NDA MATHS MOCK TEST - 194 (SOLUTION)

1. (D) $ax^2 + bx + c = 0$

Sum of Roots $\alpha + \beta = \frac{-b}{a}$

Product of Roots $\alpha\beta = \frac{c}{a}$

Equation

x^2 – (Sum of roots)x + Product of roots = 0

$\Rightarrow x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha} \times \frac{1}{\beta} = 0$

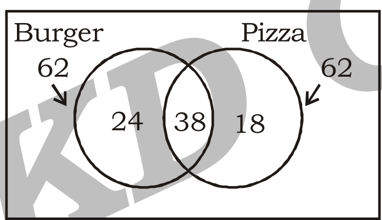
$\Rightarrow x^2 - \left(\frac{\alpha + \beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$

$\Rightarrow x^2 - \left(\frac{-b/a}{c/a}\right)x + \frac{1}{c/a} = 0$

$\Rightarrow x^2 + \frac{b}{c}x + \frac{a}{c} = 0 \Rightarrow cx^2 + bx + a = 0$

2. (C) $\frac{\sin 2A}{1 + \cos 2A} \Rightarrow \frac{2\sin A.\cos A}{2\cos^2 A}$ = tanA

(3-4) :



3. (B) Number of students who like both Burger and Pizza = (62 + 56) – 80 = 38

4. (A) Number of students who don't like Burger = 90 – 62 = 28

5. (D) $\frac{x}{y} = \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2 - (1)^2}$

$= \frac{6 + 2\sqrt{5}}{4}$

Similary, $\frac{y}{x} = \frac{6 - 2\sqrt{5}}{4}$

then, $\frac{x}{y} + \frac{y}{x} = 2 \times \frac{6}{4} = 3$

$\Rightarrow \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 = 3^2 - 2 = 7$

Now, $\left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 + 4\left(\frac{x}{y} + \frac{y}{x}\right) + 6$

= 7 + 4 × 3 + 6 = 25

6. (D) [sin59°.cos31° + cos59°.sin31°] + [cos20° .cos25° – sin20°.sin25°]

$\Rightarrow$ sin(59° + 31°) ÷ cos(20 + 25°)

[$\because$ sin(C + D) = sinC.cosD + cosC.sinD and cos(C + D) = cosC.cosD – sinC.sinD]

$\Rightarrow \frac{\sin 90°}{\cos 45°} = \sqrt{2}$

7. (B) First 6 Prime numbers = 2, 3, 5, 7, 11, 13

Mean = $\frac{\text{sum of terms}}{\text{no. of total terms}}$

Mean = $\frac{2+3+5+7+11+13}{6} = \frac{41}{6}$

8. (C) $\sin^2 30°.\cos^2 45° + 4\tan^2 30° + \sin^2 90° - 2\cos^2 90° + \frac{1}{24}$.

By Putting value

$\Rightarrow \left(\frac{1}{2}\right)^2 \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2\times 0 + \frac{1}{24}$

$\Rightarrow \frac{1}{4} \times \frac{1}{2} + \frac{4}{3} + \frac{1}{2} + 0 + \frac{1}{24}$

$\Rightarrow \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

$\Rightarrow \frac{3+32+12+1}{24} = \frac{48}{24} = 2$

9. (B) Mean = $\frac{\text{sum of terms}}{\text{no. of total terms}}$

Mean = $\frac{1+2+10+18+3+17+19}{7} = \frac{70}{7} = 10$

10. (D) $\tan^2\theta + \cot^2\theta \Rightarrow \tan^2 30° + \cot^2 30°$

$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2$

$\Rightarrow \frac{1}{3} + 3 = \frac{10}{3}$

11. (D) $\sec\theta(\cos\theta + \sin\theta) = \sqrt{2}$

$\Rightarrow \cos\theta + \sin\theta = \sqrt{2}\cos\theta$

On squaring both sides

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta.\cos\theta = 2\cos^2\theta$

$\Rightarrow 1-\sin^2\theta + 1-\cos^2\theta + 2\sin\theta.\cos\theta = 2\cos^2\theta$

$\Rightarrow \cos^2\theta + \sin^2\theta - 2\sin\theta.\cos\theta = 2 - 2\cos^2\theta$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2(1-\cos^2\theta)$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$\Rightarrow \dfrac{\sqrt{2}\sin\theta}{\cos\theta - \sin\theta} = 1$

multiply by $\sqrt{2}$

$\Rightarrow \dfrac{2\sin\theta}{\cos\theta - \sin\theta} = \sqrt{2}$

12. (C) If one root $= 5 + 3\sqrt{3}$

then 2nd root $= 5 - 3\sqrt{3}$

Sum of roots

$-\dfrac{b}{a} = 5 + 3\sqrt{3} + 5 - 3\sqrt{3}$

$\Rightarrow -\dfrac{b}{a} = 10 \Rightarrow b = -10a$

Product of roots

$\dfrac{c}{a} = \left(5+3\sqrt{3}\right)\left(5-3\sqrt{3}\right)$

$\Rightarrow \dfrac{c}{a} = -2 \Rightarrow c = -2a$

Now, $\dfrac{a^2+b^2+c^2}{a+b+c}$

$\Rightarrow \dfrac{a^2+(-10a)^2+(-2a)^2}{a-10a-2a} = \dfrac{-105a}{11}$

13. (D) Angle between the regression lines

$\tan\theta = \left\{\left(\dfrac{1-r^2}{r}\right)\left(\dfrac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)\right\}$

$\Rightarrow \tan\dfrac{\pi}{2} = \left(\dfrac{1-r^2}{r}\right)\left(\dfrac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$

$\Rightarrow \dfrac{1}{0} = \left(\dfrac{1-r^2}{r}\right)\left(\dfrac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$

$\Rightarrow r(\sigma_x^2 + \sigma_y^2) = 0$

$\therefore r = 0$

14. (A) $\therefore$ Total number of arrangements $= \dfrac{10!}{2!}$

Total number of arrangements when I's comes together = 9!

and favourable arrangements $= \dfrac{10!}{2!} - 9!$

$\therefore$ Required probability $= \dfrac{\dfrac{10!}{2!} - 9!}{\dfrac{10!}{2!}}$

$= \dfrac{(10-2)\times 9!}{10\times 9!} = \dfrac{4}{5}$

15. (D) Let A and B be the events that X and Y qualify the examination respectively,

We have, P(A) = 0.05, P(B) = 0.10 and $P(A\cap B) = 0.02$,

then P(only one of A and B will qualify the examination)

$= P(A\cap\overline{B}) + P(B\cap\overline{A})$

$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$

$= P(A) + P(B) - 2P(A\cap B)$

$= 0.05 + 0.1 - 2(0.02)$

$= 0.15 - 0.04 = 0.11$

16. (A) Let us consider the equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

If it passes through the point (1, 2) so

$5 + 2g + 4f + c = 0$

Now, the circle $x^2 + y^2 = 4$ intersects the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ orthogonally. Therefore

$2(g.0 + f.0) = c - 4 \Rightarrow c = 4$

This value of c put in equation (i) we have

$2g + 4f + 9 = 0$

So the locus of $(-g, -f)$ is

$-2x - 4y + 9 = 0 \Rightarrow 2x + 4y - 9 = 0$

17. (B) The two diameters are $x + y = 6$ and $x + 2y = 4$

The intersection point of these diameter (8, –2) which is the centre of the circle. The circle passes through the point (6, 2), therefore its radius

$= \sqrt{(8-6)^2 + (-2-2)^2} = \sqrt{20} = 2\sqrt{5}$

18. (C) The equation of the tangent at (h, h) to $x^2 + y^2 = a^2$ is $hx + hy = a^2$

Now, slope = –1

19. (A) The ellipse equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ...(i)

It is given that length of latus rectum = half of its major axis

$\Rightarrow \dfrac{2b^2}{a} = a \Rightarrow 2b^2 = a^2$ ...(ii)

Now, $b^2 = a^2\left(1-e^2\right) \Rightarrow b^2 = 2b^2\left(1-e^2\right)$

$\Rightarrow 1 - e^2 = \dfrac{1}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

20. (B) The ellipse equation $5x^2 + 9y^2 = 45$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{5} = 1$

Here $a^2 = 9$, $b^2 = 5$

The major axis is along x-axis, so the length of latus rectus = $\frac{2b^2}{a} = \frac{2\times 5}{3} = \frac{10}{3}$

21. (A) The given equation of hyperbola

$x^2 - y^2 = 9$ ...(i)

$x = 9$ meets the hyperbola (i) at P$(9, 6\sqrt{2})$ and Q$(9, -6\sqrt{2})$

Now the equation of tangent at these points are

$3x - 2\sqrt{2}\,y - 3 = 0$ and $3x + 2\sqrt{2}\,y - 3 = 0$

The combined equation of these two is

$(3x - 2\sqrt{2}\,y - 3) = 0$ and $3x + 2\sqrt{2}\,y - 3 = 0$

$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0$

22. (A) The hyperbola equation

$2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ ...(i)

Let the equation of asymptotes be

$2x^2 + 5xy + 2y^2 + 4x + 5y + \lambda = 0$ ...(ii)

This equation represents a pair of straight lines. Therefore

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Here $a = 2$, $b = 2$, $h = \frac{5}{2}$, $g = 2$, $f = \frac{5}{2}$, $c = \lambda$

$\Rightarrow 4\lambda + 25 - \frac{25}{2} - 8 - \frac{25}{4}\lambda = 0 \Rightarrow \lambda = 2$

The required equation

$2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$

23. (A) The slope of line $4y = 5x + 7$ is $\frac{5}{4}$

which is the slope of tangent. So $m = \frac{5}{4}$.

The given hyperbola $4x^2 - 9y^2 = 1$

$\Rightarrow \frac{x^2}{\frac{1}{4}} - \frac{y^2}{\frac{1}{9}} = 1$

$\Rightarrow a^2 = \frac{1}{4}$, $b^2 = \frac{1}{9}$

Now the equations of the tangents are

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$\Rightarrow y = \frac{5}{4}x \pm \sqrt{\frac{25}{64} - \frac{1}{9}}$

$\Rightarrow 24y - 30x = \pm\sqrt{61}$

24. (A) $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$

Let $xe^x = t \Rightarrow e^x(1 + x)dx = dt$

$\Rightarrow \int \frac{dt}{\cos^2 t} \Rightarrow \int \sec^2 t\, dt$

$\Rightarrow \tan t + c = \tan(xe^x) + c$

25. (A) $\int \frac{(10x^9 + 10^x.\log_e 10)}{(x^{10} + 10^x)}dx$

let $x^{10} + 10^x = t \Rightarrow (10x^9 + 10^x.\log^{10})dx = dt$

$\Rightarrow \int \frac{dt}{t} \Rightarrow \log t + c \Rightarrow \log(x^{10} + 10^x) + c$

26. (C) $\int e^x.\sin e^x dx$

Let $e^x = t \Rightarrow e^x.dx = dt$

$\Rightarrow \int \sin t.dt \Rightarrow -\cos t + c \Rightarrow -\cos(e^x) + c$

27. (B) $\int \frac{3x^2}{(x^6+1)}dx$

Let $x^3 = t \Rightarrow 3x^2\, dx = dt$

$\Rightarrow \int \frac{dt}{(t^2+1)} \Rightarrow \tan^{-1} t + c \Rightarrow \tan^{-1}(x^3) + c$

28. (A) $|z_1| = |z_2| = |z_3| = 1$(given)

Now, $|z_1| = 1 \Rightarrow |z_1|^2 = 1$

$\Rightarrow z_1\overline{z_1} = 1$

Similarly, $z_2\overline{z_2} = 1$, $z_3\overline{z_3} = 1$

Now, $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1 \Rightarrow \left|\overline{z_1} + \overline{z_2} + \overline{z_3}\right| = 1$

29. (D) Let $z = (1)^{1/n} = (\cos 2k\pi + i\sin 2k\pi)^{1/n}$

$z = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$, $k = 0, 1, 2, \ldots, n-1$

Let $z_1 = \cos\left(\frac{2k_1\pi}{n}\right) + i\sin\left(\frac{2k_1\pi}{n}\right)$

and $z_2 = \cos\left(\frac{2k_2\pi}{n}\right) + i\sin\frac{2k_2\pi}{n}$

be the two values of z s.t. they subtend $\angle$ of 90° at origin

$\therefore \frac{2k_1\pi}{n} - \frac{2k_2\pi}{n} = \pm\frac{\pi}{2} \Rightarrow 4(k_1 - k_2) = \pm n$

As k_1 and k_2 are integers and $k_1 \neq k_2$

$\therefore n = 4k$, $k \in I$

30. (A) The required probability

$= \frac{{}^4C_1 \times {}^5C_1 \times {}^3C_0 + {}^4C_1 \times {}^5C_0 \times {}^3C_1 + {}^4C_2 \times {}^5C_0 \times {}^3C_0}{{}^{12}C_2}$

$= \frac{4 \times 5 \times 1 + 4 \times 1 \times 3 + 6 \times 1 \times 1}{66}$

$= \frac{38}{66} = \frac{19}{33}$

31. (A) The required probability

$= \frac{{}^6C_2 + {}^7C_2}{{}^{13}C_2} = \frac{15+21}{78} = \frac{36}{78} = \frac{6}{13}$

32. (C) $f(\theta) = \sin\theta(\sin\theta + \sin3\theta)$

$f(\theta) = (\sin\theta + 3\sin\theta - 4\sin^3\theta).\sin\theta$

$f(\theta) = (4\sin\theta - 4\sin^3\theta)\sin\theta$

$f(\theta) = 4\sin^2\theta(1 - \sin^2\theta)$

$f(\theta) = 4\sin^2\theta.\cos^2\theta$

$f(\theta) = (2\sin\theta\cos\theta)^2 = (\sin2\theta)^2 \geq 0$

$\therefore$ Which is true for all θ.

33. (C) Given that $\alpha + \beta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2} - \beta$

$\Rightarrow \tan\alpha = \tan\left(\frac{\pi}{2} - \beta\right) \Rightarrow \tan\alpha = \cot\beta$

$\Rightarrow \tan\alpha.\tan\beta = 1 \Rightarrow 1 + \tan\alpha.\tan\beta = 2$

$\therefore \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\Rightarrow \tan\gamma = \frac{\tan\alpha - \tan\beta}{2}$ [$\because \alpha - \beta = \gamma$]

$\Rightarrow 2\tan\gamma = \tan\alpha - \tan\beta$

$\Rightarrow \tan\alpha = 2\tan\gamma + \tan\beta$

34. (B) Since the given point $(-a, 2a)$ lies on the directrix $x = -a$ of the parabola $y^2 = 4ax$. Thus the tangents are at right angle.

35. (A) The equation of any tangent to the parabola $y^2 = 4ax$ is terms of its slope m is $y = mx + \frac{a}{m}$ and the coordinate of the point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ so tangent equation is $y = mx + \frac{a}{4m}$ and the coordinates of the point of contant are $\left(\frac{a}{4m^2}, \frac{a}{2m}\right)$

It is given that $m = \tan 45 = 1$

So, the coordinates of the point of contact are $\left(\frac{a}{4}, \frac{a}{2}\right)$

36. (B) The given curve $y - e^{xy} + x = 0$

On differentiating both sides w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} - e^{xy}\left(x.\frac{dy}{dx} + y\right) + 1 = 0$

$\Rightarrow \frac{dy}{dx}(1 - x e^{xy}) = y.e^{xy} - 1$

$\Rightarrow \frac{dy}{dx} = \frac{(y.e^{xy} - 1)}{1 - x.e^{xy}}$

If curve has a vertical tangent, then

$\frac{dx}{dy} = 0 \Rightarrow \frac{1 - x.e^{xy}}{(y.e^{xy} - 1)} = 0$

$\Rightarrow 1 - x.e^{xy} = 0 \Rightarrow e^{xy} = \frac{1}{x}$

Clearly $y = 0$ and $x = 1$ satisfy the above equation.

Hence the required point (1, 0).

37. (D) $y = a(1 + \cos\theta)$, $x = a(\theta + \sin\theta)$

$\frac{dy}{d\theta} = -a\sin\theta, \frac{dx}{d\theta} = a(1 + \cos\theta)$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \Rightarrow \frac{dy}{dx} = -\frac{a\sin\theta}{a(1+\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = -\frac{\sin\theta}{1+\cos\theta}$

$\Rightarrow \frac{dy}{dx} = -\frac{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} \Rightarrow \frac{dy}{dx} = -\tan\frac{\theta}{2}$

$\left(\frac{dy}{dx}\right)_{\text{at } x=\pi/3} = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

Since the tangent makes an angle α with x-axis, then

$\tan\alpha = -\frac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \tan\frac{5\pi}{6}$

$\Rightarrow \alpha = \frac{5\pi}{6}$

38. (A) Curve $xy + ax + by = 0$ ...(i)

On differentiating both sides w.r.t. 'x'

$\Rightarrow x.\frac{dy}{dx} + y + a + b\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(x + b) = -(a + y) \Rightarrow \frac{dy}{dx} = -\frac{(a+y)}{(x+b)}$

At point (1, 1), $\frac{dy}{dx} = -\frac{(a+1)}{(b+1)}$ ...(ii)

Since, the tangent makes an angle $\tan^{-1}2$ with x-axis therefore slope of tangent is 2.

So $\left(\dfrac{dy}{dx}\right)_{1,1} = 2 \Rightarrow -\dfrac{(a+1)}{(b+1)} = 2$

$\Rightarrow -a - 1 = 2b + 2 \Rightarrow a + 2b = -3$ ...(iii)

Since point (1, 1) lie on curve (i) so

$a + b = -1$ ...(iv)

Solving equation (iii) and (iv), we get

$a = 1,\ b = -2$

39. (B) $\lim\limits_{x\to\pi/4} \dfrac{(1-\tan x)(1+\sin 2x)}{(1+\tan x)(\pi - 4x)}$ $\left[\dfrac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim\limits_{x\to\pi/4} \dfrac{(1-\tan x)(2\cos 2x)+(1+\sin 2x)(-\sec^2 x)}{(1+\tan x)(-4)+(\pi-4x)(\sec^2 x)}$

$\Rightarrow \dfrac{\left(1-\tan\frac{\pi}{4}\right)\left(2\cos\frac{\pi}{2}\right)+\left(1+\sin\frac{\pi}{2}\right)\left(-\sec^2\frac{\pi}{4}\right)}{\left(1+\tan\frac{\pi}{4}\right)(-4)+(\pi-\pi)\sec^2\frac{\pi}{4}}$

$\Rightarrow \dfrac{0+2(-2)}{2(-4)+0} = \dfrac{-4}{-8} = \dfrac{1}{2}$

40. (D) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega^4 \\ \omega^7 & \omega^8 & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & 1 & \omega \\ 1+\omega+\omega^2 & \omega^2 & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & \omega \\ 0 & \omega^2 & 1 \end{vmatrix} = 0$

41. (C) $y = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots\ldots\ldots\ldots\ldots$

$\Rightarrow y = \cos x$

On differentiating both side w.r.t. 'x'

$\Rightarrow \dfrac{dy}{dx} = -\sin x$

42. (A) Given that $f(x) = x^2 - 2x + 2$

$a = 2,\ b = \dfrac{5}{2} \Rightarrow f(a) = 2,\ f(b) = \dfrac{13}{4}$

$f'(x) = 2x - 2$

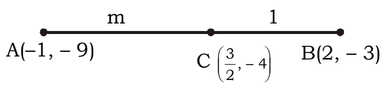
$f'(c) = 2c - 2$

Now, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow 2c - 2 = \dfrac{\frac{13}{4} - 2}{\frac{5}{2} - 2} \Rightarrow 2c - 2 = \dfrac{\frac{5}{4}}{\frac{1}{2}}$

$\Rightarrow 2c - 2 = \dfrac{5}{2} \Rightarrow 2c = \dfrac{9}{2} \Rightarrow c = \dfrac{9}{4}$

43. (D) Let ratio = m : 1



$\dfrac{m \times 2 - 1}{m+1} = \dfrac{3}{2}$, $\dfrac{-3m - 9}{m+1} = -4$

$4m - 2 = 3m + 3$, $-3m - 9 = -4m - 4$

$m = 5$, $m = 5$

Hence ratio = 5 : 1

44. (A) We know that

$(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots\ldots + {}^nC_n x^n$

On differentiating both side w.r.t. 'x'

$n(1+x)^{n-1} = 0 + {}^nC_1 + C_2(2x) + \ldots\ldots {}^nC_n (nx^{n-1})$

On putting $x = 1$

n. $2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 \ldots n\,{}^nC_n$

Hence $C_1 + C_2 + 3\,C_3 + \ldots\ldots\ldots\ldots nC_n = n.\ 2^{n-1}$

45. (C) Area = $\int_0^2 \left(e^x - e^{-x}\right) dx = \left[e^x + e^{-x}\right]_0^2$

$= e^2 + e^{-2} - e^0 - e^0 = e^2 + \dfrac{1}{e^2} - 2$

$= \left(e - \dfrac{1}{e}\right)^2$ sq. unit

46. (B) When $\theta = 180$

M = $\dfrac{60}{11}$ (H $\pm$ 6) where $+ \to$ H < 6

$- \to$ H > 6

H = 4 (between 4 and 5 O' clock)

M = $\dfrac{60}{11}$ (4 + 6)

M = $\dfrac{60}{11} \times 10 = 54\dfrac{6}{11}$

Hence time = 4 : $54\dfrac{6}{11}$

47. (C) A = {1,2,3,4,6,8,9,0}, $n = 8$

No. of proper subsets of A = $2^n - 1 = 2^8 - 1$ = 255

48. (A) Degree = 3

49. (D) Let $f(x) = \dfrac{x}{[x]}$

L.H.L. = $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{h\to 0} f(2-h)$

$= \lim\limits_{h\to 0} \dfrac{2-h}{[2-h]}$

$= \dfrac{2-0}{1} = 2$

R.H.L. = $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{h\to 0} f(2+h)$

$= \lim\limits_{h\to 0} \dfrac{2+h}{[2+h]}$

$= \dfrac{2+0}{2} = 1$

L.H.L. $\neq$ R.H.L.

Hence limit does not exist.

50. (D)

51. (A) Given that $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$

and $\vec{c} = -\hat{i} + \hat{j} - 2\hat{k}$

Now, $\vec{a}\cdot(\vec{b}-\vec{c}) + \vec{b}\cdot(\vec{c}-\vec{a}) + \vec{c}\,(\vec{a}-\vec{b})$

$\Rightarrow \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} - \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a} - \vec{c}\cdot\vec{b} = 0$

52. (B)

class	x	f	$f \times x$	$\lvert x-\bar{x}\rvert$	$\Sigma f \times \lvert x-\bar{x}\rvert$
0 – 10	5	13	65	23	299
10 – 20	15	12	180	13	156
20 – 30	25	15	375	3	45
30 – 40	35	18	630	7	126
40 – 50	45	22	990	17	374
		$\Sigma f = 80$	$\Sigma f \times x = 2240$		$\Sigma f \times \lvert x-\bar{x}\rvert = 1000$

Mean $(\bar{x}) = \dfrac{\Sigma f \times x}{\Sigma f}$

$\bar{x} = \dfrac{2240}{80} = 28$

Mean-deviation $= \dfrac{\Sigma f \times \lvert x-\bar{x}\rvert}{\Sigma f}$

$= \dfrac{1000}{80} = 12.5$

53. (B) 1 0 1

$1\times 2^0 = 1$

$0\times 2^1 = 0$

$1\times 2^2 = 4$

5

.11

$\dfrac{1}{2} = 1\times 2^{-1}$

$\dfrac{1}{4} = 1\times 2^{-2}$

$\dfrac{1}{2}+\dfrac{1}{4} = \dfrac{3}{4} = 0.75$

$(101)_2 = (5)_{10}$, $(0.11)_2 = (0.75)_{10}$

Hence $(101.11)_2 = (5.75)_{10}$

54. (C) $I = \int_{-1}^{1} \lvert x^3\rvert\, dx$

$I = -\int_{-1}^{0} x^3\, dx + \int_{0}^{1} x^3\, dx$

$I = -\left[\dfrac{x^4}{4}\right]_{-1}^{0} + \left[\dfrac{x^4}{4}\right]_{0}^{1}$

$I = -\left[0-\dfrac{1}{4}\right] + \left[\dfrac{1}{4}-0\right]$

$I = \dfrac{1}{4}+\dfrac{1}{4} = \dfrac{1}{2}$

55. (D) Sphere $x^2 + y^2 + z^2 + 18x - 12y + 6z - 18 = 0$

$u = 9,\ v = -6,\ z = 3,\ d = -18$

radius $'r' = \sqrt{u^2+v^2+w^2-d}$

$'r' = \sqrt{81+36+9+18}$

$'r' = \sqrt{144} = 12$

diameter $= 2r = 2\times 12 = 24$

56. (D) Minimum value of $(20\sin\theta + 21\cos\theta)$

$= -\sqrt{(20)^2+(21)^2}$

$= -\sqrt{400+441} = -29$

Now, min. value of $29 + 20\sin\theta + 21\cos\theta$

$= 29 - 29 = 0$

57. (A) $y = (\sin x)^{(\sin x)^{(\sin x)\ldots\ldots\infty}}$

$y = (\sin x)^y$

On taking log both side

$\log y = y \log\sin x$

On differentiating both side w.r.t. $'x'$

$\dfrac{1}{y}\dfrac{dy}{dx} = y \times \dfrac{\cos x}{\sin x} + \log\sin x \times \dfrac{dy}{dx}$

$\left(\dfrac{1}{y} - \log\sin x\right)\dfrac{dy}{dx} = y\cot x$

$\left(\dfrac{1-y\log\sin x}{y}\right)\dfrac{dy}{dx} = y\cot x$

$\dfrac{dy}{dx} = \dfrac{y^2\cot x}{1-y\log\sin x}$

58. (B) $\begin{bmatrix} a & b & c \\ m & n & o \end{bmatrix}_{2\times 3} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}_{3\times 1}$ $\quad [p \quad q]_{1\times 2}$

Hence order = 2 × 2

59. (A) differential equation

$\dfrac{dy}{dx} + y.\tan x = \sec x$

here $P = \tan x$ and $Q = \sec x$

I.F. $= e^{\int P.dx}$

$= e^{\int \tan x\, dx}$

$= e^{\int \log\sec x} = \sec x$

Solution of differential equation

$y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx$

$\Rightarrow y \times \sec x = \int \sec x.\sec x\, dx$

$\Rightarrow y \times \sec x = \tan x + c$

$\Rightarrow y = \sin x + c.\cos x$

60. (B) The required Probability $= \dfrac{1+1}{7} = \dfrac{2}{7}$

61. (B) Given that $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{7,8,3\}$, $B=\{3,8,9\}$ and $C=\{9,3,4\}$

Now, $(A\cup B)=\{3,7,8,9\}$, $(B\cap C)=\{3\}$ and $(A\cap C)=\{3\}$

$\{(A\cup B)-(B\cap C)\}\times(A\cap C)$

$=[\{3,7,8,9\}-\{3\}]\times\{3\}$

$=\{7,8,9\}\times\{3\}$

$=\{(7,3),(8,3),(9,3)\}$

62. (A) $z=\frac{2-i}{(1-2i)^2}=\frac{2-i}{-3-4i}$

$z=\frac{i-2}{3+4i}\times\frac{3-4i}{3-4i}\Rightarrow z=\frac{-2+11i}{13}$

conjugate of $z=\frac{-2-11i}{13}$

63. (D) $\begin{bmatrix}2x & 3\\ -5 & 3x\end{bmatrix}=\begin{bmatrix}4y+8 & y+6\\ 2y+1 & y-3\end{bmatrix}$

On comparing

$2x = 4y + 8$, $3 = y + 6$

$-5 = 2y + 1$, $3x = y - 3$

On solving

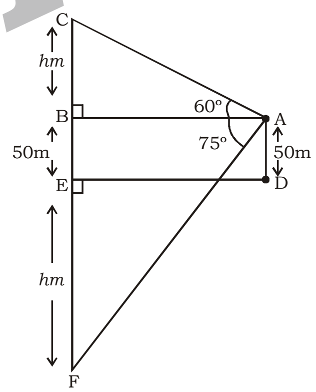
$y = -3$ and $x = -2$

64. (C) Equation $\lambda x^2 + 3x + (\lambda-1) = 0$

product of roots $= -2$

$\frac{\lambda-1}{\lambda}=-2\Rightarrow\lambda=\frac{1}{3}$

65. (C) Let BC = h m



In ΔABC

$\tan 60° = \frac{BC}{AB}$

$\sqrt{3}=\frac{h}{AB}$ ....(i)

In ΔABF

$\tan 75° = \frac{BF}{AB}$

$2+\sqrt{3}=\frac{h+50}{h/\sqrt{3}}$ ....(ii)

$2h+h\sqrt{3}=h\sqrt{3}+50\sqrt{3}\Rightarrow h=25\sqrt{3}$

height of the aeroplane above the lake level = $h + 50$

$= 25\sqrt{3} + 50 = 25(2+\sqrt{3})$m

66. (D) According to question

$\theta\times\theta\times\frac{\pi}{180}=\frac{125\pi}{4}$

$\theta^2=\frac{180\times125}{4}\Rightarrow\theta=75°$

67. (B) Word "STATEMENT"

The total no. of arrangement $=\frac{9!}{3!2!}=\frac{9!}{12}$

No. of arrangement when T's come together $=\frac{7!}{2!}=\frac{7!}{2}$

No. of arrangement when T's don't come together $=\frac{9!}{12}-\frac{7!}{2}$

$= 6\times 7! - \frac{7!}{2}=\frac{11\times7!}{2}$

68. (C) $y = \text{cosec}(\cot^{-1}x)$ ....(i)

On differentiating both side w.r.t. 'x'

$\Rightarrow\frac{dy}{dx}=-\text{cosec}(\cot^{-1}x).\cot(\cot^{-1}x).\frac{-1}{1+x^2}$

$\Rightarrow\frac{dy}{dx}=\text{cosec}(\cot^{-1}x).\frac{x}{1+x^2}$

$\Rightarrow\frac{dy}{dx}=\frac{yx}{1+x^2}$ [from eq (i)]

$\Rightarrow(1+x^2)dy = yx\,dx$

69. (B) $f(x)=\begin{cases}3x^2-4, & 2\le x<4\\ \lambda x+x^2, & 4\le x<6\end{cases}$ is continuous at $x = 4$,

then $\lim_{x\to4^-}f(x)=\lim_{x\to4^+}f(x)$

$\Rightarrow\lim_{x\to4}3x^2-4=\lim_{x\to4}\lambda x+x^2$

$\Rightarrow 3\times16-4=\lambda\times4+16$

$\Rightarrow 44 = 4\lambda + 16\Rightarrow\lambda=7$

70. (C) $\begin{bmatrix} x+7 & 13 \\ 5 & 2x \end{bmatrix} = \begin{bmatrix} y+8 & y+9 \\ y+1 & 10 \end{bmatrix}$

On comparing

$x + 7 = y + 8 \Rightarrow x - y = 1$, $13 = y + 9 \Rightarrow y = 4$

$5 = y + 1 \Rightarrow y = 4$, $2x = 10 \Rightarrow x = 5$

71. (D) Given that

$\int x^3.e^{2x}dx = ax^3.e^{2x} + bx^2.e^{2x} + cx\,e^{2x} + d\,e^{2x} + k$ ...eq.(i)

Let I = $\int x^3.e^{2x}\,dx$

I = $x^3.\int e^{2x}dx - \int\left\{\frac{d}{dx}(x^3).\int e^{2x}dx\right\}dx + k$

I = $x^3.\frac{e^{2x}}{2} - \int 3x^2.\frac{e^{2x}}{2}dx + k$

I = $\frac{1}{2}x^3.e^{2x} - \frac{3}{2}\left[x^2.\int e^{2x}dx - \int\left\{\frac{d}{dx}(x^2).\int e^{2x}dx\right\}dx\right] + k$

I = $\frac{1}{2}x^3.e^{2x} - \frac{3}{2}\left[x^2.\frac{e^{2x}}{2} - \int 2x.\frac{e^{2x}}{2}dx\right] + k$

I= $\frac{1}{2}x^3.e^{2x} - \frac{3}{4}x^2.e^{2x} + \frac{3}{2}\int x.e^{2x}\,dx + k$

I = $\frac{1}{2}x^3.e^{2x} - \frac{3}{4}x^2.e^{2x} + \frac{3}{2}\left[\frac{x.e^{2x}}{2} - \frac{1}{2}\frac{e^{2x}}{2}\right] + k$

I = $\frac{1}{2}x^3.e^{2x} - \frac{3}{4}x^2.e^{2x} + \frac{3}{4}x.e^{2x} - \frac{3}{8}e^{2x} + k$

On comparing eq(i)

$a = \frac{1}{2}$, $b = \frac{-3}{4}$, $c = \frac{3}{4}$, $d = \frac{-3}{8}$

72. (C) Given that = (2001)!

Now, $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + ...+ \frac{1}{\log_{2001} n}$

$\Rightarrow \log_n 2 + \log_n 3 + \log_n 4 +.... + \log_n 2007$

$\Rightarrow \log_n(2 \times 3 \times 4 \times ....2001)$

$\Rightarrow \log_n(2001)! \Rightarrow \log_n n = 1$

73. (B) z = -1 + $\sqrt{3}i$

arg(z) = $\tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

= $\tan^{-1}\left(-\tan\frac{\pi}{3}\right)$

= $\tan^{-1}\left[\tan\left(\frac{-\pi}{3}\right)\right] = \frac{-\pi}{3}$

74. (D) remainder = $\frac{5^7 + 11^7}{8}$

= $(-3)^7 + 3^7$

= $-3^7 + 3^7 = 0$

Hence the given number is divisible by 8.

75. (C) A = $\begin{bmatrix} \sin\theta & \cos\theta & 1 \\ \cos\theta & 1 & \sin\theta \\ 1 & \sin\theta & \cos\theta \end{bmatrix}$

$|A| = \begin{vmatrix} \sin\theta & \cos\theta & 1 \\ \cos\theta & 1 & \sin\theta \\ 1 & \sin\theta & \cos\theta \end{vmatrix}$

= $\sin\theta(\cos\theta - \sin^2\theta) - \cos\theta(\cos^2\theta - \sin\theta) + 1(\sin\theta.\cos\theta - 1)$

= $\sin\theta.\cos\theta - \sin^3\theta - \cos^3\theta + \sin\theta.\cos\theta + \sin\theta.\cos\theta - 1$

= $3\sin\theta.\cos\theta - \sin^3\theta - \cos^3\theta - 1$

76. (D) C(2n, r+1) + 2C(2n, r) + C(2n, r-1)

$\Rightarrow {}^{2n}C_{r+1} + {}^{2n}C_r + {}^{2n}C_r + {}^{2n}C_{r-1}$

We know that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{2n+1}C_{r+1} + {}^{2n+1}C_r$

$\Rightarrow {}^{2n+2}C_{r+1}$ = C(2n + 2, r + 1)

77. (B) $\cos x = \frac{1}{\sqrt{10}}$ and $\cos y = \frac{1}{\sqrt{5}}$

$\sin x = \frac{3}{\sqrt{10}}$, $\sin y = \frac{2}{\sqrt{5}}$

Now, cos(x + y) = cos x.cos y - sin x.sin y

$\Rightarrow \cos(x+y) = \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}} - \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{5}}$

$\Rightarrow \cos(x+y) = \frac{1}{\sqrt{50}} - \frac{6}{\sqrt{50}}$

$\Rightarrow \cos(x+y) = \frac{-5}{\sqrt{50}}$

$\Rightarrow \cos(x+y) = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos(x+y) = \cos\frac{3\pi}{4} \Rightarrow x + y = \frac{3\pi}{4}$

78. (D) Remainder = $\frac{11^{23} + 7^{23}}{9}$

= $(2)^{23} + (-2)^{23}$

= $2^{23} - 2^{23} = 0$

The number $11^{23} + 7^{23}$ will be divisible by 9.

79. (B) The required no. of triangles = ${}^9C_3 - {}^4C_3$ = 84 – 4 = 80

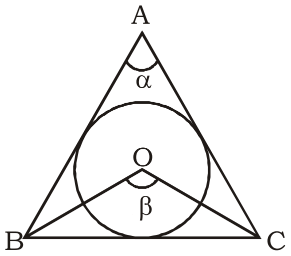
80. (B) $\cosh x + \sinh x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = e^x$

81. (D)

5	5	4	3

= 5 × 5 × 4 × 3 = 300

82. (B)



We know that

$\angle BOC = 90 + \frac{\angle BAC}{2}$

$\Rightarrow \beta = 90 + \frac{\alpha}{2}$

$\Rightarrow \sin\beta = \sin\left(90 + \frac{\alpha}{2}\right)$

$\Rightarrow \sin\beta = \cos\frac{\alpha}{2}$

$\Rightarrow \frac{2\tan\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}} = \cos\frac{\alpha}{2}$

83. (C) Let $y = \sin\left(x - \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{6}\right)$

$\Rightarrow \frac{dy}{dx} = \cos\left(x - \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right)$

for maximum and minima

$\cos\left(x - \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = 0$

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \sin\left(x - \frac{\pi}{6}\right)$

$\Rightarrow x - \frac{\pi}{6} = \frac{\pi}{2} - x + \frac{\pi}{6} \Rightarrow x = \frac{5\pi}{12}$

84. (A) P (0, -1, -2), Q(4, 1, 2), R(5, -1, 3)

$\overrightarrow{PQ}$ = (4- 0, 1+1, 2+ 2) = (4, 2, 4)

$\overrightarrow{PR}$ = (5 - 0, -1+1, 3 + 2) = (5 , 0, 5)

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 4 \\ 5 & 0 & 5 \end{vmatrix}$

$= \hat{i}\,(10 - 0) - \hat{j}\,(20 - 20) + \hat{k}\,(0 - 10)$

$= 10\hat{i} - 10\hat{k}$

$a = 10,\ b = 0,\ c = -10$

Equation of plane

$a(x - x_0) + b(y - y_0) + c\,(z - z_0) = 0$

$\Rightarrow 10(x - 0) + 0\,(y + 1) - 10(z + 2) = 0$

$\Rightarrow 10x - 10z - 20 = 0 \Rightarrow x - z = 2$

85. (B) $\begin{vmatrix} \sec\theta & \tan\theta \\ -\tan\theta & -\sec\theta \end{vmatrix}$

$\Rightarrow -\sec^2\theta + \tan^2\theta$

$\Rightarrow -(\sec^2\theta - \tan^2\theta) = -1$

86. (B) Let $y = 5^{21}$

taking log both side

$\Rightarrow \log_{10}y = 21\log_{10}5$

$\Rightarrow \log_{10}y = 21 \times 0.699$

$\Rightarrow \log_{10}y = 14.679$

The required no. of digits = 14 +1 = 15

87. (B) $x + \log_6(2^x - 1) = x\log_6 3 + \log_6 12$

$\Rightarrow x = \log_6\left(\frac{3^x \times 12}{2^x - 1}\right)$

$\Rightarrow 6^x = \frac{3^x \times 12}{2^x - 1}$

$\Rightarrow 2^x \times 3^x = \frac{3^x \times 12}{2^x - 1}$

$\Rightarrow (2^x)^2 - 2^x = 12$

$\Rightarrow (2^x - 4)(2^x + 3) = 0$

$\Rightarrow 2^x - 4 = 0, \quad 2^x + 3 \neq 0$

$\Rightarrow 2^x = 2^2 \Rightarrow x = 2$

88. (B) $n(S) = 2^5 = 32$

$n(E) = {}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3$

$n(E) = 1 + 5 + 10 + 10 = 26$

Probability $P(E) = \frac{n(E)}{n(S)} = \frac{26}{32} = \frac{13}{16}$

89. (C) $a + 3d$, $a + 6d$ and $a + 8d$ are in G.P.

Now,

$(a + 6d)^2 = (a + 3d)(a + 8d)$

$\Rightarrow a^2 + 36d^2 + 12ad = a^2 + 3ad + 8ad + 24d^2$

$\Rightarrow 12d^2 + ad = 0$

$\Rightarrow (12d + a)d = 0$

$\Rightarrow 12d + a = 0$

$\Rightarrow a = -12d,\ d \neq 0$

Common Ratio $= \frac{a + 6d}{a + 3d}$

$= \frac{-12d + 6d}{-12d + 3d}$

$= \frac{-6d}{-9d} = \frac{2}{3}$

90. (B) Equation $x^2 + ax + b = 0$

let roots $\alpha, k\alpha$

$\alpha + k\alpha = -a \Rightarrow \alpha(1+k) = -a$ ...(i)

$\alpha.k\alpha = b \Rightarrow \alpha^2 k = b$ ...(ii)

and equation

$x^2 + mx + n = 0$

let roots $\beta, k\beta$

$\beta + k\beta = -a \Rightarrow \beta(1+k) = -m$ ...(iii)

$\beta.k\beta = h \Rightarrow \beta^2 k = n$ ...(iv)

from eq(i) and eq(ii)

$\frac{\alpha}{\beta} = \frac{a}{m}$ ...(v)

from eq(ii) and eq(iv)

$\frac{\alpha^2}{\beta^2} = \frac{b}{n} \Rightarrow \left(\frac{a}{m}\right)^2 = \frac{b}{n}$ [from eq (v)]

$\Rightarrow a^2 n = m^2 b$

91. (C) $\left(\frac{-1+\sqrt{3}i}{2}\right)^{10} + \left(\frac{-1-\sqrt{3}i}{2}\right)^{10}$

We know that

$\omega = \frac{-1+\sqrt{3}i}{2}$ and $\omega^2 = \frac{-1-\sqrt{3}i}{2}$

$\Rightarrow \omega^{10} + (\omega^2)^{10}$

$\Rightarrow \omega + \omega^2 = -1$ $[\because 1 + \omega + \omega^2 = 0]$

92. (B) $S = (1^2 + 1) + (2^2 + 2) + (3^2 + 3) + \ldots\ldots\ldots\ldots$

$S = (1^2 + 2^2 + 3^2 + \ldots\ldots\ldots) + (1+2+3+\ldots\ldots\ldots)$

$S = \frac{n}{6}(n+1)(2n+1) + \frac{n(n+1)}{2}$

$S = \frac{n(n+1)}{6} \times (2n+4)$

$S = \frac{n(n+1)(n+2)}{3}$

93. (B) According to question

$$\frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{3n+1}{7n-3}$$

Now, put $\frac{n-1}{2} = 12 \Rightarrow n = 25$

$$\frac{T_{13}}{T'_{13}} = \frac{a+12d}{a'+12d'} = \frac{3\times 25+1}{7\times 25-3}$$

$$= \frac{76}{172} = \frac{19}{43}$$

94. (C) $S_n = nA + \frac{n(n+1)B}{2}$

$S_{n-1} = (n-1)A + \frac{n(n-1)B}{2}$

$T_n = S_n - S_{n-1}$

$T_n = nA + \frac{n(n+1)B}{2} - (n-1)A - \frac{n(n-1)B}{2}$

$T_n = A + Bn$

$T_{n-1} = A + B(n-1)$

Now, common difference $d = T_n - T_{n-1}$

$\Rightarrow d = A + Bn - A - B(n-1) \Rightarrow d = B$

95. (B) $S = 7 \times 7^{\frac{1}{2}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{8}} \times \ldots\ldots\ldots\ldots\infty$

$S = 7^{\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots\ldots\infty\right)}$

$S = 7^{\frac{1}{1-\frac{1}{2}}} \Rightarrow S = 7^2 = 49$

96. (B) Given that $\vec{a} = 2\hat{i} + 6\hat{j} - 3\hat{k}$ and

$\vec{b} = 8\hat{i} - 12\hat{j} + 9\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \left|\frac{2\times 8 + 6\times(-12) - 3\times 9}{\sqrt{8^2 + (-12)^2 + 9^2}}\right|$$

$$= \left|\frac{16 - 72 - 27}{17}\right| = \frac{83}{17}$$

97. (C) $n(S) = 6 \times 6$

$$E = \begin{Bmatrix} (6,1),(1,6),(5,2),(2,5) \text{ for sum} = 7 \\ (6,4),(4,6),(5,5) \text{ for sum} = 10 \end{Bmatrix}$$

$n(E) = 7$

Probability that the sum is 7 or 10

$P(E) = \frac{n(E)}{n(S)} = \frac{7}{36}$

Probability that the sum is neither 7 or 10 $= P(\overline{E}) = 1 - P(E)$

$$P(\overline{E}) = 1 - \frac{7}{36} = \frac{29}{36}$$

98. (C)

99. (C)

100. (B) $\begin{bmatrix} 1 & 2 & -4 \\ 2 & -1 & 0 \\ -3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 16 \end{bmatrix}$

Using elementary method

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 2 & -1 & 0 & 7 \\ -3 & -4 & -2 & 16 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 3R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 0 & -5 & 8 & -25 \\ 0 & 2 & -14 & 64 \end{array}\right]$$

$R_3 \to R_3 + \frac{2}{5}R_2$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 0 & -5 & 8 & -25 \\ 0 & 0 & \frac{-54}{5} & 54 \end{array}\right]$$

Now, $x + 2y - 4z = 16$ .....(i)

$5y + 8z = 25$ ....(ii)

$-\frac{54}{5}z = 54$ ...(iii)

On solving

$x = 2,\ y = -3,\ z = -5$

Hence $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix}$

101. (C) Two circles $x^2 + y^2 + 3x - y - 4 = 0$ and $x^2+y^2-2x-6y+\lambda = 0$ are orthogonal, then $2gg' + 2ff' = c + c'$

$\Rightarrow 2 \times \frac{3}{2} \times (-1) + 2 \times \left(\frac{-1}{2}\right) \times (-3) = -4 + \lambda$

$\Rightarrow -3 + 3 = -4 + \lambda \Rightarrow \lambda = 4$

102. (C) equation $bx^2 + cx + a = 0$

$\alpha + \beta = \frac{-c}{b}$ and $\alpha.\beta = \frac{a}{b}$

Now, $(b\alpha - a)(b\beta - a)$

$\Rightarrow b^2\alpha\beta - ab\beta - ab\alpha + a^2$

$\Rightarrow b^2 \times \alpha.\beta - ab(\alpha + \beta) + a^2$

$\Rightarrow b^2 \times \frac{a}{b} - ab \times \left(\frac{-c}{b}\right) + a^2$

$\Rightarrow ab + ac + a^2 \Rightarrow a(a + b + c)$

103. (B)

104. (D) $S = 1 + 11 + 111 + \ldots\ldots n$ terms

$S = \frac{1}{9}(9+99+999+\ldots\ldots n \text{ terms})$

$S = \frac{1}{9}[(10-1)+(100-1)(1000-1)+\ldots n \text{ terms}]$

$S = \frac{1}{9}[10+100+1000+\ldots\ldots n \text{ terms}]$

$-\frac{1}{9}(1+1+1+\ldots\ldots n \text{ terms})$

$S = \frac{1}{9} \times \frac{10(10^n - 1)}{10-1} - \frac{1}{9} \times n$

$S = \frac{10}{81}(10^n - 1) - \frac{n}{9}$

105. (B) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) \Rightarrow \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{4}\right)\right]$

$\Rightarrow \sin^{-1}\left[\sin\frac{\pi}{4}\right] = \frac{\pi}{4}$

106. (A) $I = \int_0^1 \frac{e^{\cot^{-1}x}}{1+x^2}dx$

Let $\cot^{-1}x = t$ when $x \to 0,\ t \to \frac{\pi}{2}$

$-\frac{1}{1+x^2}dx = dt$ $x \to 1,\ t \to \frac{\pi}{4}$

$\frac{1}{1+x^2}dx = -dt$

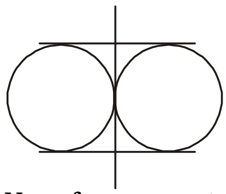
$I = \int_{\pi/2}^{\pi/4} -e^t dt$

$I = -\left[e^t\right]_{\pi/2}^{\pi/4}$

$I = -[e^{\pi/4} - e^{\pi/2}]$

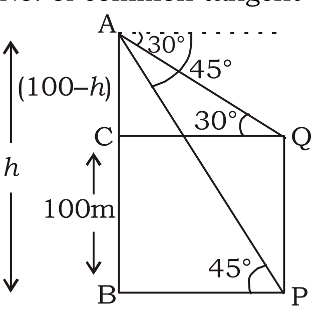
$I = e^{\pi/2} - e^{\pi/4}$

107. (C)



No. of common tangent = 3

108. (A)



Let height of the mountain = h m

In ΔABP :-

$\tan 45° = \frac{AB}{BP}$

$\Rightarrow 1 = \frac{h}{BP} = BP = h = CQ$

In ΔACQ :-

$\tan 30° = \frac{AC}{CQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100-h}{h}$

$\Rightarrow h = 100\sqrt{3} - \sqrt{3}\,h \Rightarrow h = 50(3-\sqrt{3})\,m$

109. (C) $\lim_{x\to\infty} \frac{1^2+2^2+3^2+....+n^2}{n(1+2+3+....+n)}$

$\Rightarrow \lim_{x\to\infty} \frac{\frac{n}{6}(n+1)(2n+1)}{n \times \frac{n(n+1)}{2}}$

$\Rightarrow \lim_{x\to\infty} \frac{(2n+1)}{3n}$

$\Rightarrow \lim_{x\to\infty} \frac{n\left(2+\frac{1}{n}\right)}{3n} = \frac{2}{3}$

110. (B) $f(x) = \{[x]-3\}^2 - [x-3]$

At $x = 3$

L.H.L. $= \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0} \qquad - [3-h-3]$

$= \lim_{h\to 0} \{2-3\}^2 - [0-h]$

$= \lim_{h\to 0} 1-(-1) = 2$

R.H.L. $= \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

$= \lim_{h\to 0} \qquad - [3+h-3]$

$= \lim_{h\to 0} \{3-3\}^2 - [0-h]$

$= \lim_{h\to 0} 0-(0) = 0$

L.H.L. $\neq$ R.H.L.

Hence $f(x)$ is not continuous at $x = 3$.

At $x = 4$

L.H.L. $= \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h)$

$= \lim_{h\to 0} \{[4-h]-3\}^2 - [4-h-3]$

$= \lim_{h\to 0} \{3-3\}^2 - [1-h]$

$= \lim_{h\to 0} 0-0 = 0$

R.H.L. $= \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h)$

$= \lim_{h\to 0} \{[4+h]-3\}^2 - [4+h-3]$

$= \lim_{h\to 0} \{4-3\}^2 - [1+h]$

$= \lim_{h\to 0} (1-1) = 0$

L.H.L. = R.H.L.

Hence $f(x)$ is continuous at $x = 4$.

111. (D) $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Now, $A^2 + kA + I_2 = 0$

$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + k\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 12+3k & 8+2k \\ 4+k & 4+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On comparing $k = -4$

112. (C) $\overrightarrow{OA} = \hat{i} + y\hat{j} - 3\hat{k}$

$= x\hat{i} - 3\hat{j} + 2\hat{k}$

$\overrightarrow{OC} = -\hat{i} - 4\hat{j} + 3\hat{k}$

A, B and C are collinear, then

$\overrightarrow{AB} = \lambda\overrightarrow{BC}$

$\Rightarrow (x-1)\,\hat{i} + (-3-y)\,\hat{j} + (2+3)\,\hat{k}$

$= \lambda\left[(-1-x)\hat{i} + (-4+3)\hat{j} + (3-2)\hat{k}\right]$

On comparing

$x - 1 = \lambda(-1-x) \Rightarrow x(1+\lambda) = (1-\lambda)$ ...(i)

$-3 - y = \lambda(-4+3) \Rightarrow 3 + y = \lambda$ ...(ii)

$2 + 3 = \lambda(3-2) \Rightarrow \lambda = 5$ ...(iii)

from eq(i) and eq (ii)

$x = \frac{-2}{3}$ and $y = 2$

Hence $(x, y) = \left(\frac{-2}{3}, 2\right)$

113. (A) $\vec{a} = -\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = -4\hat{i} + 3\hat{k}, \vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

$= (-1)(-4) + 1 \times 0 + (-2)\times 3 = -2$

$\vec{a}.\vec{c} = (-1)(-2) + 1(-1) + (-2)\times 5 = -9$

We know that

$(\vec{x}\times\vec{y})\times\vec{z} = (\vec{x}.\vec{z})\,\vec{y} - (\vec{x}.\vec{y})\,\vec{z}$

Now, $(\vec{a}\times\vec{c})\times\vec{b} = \lambda\vec{c} + \mu\vec{b}$

$\Rightarrow (\vec{a}.\vec{b})\,\vec{c} - (\vec{a}.\vec{c})\,\vec{b} = \lambda\vec{c} + \mu\vec{b}$

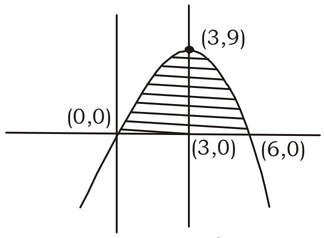
$\Rightarrow -2\vec{c} + 9\vec{b} = \lambda\vec{c} + \mu\vec{b}$

On comparing

$\lambda = -2, \mu = 9$

Hence $(\lambda, \mu) = (-2, 9)$

114. (B)



Curve $y = 6x - x^2$

Area = $\int_0^6 y\,dx$

$= \int_0^6 (6x - x^2)\,dx$

$= \left[6 \times \frac{x^2}{2} - \frac{x^3}{3}\right]_0^6$

$= \left[6 \times \frac{6^2}{2} - \frac{6^3}{3} - 0\right]$

= 108 – 72 = 36 sq.unit

115. (C) Hyperbola

$16x^2 - 4y^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{1}{4}} = 1$

$a^2 = \frac{1}{16},\ b^2 = \frac{1}{4}$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 + \frac{\frac{1}{4}}{\frac{1}{16}}}$

$\Rightarrow e = \sqrt{1+4} = \sqrt{5}$

116. (C) $\log\left(a + \sqrt{a^2 + x^2}\right) + \log\left[\frac{1}{a + \sqrt{a^2 + x^2}}\right]$

$\Rightarrow \log\left(a + \sqrt{a^2 + x^2}\right)\left(\frac{1}{a + \sqrt{a^2 + x^2}}\right)$

$[\because \log m + \log n = \log mn]$

$\Rightarrow \log 1 = 0$

117. (B) $(3\sin\theta + 4)(\sqrt{2}\sin\theta + 1) = 0$

$\sin\theta \neq \frac{-4}{3},\ \sin\theta = \frac{-1}{\sqrt{2}}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

118. (C) $y = ax\sin\left(\frac{1}{x} + b\right)$

On differentiating both side w.r.t.'x'

$\Rightarrow y_1 = ax.\cos\left(\frac{1}{x} + b\right)\left(\frac{-1}{x^2}\right) + a\sin\left(\frac{1}{x} + b\right).1$

$\Rightarrow y_1 = \frac{-a}{x}\cos\left(\frac{1}{x} + b\right) + a\sin\left(\frac{1}{x} + b\right)$

$\Rightarrow xy_1 = -a\cos\left(\frac{1}{x} + b\right) + ax\sin\left(\frac{1}{x} + b\right)$

$\Rightarrow xy_1 = -a\cos\left(\frac{1}{x} + b\right) + y$

Again, differentiating

$\Rightarrow xy_2 + y_1 = -a(-1)\sin\left(\frac{1}{x} + b\right)\left(\frac{-1}{x^2}\right) + y_1$

$\Rightarrow xy_2 + y_1 = \frac{-a}{x^2}\sin\left(\frac{1}{x} + b\right) + y_1$

$\Rightarrow x^4y_2 + x^3y_1 = -ax\sin\left(\frac{1}{x} + b\right) + x^3y_1$

$\Rightarrow x^4y_2 = -y \Rightarrow x^4y_2 + y_1 = 0$

119. (B) $\tan^{-1}\frac{4}{3} + \cos^{-1}\frac{12}{13}$

$\Rightarrow \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12}$ $\left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$

$\Rightarrow \tan^{-1}\left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}}\right)$

$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$\Rightarrow \tan^{-1}\left(\frac{63}{16}\right)$

$\Rightarrow \cos^{-1}\frac{16}{65}$ $\left[\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$

120. (A) Line $3x - 4y - 7$

$m_1 = \frac{3}{4}$

and line $3x + 5y = 9$

$m_2 = \frac{-3}{5}$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{3}{4} + \frac{3}{5}}{1 + \frac{3}{4}\left(\frac{-3}{5}\right)}\right|$

$\Rightarrow \tan\theta = \left(\frac{\frac{27}{20}}{\frac{11}{20}}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{27}{11}\right)$

NDA (MATHS) MOCK TEST - 194 (Answer Key)

1.	(D)	21.	(A)	41.	(C)	61.	(B)	81.	(D)	101.	(C)
2.	(C)	22.	(A)	42.	(A)	62.	(A)	82.	(B)	102.	(C)
3.	(B)	23.	(A)	43.	(D)	63.	(D)	83.	(C)	103.	(B)
4.	(A)	24.	(A)	44.	(A)	64.	(C)	84.	(A)	104.	(D)
5.	(D)	25.	(A)	45.	(C)	65.	(C)	85.	(B)	105.	(B)
6.	(D)	26.	(C)	46.	(B)	66.	(D)	86.	(B)	106.	(A)
7.	(B)	27.	(B)	47.	(C)	67.	(B)	87.	(B)	107.	(C)
8.	(C)	28.	(A)	48.	(A)	68.	(C)	88.	(B)	108.	(A)
9.	(B)	29.	(D)	49.	(D)	69.	(B)	89.	(C)	109.	(C)
10.	(D)	30.	(A)	50.	(D)	70.	(C)	90.	(B)	110.	(B)
11.	(D)	31.	(A)	51.	(A)	71.	(D)	91.	(C)	111.	(D)
12.	(C)	32.	(C)	52.	(B)	72.	(C)	92.	(B)	112.	(C)
13.	(D)	33.	(C)	53.	(B)	73.	(B)	93.	(B)	113.	(A)
14.	(A)	34.	(B)	54.	(C)	74.	(D)	94.	(C)	114.	(B)
15.	(D)	35.	(A)	55.	(D)	75.	(C)	95.	(B)	115.	(C)
16.	(A)	36.	(B)	56.	(D)	76.	(D)	96.	(B)	116.	(C)
17.	(B)	37.	(D)	57.	(A)	77.	(B)	97.	(C)	117.	(B)
18.	(C)	38.	(A)	58.	(B)	78.	(D)	98.	(C)	118.	(C)
19.	(A)	39.	(B)	59.	(A)	79.	(B)	99.	(C)	119.	(B)
20.	(B)	40.	(D)	60.	(B)	80.	(B)	100.	(B)	120.	(A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777