NDA MATHS MOCK TEST - 188 (SOLUTION)

1. (C) The required ways = ${}^6C_3 \times {}^{12}C_8$
$= 20 \times 495 = 9900$

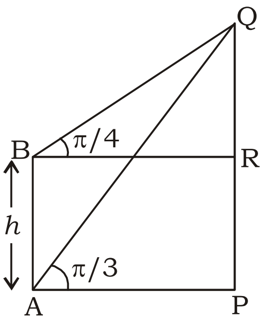
2. (D) The total numbers = $9\times9\times8\times7 = 4536$

3. (A) $\dfrac{1+\cot^2\theta}{\cot\theta} \Rightarrow \dfrac{1+\dfrac{\cos^2\theta}{\sin^2\theta}}{\dfrac{\cos\theta}{\sin\theta}}$

$\Rightarrow \dfrac{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}{\dfrac{\cos\theta}{\sin\theta}} \Rightarrow \dfrac{1}{\sin\theta.\cos\theta}$

$\Rightarrow \dfrac{2}{2\sin.\cos\theta} \Rightarrow \dfrac{2}{\sin2\theta} \Rightarrow 2\ \text{cosec}2\theta$

4. (B)



Let RQ = x

In ΔBRQ :-

$\tan\dfrac{\pi}{4} = \dfrac{QR}{BR}$

$\Rightarrow 1 = \dfrac{x}{BR} \Rightarrow BR = x = AP$

In ΔAPQ :-

$\tan\dfrac{\pi}{3} = \dfrac{PQ}{AP}$

$\Rightarrow \sqrt{3} = \dfrac{h+x}{x} \Rightarrow \sqrt{3}\,x = h + x$

$\Rightarrow (\sqrt{3}-1)x = h \Rightarrow x = \dfrac{h}{\sqrt{3}-1}$

$x = \dfrac{h(\sqrt{3}+1)}{2}$

Height of the hill = $h + x$

$= h + \dfrac{h(\sqrt{3}+1)}{2} = \dfrac{(3+\sqrt{3})h}{2}$

5. (C) $y = \tan^{-1}\left(\dfrac{3-2\tan\sqrt[3]{x}}{2+3\tan\sqrt[3]{x}}\right)$

$y = \tan^{-1}\left(\dfrac{\dfrac{3}{2}-\tan\sqrt[3]{x}}{1+\dfrac{3}{2}\tan\sqrt[3]{x}}\right)$

Let $\tan\theta = \dfrac{3}{2} \Rightarrow \theta = \tan^{-1}\dfrac{3}{2}$

$y = \tan^{-1}\left(\dfrac{\tan\theta-\tan\sqrt[3]{x}}{1+\tan\theta.\tan\sqrt[3]{x}}\right)$

$y = \tan^{-1}\left[\tan(\theta-\sqrt[3]{x})\right]$

$y = \theta - \sqrt[3]{x}$

$y = \tan^{-1}\dfrac{3}{2} - (x)^{1/3}$

On differentiating both sides

$\dfrac{dy}{dx} = 0 - \dfrac{1}{3}x^{\frac{1}{3}-1}$

$\dfrac{dy}{dx} = -\dfrac{1}{3x^{2/3}}$

6. (A) $\int_7^{10}|x-9|\,dx$

$\Rightarrow \int_7^9 -(x-9)dx + \int_9^{10}(x-9)dx$

$\Rightarrow \left[-x^2+9x\right]_7^9 + \left[x^2-9x\right]_9^{10}$

$\Rightarrow (-9^2 + 9\times9) - (-7^2 + 9\times7) + (10^2 - 9\times10) - (9^2 - 9\times9)$

$\Rightarrow 0 - 14 + 10 - 0 = -4$

7. (B) $\int e^{\ln\sin x}dx \Rightarrow \int \sin x\,dx \Rightarrow -\cos x + c$

8. (C) $u = e^{ax}\cos bx$

$\Rightarrow \dfrac{du}{dx} = e^{ax}.(-b\sin bx) + a.e^{ax}.\cos bx$

$\Rightarrow \dfrac{du}{dx} = -be^{ax}.\sin bx + a.e^{ax}.\cos bx$

and $v = e^{ax}.\sin bx$

$\Rightarrow \dfrac{dv}{dx} = b.e^{ax}.\cos bx + a.e^{ax}.\sin bx$

Now, $u\frac{du}{dx} + v.\frac{dv}{dx}$

$\Rightarrow e^{ax}\cos bx[-be^{ax}\sin bx + a.e^{ax}.\cos bx] + e^{ax}.\sin bx$

$[be^{ax}.\cos bx + ae^{ax}.\sin bx]$

$\Rightarrow -be^{2ax}.\sin bx.\cos bx + a.e^{2ax}.\cos^2 bx + b.e^{2ax}\sin bx.\cos bx + ae^{2ax}.\sin^2 bx$

$\Rightarrow a.e^{2ax}(\cos^2 bx + \sin^2 bx) = a.e^{2ax}$

9. (B) $\frac{dy}{dx} = x^3 - \frac{1}{x^2}$

$\Rightarrow dy = \left(x^3 - \frac{1}{x^2}\right)dx$

On integrating both sides

$\Rightarrow \int dy = \int\left(x^3 - \frac{1}{x^2}\right)dx$

$\Rightarrow y = \frac{x^4}{4} + \frac{1}{x} + c$ ...(i)

this equation passes through the point (–1, 2)

$\Rightarrow 2 = \frac{(-1)^4}{4} + \frac{1}{-1} + c \Rightarrow c = \frac{11}{4}$

The required equation

$y = \frac{x^4}{4} + \frac{1}{x} + \frac{11}{4}$

$\Rightarrow 4xy = x^5 + 11x + 4$

10. (B) $\log_3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}} \Rightarrow \log_3 (3)^{\frac{15}{16}} = \frac{15}{16}$

11. (B) In the expansion of $(3 + x)^6$

Total terms = 6 +1 = 7

Middle term = $T_4 = {}^6C_3\ (3)^3.(x)^6$

$= 20\times27x^6$

The required coefficient = 20×27 = 540

12. (B) $(AB)^{-1} = B^{-1}A^{-1}$

13. (C) $(1+ \omega)(1 + \omega^2)$

$\Rightarrow (-\omega^2)(-\omega)$ $[\because 1+ \omega + \omega^2 = 0]$

$\Rightarrow \omega^3 = 1$ $[\because \omega^3 = 1]$

14. (D) We know that

$\sin36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$ and $\cos36° = \frac{\sqrt{5}+1}{4}$

$\sin18° = \frac{\sqrt{5}-1}{4}$ and $\cos18° = \frac{\sqrt{10+2\sqrt{5}}}{4}$

Now, cot18°.tan36°

$\Rightarrow \frac{\cos18°}{\sin18°}.\frac{\sin36°}{\cos36°}$

$\Rightarrow \frac{\frac{\sqrt{10+2\sqrt{5}}}{4}}{\frac{\sqrt{5}-1}{4}} \times \frac{\frac{\sqrt{10-2\sqrt{5}}}{4}}{\frac{\sqrt{5}+1}{4}}$

$\Rightarrow \frac{\sqrt{100-20}}{5-1} \Rightarrow \frac{\sqrt{80}}{4} \Rightarrow \frac{4\sqrt{5}}{4} = \sqrt{5}$

15. (D) Point C divides the line joining the points A and B in ratio = $m : 1$

A.T.Q.

$\frac{m\times1+1\times(-2)}{m+1} = \frac{-1}{5}$

$\Rightarrow 5m - 10 = -m - 1$

$\Rightarrow 6m = 9 \Rightarrow m = \frac{3}{2}$

The required ratio = 3 : 2

16. (C) Given that $e = \frac{1}{2}$

and $ae = 3$

$\Rightarrow a \times \frac{1}{2} = 3 \Rightarrow a = 6$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 36\left(1-\frac{1}{4}\right)$

$\Rightarrow b^2 = 36\times\frac{3}{4} \Rightarrow b^2 = 27$

Equation of ellipse

$\frac{x^2}{6^2} = \frac{y^2}{27} = 1$

$\Rightarrow 3x^2 + 4y^2 = 108$

17. (D) $S_n = n^2 + 3n - 1$

$S_{n-1} = (n-1)^2 + 3(n-1) - 1$

$S_{n-1} = n^2 + n - 3$

$T_n = Sn - S_{n-1}$

$T_n = (n^2 + 3n + 1) - (n^2 + n - 3)$

$T_n = 2n + 2$

$T_{10} = 2\times10 + 2 = 22$

18. (D)

19. (A) AB = C

$\Rightarrow \begin{bmatrix} x-y & x \\ y & x+y \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -x+y+2x \\ -y+2x+2y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+y \\ 2x+y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

On comparing
$x + y = -2$ and $2x + y = 3$
On solving
$x = 5,\ y = -7$

$$A = \begin{bmatrix} x-y & x \\ y & x+y \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ -7 & -2 \end{bmatrix}$$

$$\text{Now, } A^2 = \begin{bmatrix} 12 & 5 \\ -7 & -2 \end{bmatrix}\begin{bmatrix} 12 & 5 \\ -7 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 12\times12+5\times(-7) & 12\times5+5\times(-2) \\ -7\times12-2\times(-7) & -7\times5-2\times(-2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 109 & 50 \\ -70 & -31 \end{bmatrix}$$

20. (D) $\frac{1}{\sin 10} - \frac{\sqrt{3}}{\cos 10}$

$$\Rightarrow \frac{\cos 10 - \sqrt{3}\sin 10}{\sin 10.\cos 10}$$

$$\Rightarrow 4\left[\frac{\frac{1}{2}\cos 10 - \frac{\sqrt{3}}{2}\sin 10}{2\sin 10.\cos 10}\right]$$

$$\Rightarrow 4\left[\frac{\cos 60.\cos 10 - \sin 60.\sin 10}{\sin 20}\right]$$

$$\Rightarrow 4\left[\frac{\cos(60+10)}{\sin(90-70)}\right]$$

$$\Rightarrow 4 \times \frac{\cos 70}{\cos 70} = 4$$

21. (C) Staight lines
$2x + 3y = 1$

slope $\Rightarrow m_1 = \frac{-2}{3}$

and $6x + 5y = 2$

slope $\Rightarrow m_2 = -\frac{6}{5}$

$$\text{Now, } \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{\frac{-2}{3} + \frac{6}{5}}{1 + \left(\frac{-2}{3}\right)\times\left(\frac{-6}{5}\right)}\right|$$

$$\Rightarrow \tan\theta = \left(\frac{\frac{8}{15}}{\frac{27}{15}}\right)$$

$$\Rightarrow \tan\theta = \frac{8}{27} \Rightarrow \theta = \tan^{-1}\left(\frac{8}{27}\right)$$

22. (C) A.T.Q,

$$\frac{4+3+y}{3} = 2 \Rightarrow y = -1$$

$$\text{and } \frac{x-6-5}{3} = 3 \Rightarrow x = 20$$

$\therefore\ x = 20,\ y = -1$

23. (D) $y = a\sin 2x + b\cos 2x$ ...(i)
On differentiating both sides w.r.t 'x'

$$\frac{dy}{dx} = 2a\cos 2x - 2b\sin 2x$$

Again, differentiating

$$\frac{d^2y}{dx^2} = -4a\sin 2x - 4b\cos 2x$$

$$\frac{d^2y}{dx^2} = -4(a\sin 2x + b\cos 2x)$$

$$\frac{d^2y}{dx^2} = -4y \quad \text{[from eq(i)]}$$

$$\frac{d^2y}{dx^2} + 4y = 0$$

24. (B) $\lim_{x\to\infty}\frac{\sin x}{x} = 0$ $[\because -1 \le \sin\theta \le 1]$

25. (A) $f(x) = \begin{cases} \frac{x^2-(k+3)x+3k}{x-3}, & x \ne 3 \\ 3, & x = 3 \end{cases}$

$$\text{L.H.L} = \lim_{x\to3^-} f(x) = \lim_{h\to0} f(3-h)$$

$$= \lim_{h\to0}\frac{(3-h)^2-(k+3)(3-h)+3k}{3-h-3}$$

$$= \lim_{h\to0}\frac{9+h^2-6h-3k-9+hk+3h+3k}{-h}$$

$$= \lim_{h\to0}\frac{h^2-3h+hk}{-h}$$

$$= \lim_{h\to0} -h+3-k$$

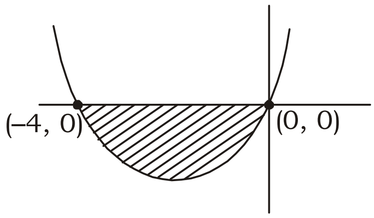
$= 3 - k$
Now, $f(3) = 3$
$\Rightarrow 3 - k = 3 \Rightarrow k = 0$

26. (C) Curve

$y = 4x + x^2$

$\Rightarrow y = 4x + x^2 + 4 - 4$

$\Rightarrow y + 4 = (x+2)^2 \Rightarrow (x+2)^2 = y + 4$



$$\text{Area} = \left|\int_4^0 y\,dx\right| = \left|\int_4^0 (4x + x^2)\,dx\right|$$

$$= \left|\left[\frac{4x^2}{2} + \frac{x^3}{3}\right]_{-4}^{0}\right|$$

$$= \left|0 + 0 - \left(4 \times \frac{(-4)^2}{2} + \frac{(-2)^3}{2}\right)\right|$$

$= |-(32 - 4)| = 28$ units

27. (B) $y = \cot^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}} \Rightarrow y = \cot^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}$

$\Rightarrow y = \cot^{-1}\left(\cot\frac{x}{2}\right) \Rightarrow y = \frac{x}{2}$

On differentiating both sides w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$

28. (D) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow \cos^{-1}\left(-\cos\frac{\pi}{6}\right)$

$\Rightarrow \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{6}\right)\right] \Rightarrow \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

29. (A) 8 sin144°.sin108°.sin72°.sin36°

$\Rightarrow$ 8 sin(180 – 36).sin(90 +18).sin(90 –18) .sin(90 – 18)

$\Rightarrow$ 8 sin36.cos18.cos18.sin36

$\Rightarrow 8\sin^2 36.\cos^2 18$

$\Rightarrow 8 \times \frac{\sqrt{10-2\sqrt{5}}}{4} \times \frac{\sqrt{10+2\sqrt{5}}}{4}$

$\Rightarrow 8 \times \frac{\sqrt{100-20}}{16} \Rightarrow 8 \times \frac{4\sqrt{5}}{16} = 2\sqrt{5}$

30. (A) We know that

$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + C_nx^n$

On differentiating both sides w.r.t. 'x'

$n(1+x)^{n-1} = 0 + C_1 + 2C_2.x + 3.C_3.x^2 + \ldots\ldots$
$\ldots\ldots + nC_nx^{n-1}$

On putting $x = -1$

$n(1-1)^{n-1} = C_1 - 2C_2 + 3C_3 + \ldots\ldots + (-1)^{n-1}.nC_n$

$0 = C_1 + 2C_2 + 3C_3 - \ldots\ldots + (-1)^{n-1}n.C_n$

Hence $C_1 - 2C_2 + 3C_3 - \ldots\ldots + (-1)^{n-1}.n.C_n = 0$

31. (D) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix}$$

$$\Rightarrow (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$R_1 \to R_1 - (R_2 + R_3)$

$$\Rightarrow (a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$C_2 \to C_2 + \frac{1}{b}C_1,\ C_3 \to C_3 + \frac{1}{c}C_1$

$$\Rightarrow (a+b+c)^2\begin{vmatrix} 2bc & 0 & 0 \\ b^2 & c+a & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$

$\Rightarrow (a+b+c)^2.2bc[(c+a)(a+b) - bc]$

$\Rightarrow 2abc.(a+b+c)^3$

32. (A) Points A($-a$, $-b$), O(0, 0), B(a, b), C(a^2, ab)

slope of OA = $\frac{-b-0}{-a-0} = \frac{b}{a}$

slope of OB = $\frac{b-0}{a-0} = \frac{b}{a}$

slope of OC = $\frac{ab-0}{a^2-0} = \frac{b}{a}$

Hence these points are collinear.

33. (A) $A^2 - B^2 = (A - B)(A + B)$

$\Rightarrow A^2 - B^2 = A^2 - BA + AB - B^2$

$\Rightarrow AB = BA$

34. (C) Let $\log_2 3, \log_4 3, \log_8 3$ are in H.P.

$\frac{1}{\log_2 3}, \frac{1}{\log_4 3}, \frac{1}{\log_8 3}$ are in A.P.

$\log_3 2, \log_3 4, \log_3 8$ are in A.P.

then, $2\log_3 4 = \log_3 2 + \log_3 8$

$\Rightarrow \log_3 4^2 = \log_3(2\times8)$

$\Rightarrow 4^2 = 2\times8$

$\Rightarrow 16 = 16$

Hence $\log_2 3, \log_4 3, \log_8 3$ are in H.P.

35. (B) m^{th} term of a H.P $= n$

m^{th} term of an A.P $= \frac{1}{n}$

$a + (m-1)d = \frac{1}{n}$ ...(i)

and n^{th} term of a H.P. $= m$

n^{th} term of an A.P. $= \frac{1}{m}$

$a + (n-1)d = \frac{1}{m}$ ...(ii)

from eq(i) and eq(ii)

$a = \frac{1}{mn}, \; d = \frac{1}{mn}$

$T_{m+n} = a + (m+n-1)d$

$T_{m+n} = \frac{1}{mn} + \frac{m+n-1}{mn}$

$T_{m+n} = \frac{m+n}{mn}$

$\therefore (m+n)^{th}$ term of a H.P. $= \frac{mn}{m+n}$

36. (B) $\int \frac{1}{x(x^6+1)}dx \Rightarrow \int \frac{x^5}{x^6(x^6+1)}dx$

Let $x^6 + 1 = t \Rightarrow 6x^5 dx = dt \Rightarrow x^5\, dx = \frac{1}{6}dt$

$\Rightarrow \frac{1}{6}\int \frac{dt}{(t-1)t}$

$\Rightarrow \frac{1}{6}\int \left[\frac{1}{t-1} - \frac{1}{t}\right]dt$

$\Rightarrow \frac{1}{6}\left[\log(t-1) - \log t\right] + c$

$\Rightarrow \frac{1}{6}\left[\log x^6 - \log(x^6-1)\right] + c$

$\Rightarrow \frac{1}{6}\log\left(\frac{x^6}{x^6-1}\right) + c$

37. (A)

2	61	1
2	30	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	

Hence $(61)_{10} = (111101)_2$

38. (D) We know that

$\sin\theta.\sin(60-\theta).\sin(60+\theta) = \frac{1}{4}\sin3\theta$

Now, sin20°.sin40°.sin60.sin80°

$\Rightarrow$ sin60°.(sin20.sin40.sin80)

$\Rightarrow \frac{\sqrt{3}}{2}\times\frac{1}{4}\sin[2\times30]$

$\Rightarrow \frac{\sqrt{3}}{2}.\sin60° \Rightarrow \frac{1}{4}\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{3}{16}$

39. (B) $I = \int \frac{x-2}{(x-3)(x-4)}dx$

$I = \int\left(\frac{-1}{x-3} + \frac{2}{x-4}\right)dx$

$I = -\log(x-3) + 2\log(x-4) + c$

$I = \log\left[\frac{(x-4)^2}{x-3}\right] + c$

40. (C) $(\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)^{-1}$

$\Rightarrow (\cos\theta + i\sin\theta)[\cos(-\theta) + i\sin(-\theta)]$

$\Rightarrow (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)$

$\Rightarrow \cos^2\theta - i^2\sin^2\theta$

$\Rightarrow \cos^2\theta + \sin^2\theta = 1$

41. (A) $\lim_{x\to0} \frac{\cos x - \cos(\sin x)}{x^3}$ $\left[\frac{0}{0}\right]$

By L-Hospital's Rule

$\lim_{x\to0} \frac{-\sin x + \cos x.\sin(\sin x)}{3x^2}$ $\left[\frac{0}{0}\right]$

Again, L-Hospital's Rule

$\Rightarrow \lim_{x\to0} \frac{-\cos x + \cos^2 x.\cos(\sin x) - \sin x.\sin(\sin x)}{6x}$

$\left[\frac{0}{0}\right]$

Again, L-Hospital's Rule

$\Rightarrow \lim_{x\to0} \frac{\sin x + \cos^2 x.\{-\sin(\sin x)\}.\cos x + \cos(\sin x).2\cos x.(\sin x) - \sin x.\cos(\sin x).\cos x - \sin(\sin x).\cos x}{6}$

$\Rightarrow \frac{0+0+0+0-0-0}{6} = 0$

42. (C) $x^a.y^b = (x+y)^{a+b}$

On taking log both sides

$a \log x + b \log y = (a+b)\log(x+y)$

On differentiating both sides w.r.t.'x'

$$\Rightarrow \frac{a}{x} + \frac{b}{y}\frac{dy}{dx} = \frac{a+b}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{b}{y}\frac{dy}{dx} - \frac{a+b}{x+y}\frac{dy}{dx} = \frac{a+b}{x+y} - \frac{a}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{b(x+y)-(a+b)y}{y(x+y)}\right) = \frac{(a+b)x - a(x+y)}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(bx-ay)}{x(bx-ay)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

43. (C) $\tan^{-1}\frac{1}{9} + \sin^{-1}\frac{4}{\sqrt{41}}$

$$\Rightarrow \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{4}{5}\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}\times\frac{4}{5}}\right)$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\tan^{-1}\left(\frac{\frac{5+36}{45}}{\frac{45-4}{45}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

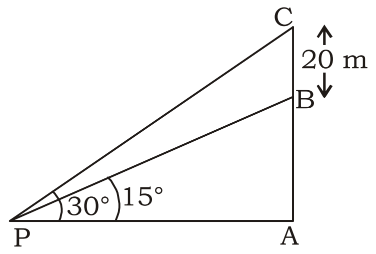
44. (D) Equation of sphere

$(x+1)^2 + (y-2)^2 + (z-3)^2 = 7^2$

$\Rightarrow x^2+1+2x+y^2+4-4y+z^2+9-6z = 49$

$\Rightarrow x^2+y^2+z^2+2x-4y-6z = 35$

45. (A)



Let AB = h m

In ΔPAB :-

$$\tan 15° = \frac{AB}{PA}$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{PA}$$

$$\Rightarrow PA = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)h \Rightarrow PA = (2+\sqrt{3})h$$

In ΔPAC:-

$$\Rightarrow \tan 30° = \frac{AC}{PA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h+20}{(2+\sqrt{3})h}$$

$\Rightarrow (2+\sqrt{3})h = \sqrt{3}\,h + 20\sqrt{3}$

$\Rightarrow 2h + \sqrt{3}\,h = \sqrt{3}\,h + 20\sqrt{3}$

$\Rightarrow 2h = 20\sqrt{3} \Rightarrow h = 10\sqrt{3}$

$\therefore$ Height of tower = $10\sqrt{3}$ m

46. (A) Parabola

$y^2 - 4y + 8x + 12 = 0$

$\Rightarrow (y-2)^2 - 4 + 8x + 12 = 0$

$\Rightarrow (y-2)^2 = -8x - 8$

$\Rightarrow (y-2)^2 = -8(x-1)$

Compare with $Y^2 = -4aX$

$4a = 5$ $a = 2$

focus $(X, Y) = (-a, 0)$

$X = a, Y = 0$

$x - 1 = -2,\quad y - 2 = 0 \Rightarrow y = 2$

$x = -1$

$\therefore$ focus = $(-1, 2)$

47. (C) $I_n = \int_1^e (\log x)^n.dx$

$$I_n = \left[(\log x)^n \int 1.dx - \int\left\{\frac{d}{dx}(\log x)^n.\int 1.dx\right\}dx\right]_1^e$$

$$I_n = \left[(\log x)^n.x - \int n.(\log x)^{n-1}\times\frac{1}{x}\times x\,dx\right]_1^e$$

$$I_n = \left[x(\log x)^n\right]_1^e - n\int(\log x)^{n-1}dx$$

$I_n = [e(\log e)^n - 1.(\log 1)^n] - nI_{n-1}$

$I_n + n\,I_{n-1} = e.1 - 0$

$I_n + n\,I_{n-1} = e$

$\therefore I_6 + 6I_5 = e$

48. (D) $\int_1^2 [x]dx \Rightarrow \int_1^{\sqrt{2}}[x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}}[x^2]dx + \int_{\sqrt{3}}^{2}[x^2]dx$

$$\Rightarrow \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}}\sqrt{2}\,dx + \int_{\sqrt{3}}^{2}\sqrt{3}\,dx$$

$$\Rightarrow [x]_1^{\sqrt{2}} + \sqrt{2}[x]_{\sqrt{2}}^{\sqrt{3}} + \sqrt{3}[x]_{\sqrt{3}}^{2}$$

$\Rightarrow \sqrt{2} - 1 + \sqrt{2}(\sqrt{3} - \sqrt{2}) + \sqrt{3}(2 - \sqrt{3})$

$\Rightarrow \sqrt{2} - 1 + \sqrt{6} - 2 + 2\sqrt{3} - 3$

$\Rightarrow \sqrt{6} + 2\sqrt{3} + \sqrt{2} - 6$

49. (C) Order = 2, Degree = 2

50. (A) $n(S) = 6 \times 6 = 36$

E = {(6, 3), (3, 6), (5, 4), (4, 5)}, $n(E) = 4$

The required Probability = $\frac{4}{36} = \frac{1}{9}$

51. (D) $\frac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x}$

$\Rightarrow \frac{2\cos\frac{4x+2x}{2}.\sin\frac{4x-2x}{2}}{2\cos\frac{4x+2x}{2}.\cos\frac{4x-2x}{2}}$

$\Rightarrow \frac{\sin x}{\cos x} = \tan x$

52. (D) Given that $\vec{a} + 3\vec{b} + 2\vec{c} = 0$

Now, $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} = \lambda(\vec{a}\times\vec{c})$

$\Rightarrow \vec{a}\times\frac{1}{3}(-\vec{a}-2\vec{c}) + \frac{1}{3}(-\vec{a}-2\vec{c})\times\vec{c} - \vec{a}\times\vec{c}$

$= \lambda(\vec{a}\times\vec{c})$

$\Rightarrow -\frac{1}{3}(\vec{a}\times\vec{a}) - \frac{2}{3}(\vec{a}\times\vec{c}) - \frac{1}{3}(\vec{a}\times\vec{c}) -$

$\frac{2}{3}(\vec{c}\times\vec{c}) - \vec{a}\times\vec{c} = \lambda(\vec{a}\times\vec{c})$

$\Rightarrow 0 - \frac{2}{3}(\vec{a}\times\vec{c}) - \frac{1}{3}(\vec{a}\times\vec{c}) - 0 - (\vec{a}\times\vec{c}) = \lambda(\vec{a}\times\vec{c})$

$\Rightarrow -2(\vec{a}\times\vec{c}) = \lambda(\vec{a}\times\vec{c}) \Rightarrow \lambda = -2$

53. (C) $I = \int_0^\infty \frac{x\log x}{(1+x^2)^2}dx$

Let $x = \tan\theta$ when $x = 0$, $t \to 0$

$dx = \sec^2\theta.d\theta$ $x = \infty$, $t = \frac{\pi}{2}$

$I = \int_0^{\pi/2}\frac{\tan\theta.\log\tan\theta}{(1+\tan^2\theta)^2}.\sec^2\theta.d\theta$

$I = \int_0^{\pi/2}\frac{\sin\theta}{\cos\theta}.\frac{\log\tan\theta}{\sec^2\theta.\sec^2\theta}.\sec^2\theta.d\theta$

$I = \int_0^{\pi/2}\sin\theta.\cos\theta.\log\tan\theta\ d\theta$ ...(i)

Prop IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2}\sin\left(\frac{\pi}{2}-\theta\right).\cos\left(\frac{\pi}{2}-\theta\right).\log\tan\left(\frac{\pi}{2}-\theta\right)d\theta$

$I = \int_0^{\pi/2}\cos\theta.\sin\theta.\log\cot\theta\, d\theta$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2}\sin\theta.\cos\theta[\log\tan\theta + \log\cot\theta]d\theta$

$2I = \int_0^{\pi/2}\sin\theta.\cos\theta.[\log 1]d\theta$

$2I = 0 \Rightarrow I = 0$

54. (B) No. of diagonals = $\frac{n(n-3)}{2}$

55. (B) $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$, $x \neq 0$

put $x = 3$

$2f(3) + 3f\left(\frac{1}{3}\right) = 9$ ...(i)

put $x = \frac{1}{3}$

$2f\left(\frac{1}{3}\right) - 3f(3) = \frac{1}{9}$ ...(ii)

from eq(i) and eq(ii)

$f(3) = \frac{53}{39}$

56. (B) $y = \cos^n x.\sin nx$

On differentiating both sides w.r.t.'x'

$\frac{dy}{dx} = \cos^n x.n.\cos nx + n.\cos^{n-1}x(-\sin x).\sin nx$

$\frac{dy}{dx} = n[\cos^n x.\cos nx - \sin x.\sin nx.\cos^{n-1}x]$

57. (C) sin135° + cos135°

$\Rightarrow$ sin(90 + 45°) + cos(90 + 45°)

$\Rightarrow$ cos45° – sin45°

$\Rightarrow \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

58. (A) 5.3, 9.3, 0, – 4.7, 7.6, 3.9, –3.2, 6.1, –4.2

On arranging in ascending order

–4.7, –4.2, –3.2, 0, 3.9, 5.3, 6.1, 7.6, 9.3

Median = 5th term = 3.9

59. (D) Given that $a = 3$, $b = 4$, $\sin A = \frac{3}{4}$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \frac{3}{4\times 3} = \frac{\sin B}{4}$

$\Rightarrow \sin B = 1 \Rightarrow B = 90°$

60. (B) $3.4\overline{17} = 3 + 0.4\overline{17} = 3 + \frac{417-4}{990}$

$= 3 + \frac{413}{990} = \frac{3383}{990}$

61. (D) Let A = 8 sinθ – 4sin²θ

A = [–2×2 sinθ×2 + (2sinθ)² + 4 – 4]

A = [(2sinθ – 2)² – 4]

A = 4 – (2 sinθ – 2)²

Maximum value of A = 4

62. (D) Differential equation

$(2 + 3x)dy + (3 - 2y)dx = 0$

$\Rightarrow (2 + 3x)dy = (2y - 3)dx$

$\Rightarrow \frac{dy}{2y-3} = \frac{dx}{2+3x}$

On integrating

$\Rightarrow \frac{\log(2y-3)}{2} = \frac{\log(2+3x)}{3} + \frac{\log c}{3}$

$\Rightarrow 3\log(2y - 3) = 2\log(2 + 3x) + 2\log c$

$\Rightarrow \log(2y - 3)^3 = \log(2 + 3x)^2.c^2$

$\Rightarrow (2y - 3)^3 = (2 + 3x)^2.C$

63. (B) Equation

$ax^2 + bx + c = 0$

$\tan 21° + \tan 24° = \frac{-b}{a}$ ...(i)

and $\tan 21°.\tan 24° = \frac{c}{a}$ ...(ii)

Now, $\tan(21° + 24°) = \frac{\tan 21 + \tan 24}{1 - \tan 21.\tan 24}$

$\Rightarrow \tan 45° = \frac{-\frac{b}{a}}{1-\frac{c}{a}}$

$\Rightarrow 1 = \frac{-b}{a-c}$

$\Rightarrow a - c = -b \Rightarrow a + b = c$

64. (D) $3x^2 - 4y^2 = 12 \Rightarrow \frac{x^2}{4} - \frac{y^2}{3} = 1$

represents a hyperbola.

65. (B) Lines $2x + y + 3 = 0$ ...(i)

and $3x + 2y = 7$ ...(ii)

On solving

$x = -13,\ y = 23$

Intersection point = (–13, 23)

Equation of line which is parallel to the line $x + 2y + 7 = 0$

$x + 2y = c$

it passes through the point (–13, 23)

$-13 + 2 \times 23 = c \Rightarrow c = 33$

The required line

$x + 2y = 33$

66. (C) $\sum_{n=1}^{9}(i^n + i^{n-1})$

$\Rightarrow (i + i^0) + (i^2 + i^1) + (i^3 + i^2) + (i^4 + i^3) + (i^5 + i^4) + (i^6 + i^5) + (i^7 + i^6) + (i^8 + i^7) + (i^9 + i^8)$

$\Rightarrow (i + 1) + (-1 + i) + (-i - 1) + (1 - i) + (i + 1) + (-1 + i)\ (-i - 1) + (1 - i) + (i + 1)$

$\Rightarrow i + 1$

67. (D) Lines $5x + 12y = 7$

and $15x + 36y = 17 \Rightarrow 5x + 12y = \frac{17}{3}$

The required Distance = $\left|\frac{\frac{17}{3} - 7}{\sqrt{5^2 + 12^2}}\right|$

$= \frac{4}{3 \times 13} = \frac{4}{39}$

68. (C) $I = \int_0^1 x(1-x)^6\,dx$

Prop.IV

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^1 (1-x).x^6\,dx$

$I = \int_0^1 (x^6 - x^7)dx$

$I = \left[\frac{x^7}{7} - \frac{x^8}{8}\right]_0^1$

$I = \frac{1}{7} - \frac{1}{8} = 0$

$I = \frac{8-7}{56} = \frac{1}{56}$

69. (B) $y = e^{3x}.\sin 4x$

On differentiating both sides w.r.t.'x'

$\frac{dy}{dx} = e^{3x}.4\cos 4x + 3.e^{3x}.\sin 4x$

$\frac{dy}{dx}\left(\text{at } x = \frac{\pi}{2}\right) = 4.e^{\frac{3\pi}{2}}.\cos 2\pi + 3e^{\frac{3\pi}{2}}.\sin 2\pi$

$\frac{dy}{dx}\left(\text{at } x = \frac{\pi}{2}\right) = 4.e^{\frac{3\pi}{2}}.1 + 3.e^{\frac{3\pi}{2}}.0$

$\frac{dy}{dx}\left(\text{at } x = \frac{\pi}{2}\right) = 4.e^{\frac{3\pi}{2}}$

70. (B) Quadratic equation $ax^2 + bx + c = 0$

$\tan30° + \tan45° = \frac{-b}{a}$

$\Rightarrow \frac{1}{\sqrt{3}} + 1 = \frac{-b}{a} \Rightarrow \frac{-b}{a} = \frac{\sqrt{3}+1}{\sqrt{3}}$ ...(i)

and $\tan30°.\tan45° = \frac{c}{a}$

$\Rightarrow \frac{1}{\sqrt{3}} \times 1 = \frac{c}{a} \Rightarrow \frac{c}{a} = \frac{1}{\sqrt{3}}$ ...(ii)

from eq(i) and eq(ii)

$\Rightarrow \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} \Rightarrow \frac{-b}{c} = \sqrt{3}+1$

$\Rightarrow -b = \sqrt{3}\,c + c \Rightarrow b + c + \sqrt{3}\,c = 0$

71. (C) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow \cos A = \frac{8^2 + 9^2 - 3^2}{2 \times 8 \times 9}$

$\Rightarrow \cos A = \frac{64 + 81 - 9}{144} \Rightarrow \cos A = \frac{136}{144} = \frac{17}{18}$

72. (D) Equation of hyperbola

$3x^2 - 16y^2 = 64 \Rightarrow \frac{x^2}{\left(\frac{8}{3}\right)^2} - \frac{y^2}{(2)^2} = 1$

Equation of line $2x + y = \lambda \Rightarrow y - 2x + \lambda$

We know that line $y = mx + c$ touches the hyperbola,

if $c^2 = a^2m^2 - b^2$

$\Rightarrow \lambda^2 = \left(\frac{8}{3}\right)^2 \times (-2)^2 - (2)^2 \Rightarrow \lambda^2 = \frac{256}{9} - 4$

$\Rightarrow \lambda^2 = \frac{229}{9} \Rightarrow \lambda = \pm\frac{2\sqrt{55}}{3}$

73. (A) A.T.Q,

$\cos\frac{\pi}{2} = \frac{-1 \times (-2) + 2 \times x + (-3) \times 4}{\sqrt{(-1)^2 + 2^2 + (-3)^2}\sqrt{(-2)^2 + x^2 + 4^2}}$

$\Rightarrow 0 = \frac{2 + 2x - 12}{\sqrt{14}\sqrt{20 + x^2}}$

$\Rightarrow 0 = 2x - 10 \Rightarrow x = 5$

74. (B) Let $x - iy = \sqrt{46 - 14\sqrt{3}i}$

On squaring both sides

$\Rightarrow (x^2 - y^2) - 2xyi = 46 - 14\sqrt{3}\,i$

On comparing

$x^2 - y^2 = 46$ and $2xy = 14\sqrt{3}$ ...(i)

Now, $(x^2 + y^2) = (x^2 - y^2) + (2xy)^2$

$\Rightarrow (x^2 + y^2)^2 = (46)^2 + (14\sqrt{3})^2$

$\Rightarrow (x^2 + y^2)^2 = 2116 + 588$

$\Rightarrow (x^2 + y^2)^2 = 2704 \Rightarrow x^2 + y^2 = 52$ ...(ii)

from eq(i) and eq(ii)

$x = \pm 7,\ y = \pm\sqrt{3}$

$\therefore \sqrt{46 - 14\sqrt{3}i} = \pm(7 - \sqrt{3}i)$

75. (A) Let $y = 1 - 2^{-x}$

$\Rightarrow 2^{-x} = (1 - y)$

On taking log both sides

$\Rightarrow -x\log2 = \log(1 - y)$

$\Rightarrow -x = \log_2(1 - y)$

$\Rightarrow x = -\log_2(1 - y)$

$\Rightarrow f^{-1}(y) = -\log_2(1 - y)$

$\Rightarrow f^{-1}(x) = -\log_2(1 - x)$

76. (A) $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1 + x^2}{2x}\right)$

I II

Function Ist is defined for all values of x. Since $\cos(-\theta) = \cos\theta$, IInd function is define only when $\left|\frac{1 + x^2}{2x}\right| \le 1$

$\Rightarrow \frac{|1 + x^2|}{|2|} \le 1 \Rightarrow |1 + x^2| \le |2x|$

$\Rightarrow |x^2 + 1| - |2x| \le 0 \Rightarrow (|x| - 1)^2 \le 0$

Since square is always +ve. So this inequility is valid only when $x = 1$or -1

Hence domain consist only two points $x = 1$ and $x = -1$

77. (C) Given quadratic equation-

$(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + (8 + 2\sqrt{5}) = 0$

Let roots are α and β

Sum of roots $\alpha + \beta = \frac{(4 + \sqrt{5})}{(5 + \sqrt{2})}$

Product of roots $\alpha.\beta = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}$

Now, H.M. $= \frac{2\alpha\beta}{(\alpha + \beta)} = \frac{2 \times \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}}{\frac{4 + \sqrt{5}}{5 + \sqrt{2}}}$

$= \frac{16 + 4\sqrt{5}}{4 + \sqrt{5}} = 4$

78. (C) Let S = $\frac{5}{13}+\frac{55}{13^2}+\frac{555}{13^3}+\ldots\infty$ ...(i)

multiplied $\frac{1}{13}$, we have-

$\frac{1}{13}S=\frac{5}{13^2}+\frac{15}{13^3}+\ldots.\infty$ ...(ii)

Subtracted eq(ii) from (i), we have-

$\frac{12}{13}S=\frac{5}{13}+\frac{50}{13^2}+\frac{500}{13^3}+\ldots\infty$

It is a G.P., common ratio $\frac{10}{13}$

$\Rightarrow \frac{12}{13}S=\frac{5/13}{\left(1-\frac{10}{13}\right)}$

$\Rightarrow \frac{12}{13}S=\frac{5}{3}\Rightarrow S=\frac{65}{36}$

79. (C) In the expansion of $\left(y^2+\frac{2}{y}\right)^5$

T_{r+1} = ${}^5C_2(y^2)^{5-r}\left(\frac{2}{y}\right)^r$

= ${}^5C_r\, 2^r.y^{10-3r}$

Now, $10 - 3r = 1$

$\Rightarrow 3r = 9 \Rightarrow r = 3$

Coefficient of y = ${}^5C_3 \times 2^3$

= 10 ×8 = 80

80. (A) $(1 + x^2)^5(1 + x)^4 = [1 + {}^5C_1(x^2) + {}^5C_2(x^2)^2 + {}^5C_3(x^2)^3 + {}^5C_4(x^2)^4 + {}^5C_5(x^2)^5] \times [1+ {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4]$

Coefficient of x^5 = ${}^5C_1.{}^4C_3 + {}^5C_2.{}^4C_1$

= 5 × 4 + 10 × 4 = 60

81. (C) In the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$

r^{th} term = ${}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-(r-1)}\left(-\frac{2}{x^2}\right)^{r-1}$

= ${}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r}(-2)^{r-1}.x^{13-3r}$

Now, $13 - 3r = 4$

$\Rightarrow 3r = 9 \Rightarrow r = 3$

82. (A) Number of words in which all the 5 letters are repeated = $(10)^5$ = 1,00,000 and the number of words in which no letters is repeated = ${}^{10}P_5$ = 30240

Hence the number of words have at least one letter repeated

= 1,00,000 – 30240 = 69760

83. (A) "COCHIN"

Total words starting with CC = 4! = 24

Total words starting with CH = 4! = 24

Total words starting with CI = 4! = 24

Total words starting with CN = 4! = 24

Now, start word will be "COCHIN"

= 24 × 4 = 96

84. (B) $\frac{\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^8}{\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)^8}$

$\Rightarrow \frac{\left[i\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)\right]^8}{\left[i\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)\right]^8}$

$\Rightarrow \frac{\left(\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)^8}{\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)^8}$

Apply Demoivre's theorem

= $\frac{\cos\pi-i\sin\pi}{\cos\pi+i\sin\pi}=\frac{-1-0}{-1+0}=1$

85. (C) $x = 1 + 2i \Rightarrow x - 1 = 2i \Rightarrow (x - 1)^2 = (2i)^2$

$\Rightarrow x^2 - 2x + 1 = -4 \Rightarrow x^2 - 2x + 5 = 0$

Now, $x^3 + 7x^2 - 13x + 16$

$\Rightarrow x(x^2 - 2x + 5) + 9(x^2 - 2x + 5) - 29$

$\Rightarrow x.0 + 9.0 - 29 = -29$

86. (A) nth term of the series

$T_n=\frac{1.3.5.....(2n-1)}{2n!}=\frac{1.2.3.4.5.6...(2n-1).2n}{2n!(2.4.6.8....2n)}$

$=\frac{(2n)!}{(2n)!.2^n.n!}=\frac{1}{2^n.n!}=\frac{1/2^n}{n!}$

Now, sum

S = $\sum_{n=1}^{\infty}\frac{1/2^n}{n!}=\frac{1/2}{1!}+\frac{(1/2)^2}{2!}+\frac{(1/2)^3}{3!}+\ldots.\infty$

= $(e^{1/2} - 1) = (\sqrt{e} - 1)$

87. (A) First we write n^{th} term of the series

Let n^{th} term of numerator is t_n.

Let us consider S = 1 + 3 + 6 + 10.. + t_n ...(i)

Again S = 1 + 3 + 6 + ... + t_{n-1} + t_n ...(ii)

Subtracted equation (ii) from equation (i)

0 = (1 + 2 + 3 + 4 + ... n term) – t_n

$\Rightarrow t_n = 1 + 2 + 3 + 4 + \ldots.n$ terms = $\frac{n(n+1)}{2}$

So, n^{th} term of the series

$$T_n = \frac{n(n+1)}{2.n!} = \frac{n+1}{2.(n-1)!} = \frac{(n-1)+2}{2.(n-1)!}$$

$$= \frac{(n-1)}{2(n-2)!} + \frac{2}{2(n-1)!} + \frac{1}{2(n-2)!} + \frac{1}{(n-1)!}$$

Now, $S = \sum_{n=1}^{\infty} T_n = \sum\left[\frac{1}{2(n-2)!} + \frac{1}{(n-1)!}\right]$

$$\Rightarrow \frac{1}{2}\sum\frac{1}{(n-2)!} + \sum -\frac{1}{(n-1)!}$$

$$\Rightarrow \frac{1}{2}e + e = \frac{3}{2}e$$

88. (A) Since A is a matrix and |A| is a matrix and |A| has certain value
So $A^{-1} = |A|^{-1}$ is not true.

89. (A) $\begin{vmatrix}(x+1) & (x+2) & (x+a)\\(x+2) & (x+3) & (x+b)\\(x+3) & (x+4) & (x+c)\end{vmatrix}$

$R_1 \to (R_1 + R_3) - 2R_2$

$$\Rightarrow \begin{vmatrix}0 & 0 & (a+c)-2b\\(x+2) & (x+3) & (x+b)\\(x+3) & (x+4) & (x+c)\end{vmatrix}$$

Since a, b, c are in A.P., So $2b = (a + c)$

$$= \begin{vmatrix}0 & 0 & 0\\(x+2) & (x+3) & (x+b)\\(x+3) & (x+4) & (x+c)\end{vmatrix} = 0$$

90. (C) $\begin{vmatrix}x-y-z & 1-x & y+z\\y-z-x & 1-y & z+x\\z-x-y & 1-z & x+y\end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix}1 & 1-x & y+z\\1 & 1-y & z+x\\1 & 1-z & x+y\end{vmatrix}$$

$C_3 \to C_3 - C_2 + C_1$

$$\Rightarrow \begin{vmatrix}1 & 1-x & x+y+z\\1 & 1-y & x+y+z\\1 & 1-z & x+y+z\end{vmatrix}$$

$$\Rightarrow (x+y+z)\begin{vmatrix}1 & 1-x & 1\\1 & 1-y & 1\\1 & 1-z & 1\end{vmatrix}$$

$\Rightarrow 0$ [$\because$ Two columns are indentical.]

91. (A) a, b, c are in G.P., then
$b^2 = ac$ ...(i)
p, q, r are in G.P., then
$q^2 = pr$ ...(ii)
from eq(i) and eq(ii)
$b^2q^2 = ac \times pr$
$(bq)^2 = ap \times cr$
Hence ap, bq, cr are in G.P.

92. (C) $I = \int \frac{1}{e^{-x}-1}\,dx \Rightarrow I = \int \frac{1}{\frac{1}{e^x}-1}\,dx$

$\Rightarrow I = \int \frac{e^x}{1-e^x}\,dx$

Let $1 - e^x = t$

$\Rightarrow -e^x\,dx = dt$

$\Rightarrow I = -\int \frac{dt}{t}$

$\Rightarrow I = -\log t + c$

$\Rightarrow I = -\log(1 - e^x) + c$

93. (D) $\frac{\sin 330°.\tan 150°.\cot 135°}{\sec 240°.\text{cosec}120°.\cos 225°}$

$$\Rightarrow \frac{\sin(360°-30°).\tan(180°-30°).\cot(180°-45°)}{\sec(180°+60°).\text{cosec}(180°-60°).\cos(180°+45°)}$$

$$\Rightarrow \frac{(-\sin 30)(-\tan 30)(-\cot 45)}{(-\sec 60)(\text{cosec}60)(-\cos 45)}$$

$$\Rightarrow \frac{-\frac{1}{2}\times\frac{1}{\sqrt{3}}\times 1}{2\times\frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{2}}} = -\frac{1}{4\sqrt{2}}$$

94. (B) Let a point $P(a\cos\theta, a\sin\theta)$ on circle $x^2 + y^2 = a^2$
Now, the equation of chord of contact of tangents from point P to the circle $x^2 + y^2 = b^2$ is
$x.a\cos\theta + y.a\sin\theta = b^2$ ...(i)
Now, if equation (i) touch the circle $x^2 + y^2 = c^2$ then (the perpendicular drawn from centre (0, 0) to (i) will be the radius)

$$\frac{b^2}{\sqrt{(a\cos\theta)^2 + (a\sin\theta)^2}} = c$$

$\Rightarrow \frac{b^2}{a} = c$

$\Rightarrow b^2 = ac$

So a, b, c are in G.P.

95. (A) Here slope $m = \tan 60 = \sqrt{3}$ and $a = \sqrt{3}$
So the equation of the tangents are
$y = mx \pm a\sqrt{1+m^2}$
$y = \sqrt{3}\,x \pm \sqrt{3}\ \sqrt{1+(\sqrt{3})^2}$
$y = \sqrt{3}\,x \pm 2\sqrt{3}$

96. (A) The equation of the normal to the circle $3x^2 + 2y^2 - 4x - 6y = 0$ at the point (0,0) is
$3(0 \times x) + 3(0 \times y) - 2(x + 0) - 3(y + 0) = 0$
$\Rightarrow 2x + 3y = 0$
Slope of the tangent $= -\frac{2}{3}$
Slope of normal $= \frac{3}{2}$
Hence, the equation of the normal at (0, 0) is $y - 0 = \frac{3}{2}(x - 0) \Rightarrow 3x - 2y = 0$

97. (A) The ellipse equation $4x^2 + 9y^2 = 36$
The equation of tangent at point P(3, –2) is $4x(3) + 9y(-2) = 36$
$\Rightarrow 12x - 18y = 36 \Rightarrow 2x - 3y = 6$

98. (B) The equation of ellipse $9x^2 + 16y^2 = 144$
$\Rightarrow \frac{9}{144}x^2 + \frac{16}{144}y^2 = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$
$\Rightarrow \frac{x^2}{(4)^2} + \frac{y^2}{(3)^2} = 1$ ...(i)
Compare this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have $a = 4,\ b = 3$
The equation of any tangent to this ellipse is $y = mx \pm \sqrt{a^2m^2 + b^2}$
$y = mx + \sqrt{16m^2 + 9}$ ...(ii)
Since it passes through the point (2, 3)
So $3 = 2m + \sqrt{16m^2 + 9}$
$\Rightarrow (3 - 2m)^2 = 16m^2 + 9 \Rightarrow 12m^2 + 12m = 0$
$\Rightarrow 12m(m + 1) = 0 \Rightarrow m = 0,\ m = -1$
These value of m put in equation (ii), we have $y = 3$ and $y = -x + 5$

99. (B) The equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)
Let (h, k) be the mid point of a focal chord of the ellipse (i)
Then the equation of the chord is $T = S_1$
$\Rightarrow \frac{xh}{a^2} + \frac{yk}{b^2} - 1 = \frac{h^2}{a^2} + \frac{k^2}{b^2} - 1$
$\Rightarrow \frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$ ...(ii)
Since it passes through the focus $(ae, 0)$ of the ellipse $\frac{hae}{a^2} + 0 = \frac{h^2}{a^2} + \frac{k^2}{b^2}$
$\Rightarrow \frac{he}{a} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$
Hence the locus of (h, k)
$\frac{xe}{a} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

100. (B) Centre of hyperbola is the mid point of the line joining two foci, therefore coordinates of the centre are (1, 5)
Now, distance between the foci = 10
$\Rightarrow 2ae = 10 \Rightarrow ae = 5$
$\Rightarrow a \times \frac{5}{4} = 5 \Rightarrow a = 4 \left[\because e = \frac{5}{4}\right]$
Now, $b^2 = a^2(e^2 - 1)$
$\Rightarrow b^2 = 16\left(\frac{25}{16} - 1\right) \Rightarrow b = 3$
Hence, the equation of hyperbola
$\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

101. (C) Eliminating "t" from $x = t^2 + 1,\ y = 2t$
We obtain $y^2 = 4x - 4$ ...(i)
Now putting $x = 2S,\ y = \frac{2}{S}$ in equatin (i), we get
$2S^3 - S^2 - 1 = 0$
$\Rightarrow (S - 1)(2S^2 + S + 1) = 0 \Rightarrow S = 1$
Putting S = 1, we get
$x = 2S,\ y = \frac{2}{S} \Rightarrow x = 2,\ y = 2$
$\therefore$ Point of intersection = (2, 2)

102. (A) Length of latus rectum
$\frac{2b^2}{a} = 12$ ...(i)
and semi-conjugate axis
$b = 2\sqrt{3}$ ...(ii)
from equation (i) and (ii)
$a = 2$
Now, eccentricity
$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{12}{4}} = 2$

103. (A) $I = \int \frac{x+\sin x}{(1+\cos x)}dx = \int \frac{x+\sin x}{2\cos^2\frac{x}{2}}.dx$

$= \int \frac{x}{2}\sec^2\frac{x}{2}dx + \int \tan\frac{x}{2}dx$

$= x.\tan\frac{x}{2} - \int \tan\frac{x}{2}.dx + \int \tan\frac{x}{2}dx + C$

$= x.\tan\frac{x}{2} + C$

104. (B) $\int \{1+2\tan x(\tan x+\sec x)^{1/2}\}dx$

$\Rightarrow \int \{1+2\tan^2 x + 2\tan x.\sec x\}^{1/2}dx$

$\Rightarrow \int \{(1+\tan^2 x)+\tan^2 x + 2\tan x.\sec x\}^{1/2}dx$

$\Rightarrow \int \{\sec^2 x + \tan^2 x + 2\tan x.\sec x\}^{1/2}dx$

$\Rightarrow \int \{(\sec x+\tan x)^2\}^{1/2}dx$

$\Rightarrow \int (\sec x + \tan x)dx$

$\Rightarrow \int \sec x\,dx + \int \tan x dx$

$\Rightarrow \log(\sec x + \tan x) + \log\sec x + C$

$\Rightarrow \log \sec x(\sec x + \tan x) + C$

105. (A) $\int (\sin 2x - \cos 2x)dx$

$\Rightarrow -\frac{1}{2}\cos 2x - \frac{1}{2}\sin 2x + C$

$\Rightarrow -\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cos 2x + \frac{1}{\sqrt{2}}\sin 2x\right] + C$

$\Rightarrow \frac{-1}{\sqrt{2}}\left[\sin\left(2x+\frac{\pi}{4}\right)\right] + C$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\left(\pi + 2x + \frac{\pi}{4}\right) + C$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\left(2x + \frac{5\pi}{4}\right) + C$ ...(i)

Now, given that

$\int (\sin 2x - \cos 2x)dx - \frac{1}{\sqrt{2}}\sin(2x - a) + b$ ...(ii)

On comparing equation (i) and (ii), we have

$a = -\frac{5\pi}{4}$ and b is any constant i.e. $b \in R$

106. (B) $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right) \Rightarrow -i\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right) = -i\omega$

and $\frac{\sqrt{3}}{2}-\frac{i}{2} \Rightarrow i\left(\frac{-1}{2}-\frac{i\sqrt{3}}{2}\right) = i\omega^2$

Now, $z = (-i\omega)^5 + (i\omega^2)^5 = -i\omega^2 + i\omega$

$\Rightarrow z = i(\omega - \omega^2) = i(i\sqrt{3}) = -\sqrt{3}$

$\therefore$ Re(z) < 0 and Im(z) = 0

107. (D) $|z-4| < |z-2|$

$\Rightarrow (x-4) + iy| < |(x-2) + iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow -8x + 16 < -4x + 4 \Rightarrow 4x - 12 > 0$

$\Rightarrow x > 3 \Rightarrow$ Re(z) > 3

108. (B) $|w| = 1 \Rightarrow \left|\frac{1-iz}{z-i}\right| = 1$

$\Rightarrow |1 - iz| = |z - i|$

$\Rightarrow |1 - i(x + iy) = |x + iy - i|$

$\Rightarrow |(y+1) - ix| = |x + i(y-1)|$

$\Rightarrow x^2 + (y+1)^2 = x^2 + (y-1)^2$

$\Rightarrow 4y = 0 \Rightarrow y = 0$

Hence z lies on the real axis.

109. (D) Possibilities of getting sum of the dice divisible by 5 = (1, 4), (2, 3), (3, 2),(4, 1) (4, 6),(5, 5),(6, 4) = 7

Possibilities of getting difference of the dice is divisible by 2 = (1, 3),(1, 5),(2,4), (2, 6),(3, 1),(3, 5),(4, 2),(4, 6),(5, 1),(5, 3), (6, 2),(6, 4) = 12

Required difference = $\left(\frac{12}{36}\right) - \left(\frac{7}{36}\right) = \frac{5}{36}$

110. (D) Required probability = $\frac{4}{52} \times \frac{5}{51} = \frac{5}{663}$

111. (C) $\sqrt{3}\,\text{cosec}20° - \sec 20° \Rightarrow \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$

$\Rightarrow \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°.\cos 20°}$

$\Rightarrow 4\left[\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20°\cos 20°}\right]$

$\Rightarrow 4\left[\frac{\sin 60°.\cos 20° - \cos 60°.\sin 20°}{\sin(2\times 20°)}\right]$

$\Rightarrow \left[\frac{4\sin(60°-20°)}{\sin 40°}\right] \Rightarrow \left[\frac{4\sin 40°}{\sin 40°}\right] = 4$

112. (B) $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$
$\Rightarrow 2\sin 2x.\cos x - 3\sin 2x = 2\cos 2x.\cos x - 3\cos 2x$
$\Rightarrow \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$
$\Rightarrow \sin 2x = \cos 2x \quad \left(as \cos x \neq \frac{3}{2}\right)$
$\Rightarrow \tan 2x = 1$
$\Rightarrow 2x = n\pi + \frac{\pi}{4}$
$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$

113. (A) Parabola $y^2 - 4y - 8x + 4 = 0$
$\Rightarrow y^2 - 4y + 4 = 8x - 4 + 4$
$\Rightarrow (y-2)^2 = 8x$
On comparing with $Y^2 = 4aX$, $a = 2$
focus (X, Y) = $(a, 0)$
X = a, Y = 0
$x = 2$, $y - 2 = 0 \Rightarrow y = 2$
Focus = (2, 2)

114. (A) The parabola $x^2 + 8x + 12y + 4 = 0$
$\Rightarrow x^2 + 8x = -12y - 4$
$\Rightarrow (x+4)^2 - 16 = -12y - 4$
$\Rightarrow (x+4)^2 = -12y + 12$
$\Rightarrow (x+4)^2 = -12(y-1)$
Now comparing with $X^2 = -4aY$, $a = 3$
vertex (X, Y) = (0, 0)
X = 0, Y = 0
$\Rightarrow x + 4 = 0, y - 1 = 0$
$\Rightarrow x = -4, y = 1$
Hence the coordinate of the vertex are (– 4, 1).

115. (A) The equation of tangent of slope m to parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$
So the quation of tangent of slope m to the parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$
Now comparing this equation with the given equation of tangent $y = mx + 1$, we have $m = 1$

116. (C) Curve $x^3 - 3xy^2 + 2 = 0$ ...(i)
On differenciating both sides w.r.t 'x'
$\Rightarrow 3x^2 - 3y^2 - 3x.2y.\frac{dy}{dx} = 0$
$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y^2}{2xy} = m_1$(say)
other curve $3x^2y - y^3 - 2 = 0$ ...(ii)
$\Rightarrow 3x^2.\frac{dy}{dx} + 6xy - 3y^2.\frac{dy}{dx} = 0$
$\Rightarrow \frac{dy}{dx}(3x^2 - 3y^2) + 6xy = 0$
$\Rightarrow \frac{dy}{dx} = -\frac{2xy}{(x^2 - y^2)} = m_2$(say)
Now $m_1.m_2 = \frac{(x^2 - y^2)}{2xy} \times \left[-\frac{2xy}{(x^2 - y^2)}\right] = -1$
Hence curve (i) and (ii) cut at 90°.

117. (C) Curve $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$
at point (1, 1), $\frac{dy}{dx} = 2 = m_1$(say)
and curve $6y = 7 - x^3$
$\Rightarrow 6.\frac{dy}{dx} = -3x^2$
$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}x^2$
at point (1, 1), $\frac{dy}{dx} = -\frac{1}{2} = m_2$(say)
Let the intersection angle is θ
So $\tan\theta = \frac{m_1 \sim m_2}{(1 + m_1 m_2)} = \frac{2 - \left(-\frac{1}{2}\right)}{1 + 2\left(-\frac{1}{2}\right)}$
$\tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2}$

118. (D) $|z - 2i| > |z + 2i|$; $z = x + iy$
$\Rightarrow |x + iy - 2i| > |x + iy + 2i|$
$\Rightarrow \sqrt{x^2 + (y-2)^2} > \sqrt{x^2 + (y+2)^2}$
On squaring
$\Rightarrow x^2 + y^2 + 4 - 4y > x^2 + y^2 + 4 + 4y$
$\Rightarrow 0 > 8y \Rightarrow y < 0$
Hence Imz < 0

119. (A) $\cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \cos^{-1}\left[\cos\left(2\pi - \frac{\pi}{4}\right)\right]$
$\Rightarrow \cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \cos^{-1}\left[\cos\frac{\pi}{4}\right]$
$\Rightarrow \cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \frac{\pi}{4}$

120. (D)

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NDA (MATHS) MOCK TEST - 188 (Answer Key)

1. (C)	21. (C)	41. (A)	61. (D)	81. (C)	101. (C)
2. (D)	22. (C)	42. (C)	62. (D)	82. (A)	102. (A)
3. (A)	23. (D)	43. (C)	63. (B)	83. (A)	103. (A)
4. (B)	24. (B)	44. (D)	64. (D)	84. (B)	104. (B)
5. (C)	25. (A)	45. (A)	65. (B)	85. (C)	105. (A)
6. (A)	26. (C)	46. (A)	66. (C)	86. (A)	106. (B)
7. (B)	27. (B)	47. (C)	67. (D)	87. (A)	107. (D)
8. (C)	28. (D)	48. (D)	68. (C)	88. (A)	108. (B)
9. (B)	29. (A)	49. (C)	69. (B)	89. (A)	109. (D)
10. (B)	30. (A)	50. (A)	70. (B)	90. (C)	110. (D)
11. (B)	31. (D)	51. (D)	71. (C)	91. (A)	111. (C)
12. (B)	32. (A)	52. (D)	72. (D)	92. (C)	112. (B)
13. (C)	33. (A)	53. (C)	73. (A)	93. (D)	113. (A)
14. (D)	34. (C)	54. (B)	74. (B)	94. (B)	114. (A)
15. (D)	35. (B)	55. (B)	75. (A)	95. (A)	115. (A)
16. (C)	36. (B)	56. (B)	76. (A)	96. (A)	116. (C)
17. (D)	37. (A)	57. (C)	77. (C)	97. (A)	117. (C)
18. (D)	38. (D)	58. (A)	78. (C)	98. (B)	118. (D)
19. (A)	39. (B)	59. (D)	79. (C)	99. (B)	119. (A)
20. (D)	40. (C)	60. (B)	80. (A)	100. (B)	120. (D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777