + \frac{\cos 2\beta + 1}{2} + \frac{\cos 2\gamma + 1}{2} = 1$
$\Rightarrow \cos 2\alpha + 1 + \cos 2\beta + 1 + \cos 2\gamma + 1 = 2$
$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
Statement II is incorrect.

31. (D) The required no. of ways = $^6C_3 \times {}^{10}C_8$

32. (C) Given that $X = \{9(n-1) : n \in N\}$
$n = 1, 2, 3, 4, \ldots$
$X = \{0, 9, 18, 27, \ldots\}$
$Y = \{4^n - 3n - 1 : n \in N\}$
$n = 1, 2, 3, 4, \ldots$
$Y = \{0, 9, 54, 243, \ldots\}$
$(X \cap Y) = \{0, 9, 54, 243\} = Y$

33. (C) Differential equation
$x\,dy - y\,dx = x^2 y\,dx$
$\Rightarrow \frac{x\,dy - y\,dx}{xy} = x\,dx$
$\Rightarrow \frac{dy}{y} - \frac{dx}{y} = x\,dx$
On integrating
$\Rightarrow \log y - \log x = \frac{x^2}{2} + c$
$\Rightarrow \log\frac{y}{x} = \frac{x^2}{2} + c$

34. (B) The required no. of triangles = $^{11}C_3 - {}^4C_3$
$= 165 - 4 = 161$

35. (B) $\sin^{-1}(\log_3 2x)$
Here $-1 \le \log_3 2x \le 1 \Rightarrow 3^{-1} \le 2x \le 3^1$
$\Rightarrow \frac{1}{3} \le 2x \le 3 \Rightarrow \frac{1}{6} \le x \le \frac{3}{2}$
Domain = $\left[\frac{1}{6}, \frac{3}{2}\right]$

36. (C) Series $\frac{1^2}{2}+\frac{1^2+2^2}{2+4}+\frac{1^2+2^2+3^2}{2+4+6}+.......$

$T_n = \frac{1^2+2^2+3^2+.....+n^2}{2+4+6+.....+n}$

$T_n = \frac{1^2+2^2+3^2+.....+n^2}{2(1+2+3+.....+n)}$

$T_n = \frac{\frac{n}{6}(n+1)(2n+1)}{2\times\frac{n(n+1)}{2}} = \frac{2n+1}{6}$

37. (A) $f(x) = x^2 + 5x - 6$

$f'(x) = 2x + 5 \Rightarrow f'(c) = 2c + 5$

$a = -1,\ b = \frac{1}{2}$

$f(a) \Rightarrow f(-1) = (-1)^2 + 5(-1) - 6 = -10$

$f(b) \Rightarrow f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 5\times\frac{1}{2} - 6 = \frac{-13}{4}$

Now, $f'(c) = \frac{f(b)-f(a)}{b-a}$

$\Rightarrow 2c + 5 = \frac{\frac{-13}{4}+10}{\frac{1}{2}+1} \Rightarrow 2c + 5 = \frac{9}{2}$

$\Rightarrow 4c + 10 = 9 \Rightarrow c = \frac{-1}{4}$

38. (B) $0.1\overline{37} = \frac{137-1}{990} = \frac{136}{990} = \frac{68}{495}$

39. (B) Given that $f(x) = 3x + 7$

x	$f(x)$
1	10
2	13
⋮	⋮

so on

So function is injective but not surjective.

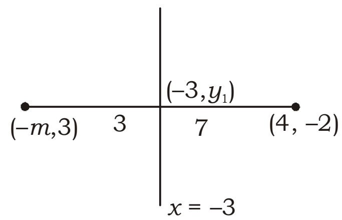
40. (C)

41. (A) A = {1, 2, 4, 5, 7, 8, 11,13} ; $n = 8$

Number of proper subsets = $2^n - 1$

$= 2^8 - 1 = 255$

42. (B)



Now, $\frac{3\times4+7\times(-m)}{3+7} = -3$

$\Rightarrow \frac{12-7m}{10} = -3 \Rightarrow 12-7m = -30 \Rightarrow m = 6$

43. (C) $I = \int \frac{\ln x}{x}\, dx$

$I = \ln x\int\frac{1}{x}\,dx - \int\left\{\frac{d}{dx}(\ln x).\int\frac{1}{x}\,dx\right\}dx$

$I = (\ln x)(\ln x) - \int\frac{1}{x}.\ln x\,dx + 2c$

$I = (\ln x)^2 - I + 2c$

$2I = (\ln x)^2 + 2c \Rightarrow I = \frac{(\ln x)^2}{2} + c$

44. (B) In the expansion $\left(3\sqrt{x}+\frac{1}{6x}\right)^7$

Middle terms = $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2}+1\right)^{th}$

$= 4^{th}$ and 5^{th}

$T_4 = T_{3+1} = {}^7C_3\,(3\sqrt{x})^4\left(\frac{1}{6x}\right)^3$

$= 35\times\frac{3^4}{6^3}x^{-1} = \frac{105}{8}\ x^{-1}$

$T_5 = T_{4+1} = {}^7C_4\,(3\sqrt{x})^3\left(\frac{1}{6x}\right)^4$

$= 35\times\frac{3^3}{6^4}x^{-5/2} = \frac{35}{48}x^{-5/2}$

The required sum = $\frac{105}{8} + \frac{35}{48}$

$= \frac{630+35}{48} = \frac{665}{48}$

45. (D) equation whose roots are 4 and – 6

$(x-4)(x+6) = 0$

$\Rightarrow x^2 + 2x - 24 = 0$

Original equation

$x^2 - 2x - 24 = 0$

$\Rightarrow x^2 - 6x + 4x - 24 = 0$

$\Rightarrow (x-6)(x+4) = 0$

Roots of originalar equation = 6, – 4

46. (C) Line $(3x - 4y + 6) + \lambda(x + 2y + 1) = 0$

$(3+\lambda)x + (-4+2\lambda)y + 6 + \lambda = 0$

$y = -\left(\frac{3+\lambda}{-4+2\lambda}\right)x - \frac{6+\lambda}{-4+2\lambda}$

it is parallel to x-axis i.e.

m = 0

$\Rightarrow -\left(\frac{3+\lambda}{-4+2\lambda}\right) = 0$

$\Rightarrow \lambda + 3 = 0 \Rightarrow \lambda = -3$

47. (B) $A_1 = \int_0^{\pi/4} \cos x \ dx \Rightarrow A_1 = [\sin x]_0^{\pi/4}$

$\Rightarrow A_1 = \sin\frac{\pi}{4} - \sin 0 \Rightarrow A_1 = \frac{1}{\sqrt{2}}$

and $A_2 = \int_0^{\pi/4} \sin 2x \ dx \Rightarrow A_2 = -\left[\frac{\cos 2x}{2}\right]_0^{\pi/4}$

$\Rightarrow A_2 = -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right]$

$\Rightarrow A_2 = -\frac{1}{2}[0-1] = \frac{1}{2}$

Now, $\frac{A_1}{A_2} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} \Rightarrow \frac{A_1}{A_2} = \frac{\sqrt{2}}{1}$

Hence $A_1 : A_2 = \sqrt{2} : 1$

48. (C) (a, b), $(c - d)$ and $(a - c, b - d)$ are collinear,

then $\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$

$\Rightarrow a(d - b + d) - b(c - a + c) + 1(bc - cd - ad + cd) = 0$

$\Rightarrow ad - ab + ad - bc + ab - bc + bc - ad = 0$

$\Rightarrow ad - bc = 0 \Rightarrow ad = bc$

49. (A) $\frac{1 - \cos\alpha - \sin\alpha}{2\cos\alpha} = y$

$\Rightarrow \frac{1 - \cos\alpha - \sin\alpha}{2\cos\alpha} \times \frac{1 + \cos\alpha + \sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$

$\Rightarrow \frac{1^2 - (\cos\alpha + \sin\alpha)^2}{2\cos\alpha(1 + \cos\alpha + \sin\alpha)} = y$

$\Rightarrow \frac{1 - (\cos^2\alpha + \sin^2\alpha + 2\sin\alpha.\cos\alpha)}{2\cos\alpha(1 + \cos\alpha + \sin\alpha)} = y$

$\Rightarrow \frac{1 - (1 + 2\sin\alpha.\cos\alpha)}{2\cos\alpha(1 + \cos\alpha + \sin\alpha)} = y$

$\Rightarrow \frac{1 - 1 - 2\sin\alpha.\cos\alpha}{2\cos\alpha(1 + \cos\alpha + \sin\alpha)} = y$

$\Rightarrow \frac{-2\sin\alpha.\cos\alpha}{2\cos\alpha(1 + \cos\alpha + \sin\alpha)} = y$

$\Rightarrow \frac{-\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$

$\Rightarrow \frac{\sin\alpha}{1 + \cos\alpha + \sin\alpha} = -y$

50. (B) Given that

$\sin A = k.\sin B \Rightarrow \frac{\sin A}{\sin B} = \frac{k}{1}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{k+1}{k-1}$

$\Rightarrow \frac{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}.\sin\frac{A-B}{2}} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} = \frac{k-1}{k+1}$

51. (A) cos80° + cos40° – cos20°

$\Rightarrow 2\cos\frac{80+40}{2}.\cos\frac{80-40}{2} - \cos 20$

$\Rightarrow$ 2cos60.cos20 – cos20

$\Rightarrow 2 \times \frac{1}{2}\cos 20 - \cos 20$

$\Rightarrow$ cos20 – cos20 = 0

52. (B) $7x^2 + y^2 = k(x^2 - y^2 - 4x + 3y)$

$(7 - k)x^2 + (1 + k)y^2 + 4kx - 3ky = 0$ is a circle, then

Coefficient of x^2 = coefficient of y^2

$\Rightarrow 7 - k = 1 + k$

$\Rightarrow 6 = 2k \Rightarrow k = 3$

53. (C) $\begin{vmatrix} 2\cos^2\frac{\alpha}{2} & \sin\alpha & 1 \\ 2\cos^2\frac{\beta}{2} & \sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}$

$\Rightarrow 2\cos^2\frac{\alpha}{2}(\sin\beta - 0) - \sin\alpha\left(2\cos^2\frac{\beta}{2} - 1\right)$
$+ 1(0 - \sin\beta)$

$\Rightarrow 2\cos^2\frac{\alpha}{2}.\sin\beta - \sin\alpha.\cos\beta - \sin\beta$

$\Rightarrow 2\cos^2\frac{\alpha}{2}.\sin\beta - \sin\beta - \sin\alpha.\cos\beta$

$\Rightarrow \sin\beta\left(2\cos^2\frac{\alpha}{2} - 1\right) - \sin\alpha.\cos\beta$

$\Rightarrow \sin\beta.\cos\alpha - \sin\alpha.\cos\beta$

$\Rightarrow \sin(\beta - \alpha)$

54. (C) $\int_{-\pi/2}^{\pi/2} \frac{\sin x}{x^4} dx = 0$ [∵ function is odd.]

55. (C) A.T.Q.

$\frac{2b^2}{a} = \frac{1}{2} \times 2b \Rightarrow \frac{2b^2}{a} = b \Rightarrow 2b = a$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 - \frac{b^2}{4b^2}}$

$\Rightarrow e = \sqrt{1 - \frac{1}{4}} \Rightarrow e = \sqrt{\frac{3}{4}} \Rightarrow e = \frac{\sqrt{3}}{2}$

56. (B) $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Now, $(\vec{a} - 2\vec{b}) \times (2\vec{a} + \vec{b})$

$\Rightarrow 2(\vec{a} \times \vec{a}) - 4(\vec{b} \times \vec{a}) + (\vec{a} \times \vec{b}) - 2(\vec{b} \times \vec{b})$

$\Rightarrow 0 + 4(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{b}) - 0 = 5(\vec{a} \times \vec{b})$

57. (C) $z = \frac{4-3i}{3+4i} - \frac{3+4i}{4-3i}$

$z = \frac{4-3i}{3+4i} \times \frac{3-4i}{3-4i} - \frac{3+4i}{4-3i} \times \frac{4+3i}{4+3i}$

$z = \frac{12 - 9i - 16i + 12i^2}{9 - 16i^2} - \frac{12 + 16i + 9i + 12i^2}{16 - 9i^2}$

$z = \frac{12 - 25i - 12}{9 + 16} - \frac{12 + 25i - 12}{16 + 9}$

$z = \frac{-25i}{25} - \frac{25i}{25}$

$z = -i - i = -2i$

58. (B) We know that

curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Area = $\frac{a^2}{6}$

Now, $\sqrt{x} + \sqrt{y} = 2 \Rightarrow \sqrt{x} + \sqrt{y} = \sqrt{4}$

The required area = $\frac{4^2}{6} = \frac{8}{3}$ sq. unit

59. (C) $\begin{vmatrix} a^2+b^2 & a+b & \lambda \\ b^2+c^2 & b+c & \lambda \\ c^2+a^2 & c+a & \lambda \end{vmatrix} = (a-b)(b-c)(c-a)$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ c^2-a^2 & c-a & 0 \\ c^2-b^2 & c-b & 0 \end{vmatrix} = (a-b)(b-c)(c-a)$

$\Rightarrow (c-a)(c-b) \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ c+a & 1 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$(c-b)(c-a)$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ c+a & 1 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ a-b & 0 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$\Rightarrow (a-b) \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ 1 & 0 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ 1 & 0 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -1$

$\Rightarrow (a^2 + b^2) \times 0 - (a + b) \times 0 + \lambda(1 - 0) = -1$

$\Rightarrow \lambda = -1$

60. (B) In ΔABC, $\frac{1}{b+c} + \frac{1}{a+c} = \frac{3}{a+b+c}$

$\Rightarrow \frac{a+b+2c}{(b+c)(a+c)} = \frac{3}{a+b+c}$

$\Rightarrow a^2 + ab + 2ac + ab + b^2 + 2bc + ac + bc + 2c^2$

$= 3ab + 3bc + 3ca + 3c^2$

$\Rightarrow a^2 + b^2 - c^2 = ab$

$\Rightarrow \frac{a^2 + b^2 - c^2}{ab} = 1$

$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}$

$\Rightarrow \cos C = \cos\frac{\pi}{3} \Rightarrow C = \frac{\pi}{3}$

61. (B) $\frac{a\omega^7 + b\omega^9 + c\omega^{14}}{b\omega^{12} + a\omega^{10} + c\omega^{11}}$

$\Rightarrow \frac{a\omega^{3\times2+1} + b(\omega^3)^3 + c\omega^{3\times4+2}}{b(\omega^3)^4 + a\omega^{3\times3+1} + c\omega^{3\times3+2}}$

$\Rightarrow \frac{a\omega + b + c\omega^2}{b + a\omega + c\omega^2} \Rightarrow \frac{b + a\omega + c\omega^2}{b + a\omega + c\omega^2} = 1$

62. (B) $n(S) = {}^{12}C_3 = 220$

$n(E) = {}^3C_1 \times {}^4C_2 \times {}^5C_0 + {}^3C_1 \times {}^4C_1 \times {}^5C_1 + {}^3C_1\times {}^4C_0\times {}^5C_2 + {}^3C_2\times{}^4C_1\times{}^5C_0+{}^3C_2\times {}^4C_0\times {}^5C_1 + {}^3C_3\times {}^4C_0\times {}^5C_0$

$n(E) = 3\times6\times1 + 3\times4\times5 + 3\times1\times10 + 3\times4\times1 + 3\times1\times5 + 1\times1\times1$

$n(E) = 18 + 60 + 30 + 12 + 15 + 1 = 136$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$= \frac{136}{220} = \frac{34}{55}$

63. (D) $\begin{vmatrix} x & 3i & i \\ y & -2 & 3i \\ 0 & -i & i \end{vmatrix} = 9 + 12i$

$\Rightarrow x(-2i + 3i^2) - 3i(yi) + 1(-yi) = 9+12i$

$\Rightarrow -2xi - 3x + 3y - yi = 9+12i$

$\Rightarrow (-3x + 3y) + (-2x - y)i = 9 + 12i$

On comparing

$-3x + 3y = 9$ ...(i)

and $-2x - y = 12$ ...(ii)

from eq(i) and eq(ii)

$x = -5$ and $y = -2$

64. (B) $I = \int e^{a\ln x}dx \Rightarrow I = \int e^{\ln x^a}dx$

$I = \int x^a dx \Rightarrow I = \frac{x^{a+1}}{a+1} + c$

65. (C) $\vec{a} = 3\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \lambda\hat{j} + 10\hat{k}$ are perpendicular, then

$\vec{a}.\vec{b} = 0$

$\Rightarrow (3\hat{i} + 4\hat{j} - \hat{k}).(-2\hat{i} + \lambda\hat{j} + 10\hat{k}) = 0$

$\Rightarrow -6 + 4\lambda - 10 = 0$

$\Rightarrow 4\lambda - 16 = 0 \Rightarrow \lambda = 4$

66. (C) Let $y = \text{cosec}^2(\cot^{-1}x)$

$y = 1 + [\cot(\cot^{-1}x)]^2$

$y = 1 + x^2$

On differentiating both sides w.r.t. 'x'

$\frac{dy}{dx} = 2x$

67. (B) $S_n = n(n + 3)$

$S_n = n^2 + 3n$

$S_{n-1} = (n-1)^2 + 3(n-1)$

$S_{n-1} = n^2 + 1 - 2n + 3n - 3$

$S_{n-1} = n^2 + n - 2$

Now, $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = S_n - S_{n-1}$

$\Rightarrow T_n = n^2 + 3n - (n^2 + n - 2)$

$\Rightarrow T_n = 2n + 2$

Now, $T_6 = 2 \times 6 + 2 = 14$

68. (B) curve $y = x^2 - 5x + 4$

it cuts x–axis i.e $y = 0$

$x^2 - 5x + 4 = 0$

$\Rightarrow (x - 4)(x - 1) = 0$

$\Rightarrow x = 1, 4$

Hence two tangents are parallel to x-axis for the curve.

69. (A) $\lim_{x\to\infty} \frac{x^3 + 5x^2 - 6}{1 + 2x - 4x^3} \Rightarrow \lim_{x\to\infty} \frac{x^3\left(1 + \frac{5}{x} - \frac{6}{x^3}\right)}{x^3\left(\frac{1}{x^3} + \frac{2}{x^2} - 4\right)}$

$\Rightarrow \lim_{x\to\infty} \frac{1 + \frac{5}{x} - \frac{6}{x^3}}{-4 + \frac{1}{x^3} + \frac{2}{x^2}} \Rightarrow \frac{1 + 0 - 0}{-4 + 0 + 0} = \frac{-1}{4}$

70. (A)

71. (B) $[(3x - 4y)^3(3x + 4y)^3]^4$

$\Rightarrow [(3x - 4y)(3x + 4y)]^{12}$

$\Rightarrow (9x^2 - 16y^2)^{12}$

Total terms = 12 + 1 = 13

72. (B) Differential equation

$\frac{dy}{dx} = \sin|y - x| + 1 \Rightarrow \frac{dy - dx}{dx} = \sin|y - x|$

$\Rightarrow \frac{dy - dx}{\sin|y - x|} = dx \Rightarrow \int \frac{d(y - x)}{\sin(y - x)} = \int dx$

$\Rightarrow \int \text{cosec}|y - x|\, d(y - x) = \int dx$

$\Rightarrow \log|\text{cosec}(y - x) - \cot(y - x)| = x + C$

$\Rightarrow e^x|\text{cosec}(y - x) - \cot(y - x)| = c$

73. (B) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and $C = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, $A\sin\alpha + B\cos\alpha$

$\Rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\sin\alpha + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cos\alpha$

$\Rightarrow \begin{bmatrix} 0 & \sin\alpha \\ -\sin\alpha & 0 \end{bmatrix} + \begin{bmatrix} \cos\alpha & 0 \\ 0 & \cos\alpha \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = C$

Hence $A\sin\alpha + B\cos\alpha = C$

74. (B) $|x^2 - x - 6| = x + 2$

$\Rightarrow x^2 - x - 6 = x + 2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = -2, 4$

and $-x^2 + x + 6 = x + 2$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Hence roots are 2, –2, 4.

75. (C) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\times\frac{1}{5}}\right] + \tan^{-1}\left[\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{5+3}{15-1}\right] + \tan^{-1}\left[\frac{8+7}{56-1}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{8}{14}\right] + \tan^{-1}\left[\frac{15}{55}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{4}{7}\right] + \tan^{-1}\left[\frac{3}{11}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{3}{11}}\right] \Rightarrow \tan^{-1}\left[\frac{44+21}{77-12}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{65}{65}\right] = \tan^{-1}(1) = \frac{\pi}{4}$$

76. (B) A.T.Q.

$$180 - \frac{360}{n} = 140 \Rightarrow 40 = \frac{360}{n} \Rightarrow n = 9$$

No. of diagonals $= \frac{n(n-3)}{2}$

$$= \frac{9\times(9-3)}{2} = 9 \times 3 = 27$$

77. (B) $2\cos\frac{12\pi}{13}.\cos\frac{4\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$$\Rightarrow 2\cos\frac{12\pi}{13}.\cos\frac{4\pi}{13} + 2\cos\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}.\cos\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}$$

$$\Rightarrow 2\cos\frac{12\pi}{13}.\cos\frac{4\pi}{13} + 2\cos\frac{4\pi}{13}.\cos\frac{\pi}{13}$$

$$\Rightarrow -2\cos\frac{\pi}{13}.\cos\frac{4\pi}{13} + 2\cos\frac{4\pi}{13}.\cos\frac{\pi}{13} = 0$$

78. (B) $2\sin^{-1}x = \sin^{-1}\left[2x\sqrt{1-x^2}\right]$

We know that

$$-\frac{\pi}{2} \le \sin^{-1}\theta \le \frac{\pi}{2}$$

Now, $-\frac{\pi}{2} \le \sin^{-1}\left[2x\sqrt{1-x^2}\right] \le \frac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}x \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$$

$$x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

79. (C) $I = \int_3^4 \frac{dx}{x^2 - 2x}$

$$I = \int_3^4 \frac{dx}{x(x-2)}$$

$$I = \int_3^4 \frac{1}{2}\left(\frac{1}{x-2} - \frac{1}{x}\right) dx$$

$$I = \frac{1}{2}\left[\ln(x-2)\right]_3^4 - \frac{1}{2}\left[\ln x\right]_3^4$$

$$I = \frac{1}{2}\left[\ln(4-2) - \ln(3-2)\right] - \frac{1}{2}\left[\ln 4 - \ln 3\right]$$

$$I = \frac{1}{2}\ln 2 - \ln 2 + \frac{1}{2}\ln 3$$

$$I = \frac{1}{2}\ln 3 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln\frac{3}{2}$$

80. (C) $\frac{1}{bc}, \frac{1}{ca}$ and $\frac{1}{ab}$ are in A.P.,

then $\frac{2}{ca} = \frac{1}{bc} + \frac{1}{ab}$

$$\Rightarrow \frac{2}{ca} = \frac{a+c}{abc}$$

$\Rightarrow 2b = a + c$

Hence a, b and c are in A.P.

81. (B) $I = \int \cot^2 x . \text{cosec}^4 x \; dx$

$I = \int \cot^2 x . \text{cosec}^2 x . \text{cosec}^2 x \; dx$

$I = \int \cot^2 x . (1 + \cot^2 x) . \text{cosec}^2 x \; dx$

$I = \int (\cot^2 x + \cot^4 x) . \text{cosec}^2 x \; dx$

Let $\cot x = t$

$\Rightarrow - \text{cosec}^2 x \, dx = dt \Rightarrow \text{cosec}^2 x \, dx = - dt$

$I = -\int (t^2 + t^4) \; dt$

$I = -\left[\frac{t^3}{3} + \frac{t^5}{5}\right] + c$

$I = -\left[\frac{\cot^3 x}{3} + \frac{\cot^5 x}{5}\right] + c$

82. (B) Data 21, 21 , 20, 21, 22, 23, 20, 20, 21, 21, 24, 25

Mode = 21

83. (A) $y = \tan^{-1}\left(\frac{3 - 2\tan\sqrt{x}}{2 + 3\tan\sqrt{x}}\right)$

$y = \tan^{-1}\left(\frac{\frac{3}{2} - \tan\sqrt{x}}{1 + \frac{3}{2}\tan\sqrt{x}}\right)$

Let $\frac{3}{2} = \tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{3}{2}\right)$

$y = \tan^{-1}\left(\frac{\tan\phi - \tan\sqrt{x}}{1 + \tan\phi . \tan\sqrt{x}}\right)$

$y = \tan^{-1}[\tan(\phi - \sqrt{x})]$

$y = \phi - \sqrt{x}$

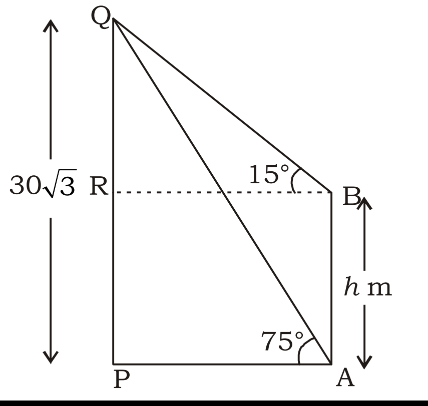
$y = \tan^{-1}\left(\frac{3}{2}\right) - \sqrt{x}$

On differentiating both sides w.r.t.'x'

$\frac{dy}{dx} = 0 - \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$

84. (B)



Let AB = h m = PR

and QR = $30\sqrt{3} - h$

In ΔQRB

$\tan 15° = \frac{QR}{RB} \Rightarrow \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{30\sqrt{3} - h}{RB}$ ...(i)

In ΔAPQ

$\tan 75° = \frac{PQ}{PA} \Rightarrow \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{30\sqrt{3}}{RB}$ ...(ii)

from eq(i) and eq(ii)

$\frac{(\sqrt{3} - 1)^2}{(\sqrt{3} + 1)^2} = \frac{30\sqrt{3} - h}{30\sqrt{3}}$

$\Rightarrow \frac{4 - 2\sqrt{3}}{4 + 2\sqrt{3}} = \frac{30\sqrt{3} - h}{30\sqrt{3}}$

$\Rightarrow \frac{2 - \sqrt{3}}{2 + \sqrt{3}} = \frac{30\sqrt{3} - h}{30\sqrt{3}}$

$\Rightarrow 60\sqrt{3} - 90 = 60\sqrt{3} - 2h + 90 - \sqrt{3}\,h$

$\Rightarrow (2 - \sqrt{3})h = 180 \Rightarrow h = \frac{180}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$\Rightarrow h = 180(2 + \sqrt{3})$

Hence height of the pole = $180(2 + \sqrt{3})$ m

85. (C) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix}$

$\vec{a} \times \hat{i} = \hat{i}(0) - \hat{j}(-a_3) + \hat{k}(-a_2)$

$\vec{a} \times \hat{i} \Rightarrow a_3\hat{j} - a_2\hat{k}$

$|\vec{a} \times \hat{i}|^2 = |a_3\hat{j} - a_2\hat{k}|^2$

$|\vec{a} \times \hat{i}|^2 = a_3^2 + a_2^2$

Similarly

$|\vec{a} \times \hat{j}|^2 = a_1^2 + a_3^2$ and $|\vec{a} \times \hat{k}|^2 = a_1^2 + a_2^2$

Now, $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$

$\Rightarrow a_3^2 + a_2^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$

$\Rightarrow 2(a_1^2 + a_2^2 + a_3^2) = 2|\vec{a}|^2$

86. (C) Let the equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

it passes through the point (1, 2)

$1^2 + 2^2 + 2g\times2 + 2f\times2 + c = 0$

$\Rightarrow 4g + 4f + c = -5$ ...(ii)

eq(i) passes through the point (–1, 0)

$(-1)^2 + 0^2 + 2g\times(-1) + 2f\times0 + c = 0$

$\Rightarrow -2g + c = -1$ ...(iii)

eq(i) passes through the point (3, – 4)

$3^2 + (-4)^2 + 2g\times3 + 2f\times(-4) + c = 0$

$\Rightarrow 6g - 8f + c = -25$ ...(iv)

On solving eq(ii), (iii) and (iv)

$g = \frac{-8}{5}, f = \frac{7}{5}, c = \frac{-21}{5}$

from eq(i)

$x^2 + y^2 + 2\left(\frac{-8}{5}\right)x + 2\left(\frac{7}{5}\right)y - \frac{21}{5} = 0$

$\Rightarrow 5x^2 + 5y^2 - 16x + 14y - 21 = 0$

87. (B) $y = (\sec x)^{(\sec x)\ldots\ldots\infty}$

$\Rightarrow y = (\sec x)^y$

taking log both sides

$\Rightarrow \log y = y \log\sec x$ ...(i)

On differentiating both sides w.r.t.'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y\times\frac{\sec x.\tan x}{\sec x} + \frac{dy}{dx}.\log\sec x$

$\Rightarrow \frac{dy}{dx} = y^2.\tan x + \frac{dy}{dx}(y\log\sec x)$

$\Rightarrow \frac{dy}{dx} = y^2.\tan x + \log y.\frac{dy}{dx}$ [from eq(i)]

$\Rightarrow (1-\log y)\frac{dy}{dx} = y^2.\tan x$

$\Rightarrow \frac{dy}{dx} = \frac{y^2.\tan x}{1-\log y}$

88. (D) Equation

$x^2 + \alpha x + \beta = 0$

Roots are α and β,

then $\alpha + \beta = -\alpha$

$\Rightarrow 2\alpha + \beta = 0$ ...(i)

and $\alpha.\beta = \beta \Rightarrow \alpha = 1$

from eq(i)

$2\times1 + \beta = 0 \Rightarrow \beta = -2$

New quadratic equation

$\Rightarrow -x^2 - \alpha x - \beta \Rightarrow -x^2 - x + 2$

$\Rightarrow -x^2 - x - \frac{1}{4} + \frac{1}{4} + 2$

$\Rightarrow -\left(x+\frac{1}{2}\right)^2 + \frac{9}{4}$

Hence greatest value of the equation $= \frac{9}{4}$

89. (B) Equation

$5x^2 + 4 = 0$

Roots are sinα and sinβ,

then $\sin\alpha + \sin\beta = 0$

and $\sin\alpha.\sin\beta = \frac{4}{5} \Rightarrow \operatorname{cosec}\alpha.\operatorname{cosec}\beta = \frac{5}{4}$

90. (A) Let $z = \begin{bmatrix} \omega & \omega^2 & 1+\omega^2 \\ 1 & \omega & \omega+\omega^2 \\ \omega^2 & 1 & 1+\omega \end{bmatrix}$

$|z| = \begin{vmatrix} \omega & \omega^2 & 1+\omega^2 \\ 1 & \omega & \omega+\omega^2 \\ \omega^2 & 1 & 1+\omega \end{vmatrix}$

$C_1 \rightarrow C_1 + C_3$

$|z| = \begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & 1+\omega^2 \\ 1+\omega+\omega^2 & \omega & \omega+\omega^2 \\ 1+\omega+\omega^2 & 1 & 1+\omega \end{vmatrix}$

$|z| = \begin{vmatrix} 0 & \omega^2 & 1+\omega^2 \\ 0 & \omega & \omega+\omega^2 \\ 0 & 1 & 1+\omega \end{vmatrix}$

$|z| = 0 = 1 + \omega + \omega^2$

91. (A) $\frac{d}{dx}\left(\frac{\sin^{-1}x}{\sqrt{1-x^2}}\right)$

$\Rightarrow \frac{\sqrt{1-x^2}\times\frac{1}{\sqrt{1-x^2}} - \left(\sin^{-1}x\right)\times\frac{-2x}{2\sqrt{1-x^2}}}{\left(\sqrt{1-x^2}\right)^2}$

$\Rightarrow \frac{1+\frac{x}{\sqrt{1-x^2}}\sin^{-1}x}{1-x^2} \Rightarrow \frac{\sqrt{1-x^2}+x.\sin^{-1}x}{\left(1-x^2\right)^{3/2}}$

92. (C) $AA^T = 1 \Rightarrow |AA^T| = 1$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

93. (C) $\lim_{x\to\infty}\frac{2x^4+5x^3+6x}{5x^5+3x+4x^2} \Rightarrow \lim_{x\to\infty}\frac{x^4\left(2+\frac{5}{x}+\frac{6}{x^2}\right)}{x^5\left(5+\frac{3}{x^4}+\frac{4}{x^3}\right)}$

$\Rightarrow \lim_{x\to\infty}\frac{\left(2+\frac{5}{x}+\frac{6}{x^2}\right)}{x\left(5+\frac{3}{x^4}+\frac{4}{x^3}\right)} = \frac{1}{\infty} = 0$

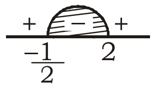
94. (B) $\int_1^2 \{k^2+(1-k)x+2x^3\}\,dx \le 10$

$\Rightarrow \left[k^2x+(1-k)\frac{x^2}{2}+\frac{2x^4}{4}\right]_1^2 \le 10$

$\Rightarrow (2k^2+(1-k)\times 2+8)-\left(k^2+(1-k)\times\frac{1}{2}+\frac{1}{2}\right) \le 10$

$\Rightarrow k^2-\frac{3k}{2}+9 \le 10 \Rightarrow 2k^2-3k+18 \le 20$

$\Rightarrow 2k^2-3k-2 \le 0 \Rightarrow (2k+1)(k-2) \le 0$



Hence $\frac{-1}{2} \le k \le 2$

95. (A) $\sin\left(\sin^{-1}\frac{12}{13}+\cos^{-1}x\right) = 1$

$\Rightarrow \sin^{-1}\frac{12}{13} + \cos^{-1}x = \sin^{-1}(1)$

$\Rightarrow \sin^{-1}\frac{12}{13} + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{12}{13}$

$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{12}{13} \Rightarrow x = \frac{12}{13}$

96. (A) $f(x) = \frac{1}{\sqrt{55+x^2}}$

On differentiating both sides w.r.t. 'x'

$\Rightarrow f'(x) = \frac{-(2x)}{2(55+x^2)^{\frac{3}{2}}} \Rightarrow f'(x) = \frac{-x}{(55+x^2)^{\frac{3}{2}}}$

Now, $\lim_{x\to 3}\frac{f(3)-f(x)}{x^3-27}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 3}\frac{-f'(x)}{3x^2} \Rightarrow \lim_{x\to 3}\frac{x}{3x^2(55+x^2)^{\frac{3}{2}}}$

$\Rightarrow \lim_{x\to 3}\frac{1}{3x(55+x^2)^{\frac{3}{2}}}$

$\Rightarrow \frac{1}{3\times 3(55+3^2)^{\frac{3}{2}}} = \frac{1}{4608}$

97. (C) $I = \int_0^1 \frac{x^8}{\sqrt{1-x^6}}\,dx$

$I = \int_0^1 \frac{x^6.x^2}{\sqrt{1-(x^3)^2}}\,dx$

Let $x^3 = \sin\theta$ when $x = 0,\ \theta = 0$

$\Rightarrow 3x^2\,dx = \cos\theta\,d\theta$ $x = 1,\ \theta = \frac{\pi}{2}$

$\Rightarrow x^2\,dx = \frac{1}{3}\cos\theta\,d\theta$

$I = \int_0^{\pi/2}\frac{1}{3}\frac{\sin^2\theta.\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$

$I = \frac{1}{3}\int_0^{\pi/2}\frac{\sin^2\theta.\cos\theta}{\cos\theta}\,d\theta$

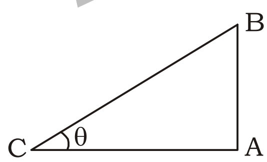
$I = \frac{1}{3}\int_0^{\pi/2}\sin^2\theta\,d\theta$

$I = \frac{1}{3}\int_0^{\pi/2}\frac{1-\cos 2\theta}{2}\,d\theta$

$I = \frac{1}{6}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$I = \frac{1}{6}\left[\frac{\pi}{2}-0\right] = \frac{\pi}{12}$

98. (B)



Let angle of elevation = θ

AB = h, AC = $\sqrt{3}\,h$

In ΔABC:-

$\tan\theta = \frac{AB}{AC} \Rightarrow \tan\theta = \frac{h}{\sqrt{3}h}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \tan\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$

Angle of elevation = $\frac{\pi}{6}$

99. (C) Plane $3x + 4y - 5z + 11 = 0$ and point $(-1, 2, 4)$

Distance = $\left|\frac{3\times(-1)+4\times 2-5\times 4}{\sqrt{3^2+4^2+(-5)^2}}\right|$

$= \frac{15}{5\sqrt{2}} = \frac{3}{\sqrt{2}}$

100. (B) When θ = 180°

$M = \frac{60}{11}(H \pm 6)$ where +H < 6
−H > 6

H = 7(between 7 and 8 o'clock)

$M = \frac{60}{11}(7-6)$

$M = \frac{60}{11} \times 1 = 6\frac{5}{11}$

Hence time = $7 : 5\frac{5}{11}$

101. (C) $\sin\frac{\pi}{12} < \tan\frac{\pi}{12} < \cos\frac{\pi}{12}$

102. (B)

103. (C) 1

104. (C) $\lim_{x\to 2}\left[\frac{x^3-2x^2}{x^3-3x+2}\right]$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 2}\frac{3x^2-4x}{3x^2-3} \Rightarrow \frac{3\times 2^2-4\times 2}{3\times 2^2-3}$

$\Rightarrow \frac{12-8}{12-3} = \frac{4}{9}$

105. (B) Equation $ax^2 + bx + c = 0$

roots = α and $\frac{1}{\alpha}$

Now, $\alpha.\frac{1}{\alpha} = \frac{c}{a} \Rightarrow 1 = \frac{c}{a} \Rightarrow c = a$

106. (A) Let x and y are two persons and they hit a target with the probability A and B respectively.

$\therefore$ P(A) = $\frac{1}{5}$ and P(B) = $\frac{1}{4}$

P (Probability of hitting the target by any one x or y)

$\Rightarrow P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$\Rightarrow P(A).P(\overline{B}) + P(\overline{A}).P(B)$

$\Rightarrow \frac{1}{5}\times\frac{3}{4} + \frac{4}{5}\times\frac{1}{4}$

$\Rightarrow \frac{3}{20} + \frac{4}{20} = \frac{7}{20}$

107. (A) $x^3 - 1 = (x-1)(x^2+x+1)$
$x^3 - 1 = (x-1)(x-\omega)(x-\omega^2)$

108. (D) In ΔABC, $\overrightarrow{AB} = 3\hat{i}+\hat{j}-\hat{k}$, $\overrightarrow{AC} = 3\hat{i}-2\hat{j}+5\hat{k}$

Now, $\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -1\\ 3 & -2 & 5\end{vmatrix}$

$\Rightarrow \overrightarrow{AB}\times\overrightarrow{AC} = \hat{i}(5-2) - \hat{j}(15+3) + \hat{k}(-6-3)$

$\Rightarrow \overrightarrow{AB}\times\overrightarrow{AC} = 3\hat{i} - 18\hat{j} - 9\hat{k}$

Area of ΔABC = $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{3^2+(-18)^2+(-9)^2}$

$= \frac{1}{2}\sqrt{9+324+81} = \frac{1}{2}\sqrt{414}$

$= \frac{1}{2}\times 3\sqrt{46} = \frac{3}{2}\sqrt{46}$

109. (B) $3^c = \left(3\times\frac{180}{\pi}\right)^0 = \left(\frac{540\times 7}{22}\right)^0$

$= \left(\frac{1890}{11}\right)^0 = 171° 49' 5''$

110. (A) We know that

Minimum value of $\left(ax^2 + \frac{b}{x^2}\right) = 2\sqrt{ab}$

So minimum value of $(27\sin^2\theta + 12\,\text{cosec}^2\theta)$

$= 2\sqrt{27\times 12} = 2\sqrt{9\times 3\times 3\times 4}$

$= 2\times 3\times 3\times 2 = 36$

111. (C) Differential equation

$\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{1-x^2} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{1-x^2}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \frac{-dx}{\sqrt{1-x^2}}$

On integrating

$\Rightarrow \sin^{-1}y = \cos^{-1}x + c \Rightarrow \sin^{-1}y - \cos^{-1}x = c$

112. (C) $\frac{1+\cos\theta}{1-\cos\theta} = 3 \Rightarrow 1+\cos\theta = 3 - 3\cos\theta$

$\Rightarrow 4\cos\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

113. (D) $\int_0^{\pi}|\cos x|\,dx \Rightarrow 2\int_0^{\pi/2}\cos x\,dx$

$\Rightarrow 2[\sin x]_0^{\pi/2} \Rightarrow 2\left[\sin\frac{\pi}{2} - \sin 0\right]$

$\Rightarrow 2[1-0] = 2$

114. (B) Equations $\alpha x + (1+\beta)y = 3$ and $(1+\alpha)x + \beta y = 2$ has a unique solution, then

$$\frac{\alpha}{1+\alpha} \neq \frac{1+\beta}{\beta} \Rightarrow \alpha\beta \neq (1+\alpha)(1+\beta)$$

$\Rightarrow \alpha\beta \neq 1 + \alpha + \beta + \alpha\beta$

$\Rightarrow 0 \neq 1 + \alpha + \beta \Rightarrow \alpha + \beta \neq -1$

115. (A) $I = \int \cos(\log_e x)\,dx$

Let $\log_e x = t$

$\Rightarrow x = e^t \Rightarrow dx = e^t\,dt$

$I = \int \cos t\, e^t\, dt$

$I = \cos t \int e^t\,dt - \int\left\{\frac{d}{dt}(\cos t).\int e^t\,dt\right\}dt$

$I = \cos t.e^t - \int(-\sin t).e^t\,dt$

$I = e^t.\cos t + \int \sin t.e^t\,dt$

$I = e^t.\cos t + \sin t.\int e^t\,dt - \int\left\{\frac{d}{dt}(\sin t)\int e^t\,dt\right\}dt$

$I = e^t.\cos t + \sin t.e^t - \int \cos t.e^t\,dt$

$I = e^t.\cos t + \sin t.e^t - I + c$

$2I = e^t(\cos t + \sin t) + c$

$I = \frac{e^t(\cot t + \sin t)}{2} + c$

$I = \frac{1}{2}x[\cos(\log_e x) + \sin(\log_e x)] + c$

116. (B) $\lim_{x\to\pi/4}\frac{\tan^2 x - \cot x}{\cos\left(x+\frac{\pi}{4}\right)}$ $\left[\frac{0}{0}\right]$ from

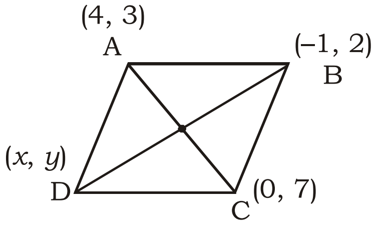
by L-Hospital's Rule

$\Rightarrow \lim_{x\to\pi/4}\frac{2\tan x.\sec^2 x + \text{cosec}^2 x}{-\sin\left(x+\frac{\pi}{4}\right)}$

$\Rightarrow \frac{2\tan\frac{\pi}{4}.\sec^2\frac{\pi}{4} + \text{cosec}^2\frac{\pi}{4}}{-\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)}$

$\Rightarrow \frac{2\times1\times(\sqrt{2})^2 + (\sqrt{2})^2}{-1} \Rightarrow \frac{4+2}{-1} = -6$

117. (C) Let fourth vertex = (x, y)



Diagonals of a parallelogram are perpendicular bisector to each other,

then $\frac{x-1}{2} = \frac{4+0}{2} \Rightarrow x = 5$

and $\frac{y+2}{2} = \frac{3+7}{2} \Rightarrow y = 8$

$\therefore$ fourth vertex = (5, 8)

118. (C) differential equation

$\frac{dy}{dx} = \tan^2(x+y)$

Let $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\Rightarrow \frac{dt}{dx} - 1 = \tan^2 t \Rightarrow \frac{dt}{dx} = 1 + \tan^2 t$

$\Rightarrow \frac{dt}{dx} = \sec^2 t \Rightarrow 2\cos^2 t\,dt = 2dx$

$\Rightarrow (1 + \cos 2t)dt = 2dx$

On Integrating

$\Rightarrow t + \frac{\sin 2t}{2} = 2x + \frac{c}{2} \Rightarrow 2t + \sin 2t = 4x + c$

$\Rightarrow 2(x+y) + \sin 2(x+y) = 4x + c$

$\Rightarrow \sin 2(x+y) = 2(x-y) + c$

119. (A) A = {1, 2, 3, 4} and B = {1, 2, 5}

(A × B) = {(1,1), (1, 2), (1, 5), (2, 1), (2, 2), (2, 5), (3, 1), (3, 2), (3, 5), (4, 1), (4, 2), (4, 5)}

(B × A) = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2,2), (2, 3), (2, 4), (5, 1) (5, 2), (5, 3), (5, 4)}

Now, (A × B) $\cap$ (B × A)

$\Rightarrow$ {(1, 1), (1, 2), (2,1), (2, 2)}

120. (C) $f(x) = y = \frac{3^x + 3^{-x}}{3^x - 3^{-x}}$

by Componendo and Dividendo Rule

$\Rightarrow \frac{y+1}{y-1} = \frac{2.3^x}{2.3^{-x}} \Rightarrow \frac{y+1}{y-1} = 3^{2x}$

$\Rightarrow 2x = \log_3\left(\frac{y+1}{y-1}\right)$

$\Rightarrow x = \frac{1}{2}\log_3\left(\frac{y+1}{y-1}\right)$

$\Rightarrow f^{-1}(y) = \frac{1}{2}\log_3\left(\frac{y+1}{y-1}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log_3\left(\frac{x+1}{x-1}\right)$

NDA (MATHS) MOCK TEST - 198 (Answer Key)

1.	(B)	21.	(B)	41.	(A)	61.	(B)	81.	(D)	101.	(C)
2.	(B)	22.	(C)	42.	(B)	62.	(B)	82.	(B)	102.	(B)
3.	(C)	23.	(B)	43.	(C)	63.	(D)	83.	(A)	103.	(C)
4.	(B)	24.	(C)	44.	(B)	64.	(B)	84.	(D)	104.	(C)
5.	(B)	25.	(A)	45.	(D)	65.	(C)	85.	(C)	105.	(B)
6.	(D)	26.	(B)	46.	(C)	66.	(C)	86.	(C)	106.	(A)
7.	(B)	27.	(C)	47.	(B)	67.	(B)	87.	(B)	107.	(A)
8.	(D)	28.	(C)	48.	(C)	68.	(B)	88.	(D)	108.	(D)
9.	(B)	29.	(C)	49.	(A)	69.	(A)	89.	(B)	109.	(B)
10.	(B)	30.	(B)	50.	(B)	70.	(A)	90.	(A)	110.	(A)
11.	(B)	31.	(D)	51.	(A)	71.	(B)	91.	(A)	111.	(C)
12.	(A)	32.	(C)	52.	(B)	72.	(B)	92.	(C)	112.	(C)
13.	(C)	33.	(C)	53.	(C)	73.	(B)	93.	(C)	113.	(D)
14.	(C)	34.	(B)	54.	(C)	74.	(B)	94.	(B)	114.	(A)
15.	(B)	35.	(B)	55.	(C)	75.	(C)	95.	(A)	115.	(A)
16.	(A)	36.	(C)	56.	(B)	76.	(B)	96.	(A)	116.	(B)
17.	(B)	37.	(A)	57.	(C)	77.	(B)	97.	(C)	117.	(C)
18.	(C)	38.	(B)	58.	(B)	78.	(B)	98.	(B)	118.	(C)
19.	(D)	39.	(B)	59.	(C)	79.	(C)	99.	(C)	119.	(A)
20.	(A)	40.	(C)	60.	(B)	80.	(C)	100.	(B)	120.	(C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777