NDA MATHS MOCK TEST - 158 (SOLUTION)

1. (C) (a, b), $(c-d)$ and $(a-c, b-d)$ are collinear,

then $\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$

$\Rightarrow a(d-b+d) - b(c-a+c) + 1(bc-cd-ad+cd) = 0$

$\Rightarrow ad - ab + ad - bc + ab - bc + bc - ad = 0$

$\Rightarrow ad - bc = 0 \Rightarrow ad = bc$

2. (A) $\frac{1-\cos\alpha-\sin\alpha}{2\cos\alpha} = y$

$\Rightarrow \frac{1-\cos\alpha-\sin\alpha}{2\cos\alpha} \times \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

$\Rightarrow \frac{1^2-(\cos\alpha+\sin\alpha)^2}{2\cos\alpha(1+\cos\alpha+\sin\alpha)} = y$

$\Rightarrow \frac{1-(\cos^2\alpha+\sin^2\alpha+2\sin\alpha.\cos\alpha)}{2\cos\alpha(1+\cos\alpha+\sin\alpha)} = y$

$\Rightarrow \frac{1-(1+2\sin\alpha.\cos\alpha)}{2\cos\alpha(1+\cos\alpha+\sin\alpha)} = y$

$\Rightarrow \frac{1-1-2\sin\alpha.\cos\alpha}{2\cos\alpha(1+\cos\alpha+\sin\alpha)} = y$

$\Rightarrow \frac{-2\sin\alpha.\cos\alpha}{2\cos\alpha(1+\cos\alpha+\sin\alpha)} = y$

$\Rightarrow \frac{-\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

$\Rightarrow \frac{\sin\alpha}{1+\cos\alpha+\sin\alpha} = -y$

3. (B) Given that

$\sin A = k.\sin B$

$\Rightarrow \frac{\sin A}{\sin B} = \frac{k}{1}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\sin A+\sin B}{\sin A-\sin B} = \frac{k+1}{k-1}$

$\Rightarrow \frac{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}.\sin\frac{A-B}{2}} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} = \frac{k-1}{k+1}$

4. (A) $\cos80° + \cos40° - \cos20°$

$\Rightarrow 2\cos\frac{80+40}{2}.\cos\frac{80-40}{2} - \cos20$

$\Rightarrow 2\cos60.\cos20 - \cos20$

$\Rightarrow 2\times\frac{1}{2}\cos20 - \cos20$

$\Rightarrow \cos20 - \cos20 = 0$

5. (B) $7x^2 + y^2 = k(x^2 - y^2 - 4x + 3y)$

$(7-k)x^2 + (1+k)y^2 + 4kx - 3ky = 0$ is a circle, then

Coefficient of x^2 = coefficient of y^2

$\Rightarrow 7 - k = 1 + k$

$\Rightarrow 6 = 2k \Rightarrow k = 3$

6. (C) $\begin{vmatrix} 2\cos^2\frac{\alpha}{2} & \sin\alpha & 1 \\ 2\cos^2\frac{\beta}{2} & \sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}$

$\Rightarrow 2\cos^2\frac{\alpha}{2}(\sin\beta - 0) - \sin\alpha\left(2\cos^2\frac{\beta}{2} - 1\right) + 1(0 - \sin\beta)$

$\Rightarrow 2\cos^2\frac{\alpha}{2}.\sin\beta - \sin\alpha.\cos\beta - \sin\beta$

$\Rightarrow 2\cos^2\frac{\alpha}{2}.\sin\beta - \sin\beta - \sin\alpha.\cos\beta$

$\Rightarrow \sin\beta\left(2\cos^2\frac{\alpha}{2} - 1\right) - \sin\alpha.\cos\beta$

$\Rightarrow \sin\beta.\cos\alpha - \sin\alpha.\cos\beta$

$\Rightarrow \sin(\beta - \alpha)$

7. (C) $\int_{-\pi/2}^{\pi/2} \frac{\sin x}{x^4} dx = 0$ [$\because$ function is odd.]

8. (D) $\lim_{x\to3} \frac{\sqrt{2+\sqrt{1+x}}-2}{3-x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to3} \frac{\frac{1}{2\sqrt{2+\sqrt{1+x}}} \times \frac{1}{2\sqrt{1+x}} - 0}{-1}$

$\Rightarrow \lim_{x\to3} -\frac{1}{4} \times \frac{1}{\sqrt{1+x}\sqrt{2+\sqrt{1+x}}}$

$\Rightarrow -\frac{1}{4} \times \frac{1}{\sqrt{1+3}\sqrt{2+\sqrt{1+3}}}$

$\Rightarrow -\frac{1}{4} \times \frac{1}{2\times\sqrt{2+2}}$

$\Rightarrow -\frac{1}{4} \times \frac{1}{2\times2} = -\frac{1}{16}$

9. (B) $y = e^{\sqrt{x}} + \frac{1}{e^{\sqrt{x}}}$

On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}} + e^{-\sqrt{x}} \times \left(\frac{-1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}} - \frac{1}{e^{\sqrt{x}}} \times \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}\left(e^{\sqrt{x}} - \frac{1}{e^{\sqrt{x}}}\right)$$

10. (C) $\int_0^2 x^m(2-x)^n dx + \lambda\int_0^2 x^n(2-x)^m dx = 0$

Prop.IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\Rightarrow \int_0^2 (2-x)^m x^n dx + \lambda\int_0^2 x^n(2-x)^m dx = 0$$

$$\Rightarrow (\lambda+1)\int_0^2 x^n(2-x)^m dx = 0$$

$\Rightarrow \lambda + 1 = 0 \Rightarrow \lambda = -1$

11. (C) Given that A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now, $A^2 + 7I_2 = 5A$

$\Rightarrow A^{-1}A^2 + 7A^{-1}I_2 = 5A^{-1}A$

$\Rightarrow A + 7A^{-1} = 5I$

$\Rightarrow 7A^{-1} = 5I - A$

$$\Rightarrow 7A^{-1} = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

12. (B) $\tan A = \frac{1}{7}$ and $\tan B = \frac{1}{3}$

Now, $\tan 2B = \frac{2\tan B}{1-\tan^2 B}$

$$\Rightarrow \tan 2B = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}}$$

$$\Rightarrow \tan 2B = \frac{\frac{2}{3}}{\frac{8}{9}} \Rightarrow \tan 2B = \frac{3}{4}$$

Now, $\tan(A+2B) = \frac{\tan A + \tan 2B}{1-\tan A.\tan^2 B}$

$$\Rightarrow \tan(A+2B) = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}$$

$$\Rightarrow \tan(A+2B) = \frac{\frac{4+21}{28}}{\frac{28-3}{28}}$$

$$\Rightarrow \tan(A+2B) = \frac{25}{25}$$

$\Rightarrow \tan(A+2B) = 1$

$\Rightarrow \tan(A+2B) = \tan 45° \Rightarrow A + 2B = 45°$

13. (B) $f(x) = \sqrt{x-\sqrt{x-\sqrt{x-\sqrt{x.....\infty}}}}$

$\Rightarrow f(x) = \sqrt{x-f(x)}$

On squaring

$\Rightarrow [f(x)]^2 = x - f(x)$

On differentiating both side

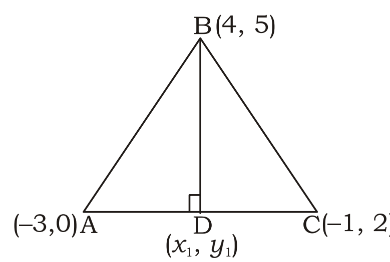
$\Rightarrow 2f(x).\ f'(x) = 1 - f'(x)$

$\Rightarrow 2f(x).f'(x) + f'(x) = 1$

$\Rightarrow f'(x)[2f(x)+1] = 1$

$$\Rightarrow f'(x) = \frac{1}{2f(x)+1}$$

14. (C)



Let D = (x_1, y_1)

Slope of line AC(m_1) = $\frac{2-0}{-1+3}$ = 1

Slope of line BD(m_2) = $\frac{y_1-5}{x_1-4}$

Now, $m_1 \times m_2 = -1$

$\Rightarrow 1 \times \frac{y_1-5}{x_1-4} = -1$

$\Rightarrow x_1 + y_1 = 9$ …(i)

Equation of line AC

$y - 2 = \frac{2-0}{-1+3}(x+1)$

$\Rightarrow x - y = -3$

Point D(x_1, y_1) lies on the line AC

$\Rightarrow x_1 - y_1 = -3$ …(ii)

from eq(i) and eq(ii)

$x_1 = 3,\ y_1 = 6$

Co-ordinate of foot of altitude = (3, 6)

15. (C) Let circumcentre(P) of $\Delta ABC = (x_1, y_1)$

AP = BP = CP

Now, $AP^2 + BP^2$

$\Rightarrow (x_1 + 3)^2 + (y_1 - 0)^2 = (x_1 - 4)^2 + (y_1 - 5)^2$

On solving

$\Rightarrow 7x_1 + 5y_1 = 16$ ...(i)

Now, $AP^2 = CP^2$

$\Rightarrow (x_1+3)^2 + (y_1 - 0)^2 = (x_1+1)^2 + (y_1 - 2)^2$

$\Rightarrow x_1^2 + 9 + 6x_1 + y_1^2 = x_1^2 + 1 + 2x_1 + y_1^2 + 4 - 9y_1$

$\Rightarrow 9 + 6x_1 = 1 + 2x_1 + 4 - 4y_1$

$\Rightarrow 4x_1 + 4y_1 = -4$

$\Rightarrow x_1 + y_1 = -1$ ...(ii)

from eq(i) and eq(ii)

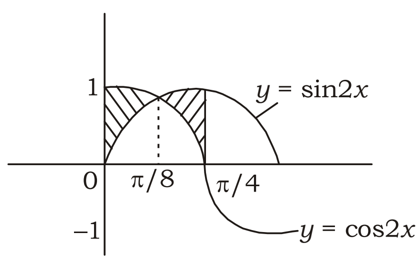
$x_1 = \frac{21}{2}$ and $y_1 = \frac{-23}{2}$

Hence circumcentre of $\Delta ABC = \left(\frac{21}{2}, \frac{-23}{2}\right)$

16. (B) Centroid of $\Delta ABC = \left[\frac{(-3)+4+(-1)}{3}, \frac{0+5+2}{3}\right]$

Centroid of $\Delta ABC = \left(0, \frac{7}{3}\right)$

17. (B)



Area = $\int_0^{\pi/8} (\cos 2x - \sin 2x)\,dx$

Area = $\left[\frac{\sin 2x}{2} + \frac{\cos 2x}{2}\right]_0^{\pi/8}$

Area = $\frac{1}{2}\left[\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)\right]$

Area = $\frac{1}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$

Area = $\frac{\sqrt{2}-1}{2}$ sq.unit

18. (A) Area = $\int_{\pi/8}^{\pi/4} (\sin 2x - \cos 2x)\,dx$

Area = $\left[\frac{-\cos 2x}{2} - \frac{\sin 2x}{2}\right]_{\pi/8}^{\pi/4}$

Area = $\frac{-1}{2}\left[\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$

Area = $\frac{-1}{2}\left[(0+1) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

Area = $\frac{-1}{2}\left[1 - \sqrt{2}\right] = \frac{\sqrt{2}-1}{2}$ sq.unit

19. (B) $\sin\left[\tan^{-1}\left\{\tan\left(\frac{17\pi}{4}\right)\right\}\right]$

$\Rightarrow \sin\left[\tan^{-1}\left\{\tan\left(2 \times 2\pi + \frac{\pi}{4}\right)\right\}\right]$

$\Rightarrow \sin\left[\tan^{-1}\left\{\tan\frac{\pi}{4}\right\}\right]$

$\Rightarrow \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

20. (D) $A = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 18 & 18 \\ 18 & 18 \end{bmatrix}$

$A^2 = 2\begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix} = 2\begin{bmatrix} 3^2 & 3^2 \\ 3^2 & 3^2 \end{bmatrix}$

$A^3 = A^2.A$

$A^3 = 2\begin{bmatrix} 9 & 9 \\ 9 & 9 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

$A^3 = 2\begin{bmatrix} 54 & 54 \\ 54 & 54 \end{bmatrix}$

$A^3 = 2^2\begin{bmatrix} 27 & 27 \\ 27 & 27 \end{bmatrix} = 2^2\begin{bmatrix} 3^3 & 3^3 \\ 3^3 & 3^3 \end{bmatrix}$

Similarly

$A^n = 2^{n-1}\begin{bmatrix} 3^n & 3^n \\ 3^n & 3^n \end{bmatrix}$

21. (B) $\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right)$

$\Rightarrow \tan^{-1}\left[\frac{\frac{a}{b} + \frac{a+b}{a-b}}{1 - \frac{a}{b} \times \frac{a+b}{a-b}}\right]$

$\Rightarrow \tan^{-1}\left[\frac{a^2 - ab + ab + b^2}{ab - b^2 - a^2 - ab}\right]$

$\Rightarrow \tan^{-1}\left[\frac{a^2 + b^2}{-(a^2 + b^2)}\right]$

$\Rightarrow \tan^{-1}(-1)$

$\Rightarrow \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$

22. (A) Let X and Y are two persons and they hit a target with the probability A and B respectively.

$\therefore$ P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{4}$

P(Probability of hitting the target by anyone X or Y)

$\Rightarrow P(A\cap \overline{B}) + P(\overline{A} \cap B)$

$\Rightarrow P(A).P(\overline{B}) + P(\overline{A}).P(B)$

$\Rightarrow \frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{1}{4}$

$\Rightarrow \frac{1}{4}+\frac{1}{6} = \frac{5}{12}$

23. (C) $x = 1+\left(\frac{y}{5}\right)+\left(\frac{y}{5}\right)^2+\left(\frac{y}{5}\right)^3+....$ where $|y| < 5$

$\Rightarrow x = \frac{1}{1-\frac{y}{5}} \Rightarrow x = \frac{5}{5-y}$

$\Rightarrow 5x - xy = 5 \Rightarrow xy = 5x - 5$

$\Rightarrow y = \frac{5x-5}{x}$

24. (B) sin(–1140) = – sin(1140)

= – sin(3×360 + 60)

= – sin60 = $\frac{-\sqrt{3}}{2}$

25. (D) $\lim_{x\to 0}\frac{\sin x - \tan x}{x}$ $\left[\frac{0}{0}\right]$ from

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0}\frac{\cos x+\sec^2 x}{1}$

$\Rightarrow \cos 0 + \sec^2 0$

$\Rightarrow 1 + 1 = 2$

26. (A) Let $z = \begin{bmatrix} \omega & \omega^2 & 1+\omega^2 \\ 1 & \omega & \omega+\omega^2 \\ \omega^2 & 1 & 1+\omega \end{bmatrix}$

$|z| = \begin{vmatrix} \omega & \omega^2 & 1+\omega^2 \\ 1 & \omega & \omega+\omega^2 \\ \omega^2 & 1 & 1+\omega \end{vmatrix}$

$C_1 \to C_1 + C_3$

$|z| = \begin{vmatrix} \omega+1+\omega^2 & \omega^2 & 1+\omega^2 \\ 1+\omega+\omega^2 & \omega & \omega+\omega^2 \\ \omega^2+1+\omega & 1 & 1+\omega \end{vmatrix}$

$|z| = \begin{vmatrix} 0 & \omega^2 & 1+\omega^2 \\ 0 & \omega & \omega+\omega^2 \\ 0 & 1 & 1+\omega \end{vmatrix}$

$|z| = 0 = 1 + \omega + \omega^2$

27. (B) $A = \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix}$

Co-factors of A-

$C_{11} = (-1)^{1+1}(1) = 1$, $C_{12} = (-1)^{1+2}(4) = -4$

$C_{21} = (-1)^{2+1}(3) = -3$, $C_{22} = (-1)^{2+2}(-2) = -2$

$C = \begin{bmatrix} 1 & -4 \\ -3 & -2 \end{bmatrix}$

$AdjA = C^T = \begin{bmatrix} 1 & -3 \\ -4 & -2 \end{bmatrix}$

then $A(AdjA) = \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -4 & -2 \end{bmatrix}$

$\Rightarrow A(AdjA) = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$

$\Rightarrow A(AdjA) = -14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -14I_2$

28. (B) word "SELECTION"

Total no. of arrangements = $\frac{9!}{2!}$

when 'E' come together

the total no. of arrangements = 8!

when 'E' do not come together

The total no. of arrangement = $\frac{9!}{2!} - 8!$

= $\frac{7}{2}\times 8!$

The required probability = $\frac{\frac{7}{2}\times 8!}{\frac{9!}{2!}} = \frac{7}{9}$

29. (C) Equation of line is $ax\tan\alpha - by\sec\alpha = ab$

Perpendicular distance from point $(0, \sqrt{a^2+b^2})$

$d_1 = \left|\frac{0 - b\sec\alpha\left(\sqrt{a^2+b^2}\right) - ab}{\sqrt{(a\tan\alpha)^2+(-b\sec\alpha)^2}}\right|$

$d_1 = \frac{b\left(\sqrt{a^2+b^2}\right)\sec\alpha - ab}{\sqrt{a^2\tan^2\alpha + b^2\sec^2\alpha}}$

Similarly perpendicular distance from point $(0, -\sqrt{a^2+b^2})$

$d_2 = \left|\frac{0 - b\sec\alpha\left(-\sqrt{a^2+b^2}\right) - ab}{\sqrt{(a\tan\alpha)^2+(-b\sec\alpha)^2}}\right|$

$d_2 = \frac{b\left(\sqrt{a^2+b^2}\right)\sec\alpha - ab}{\sqrt{a^2\tan^2\alpha + b^2\sec^2\alpha}}$

Now, $d_1 \times d_2 = \frac{\left[b(\sqrt{a^2+b^2})\sec\alpha + ab\right]\left[b\sqrt{a^2+b^2}\sec\alpha - ab\right]}{\sqrt{a^2\tan^2\alpha+b^2\sec^2\alpha}\sqrt{a^2\tan^2\alpha+b^2\sec^2\alpha}}$

$\Rightarrow d_1 \times d_2 = \frac{b^2(a^2+b^2)\sec^2\alpha - a^2b^2}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2[a^2\sec^2\alpha + b^2\sec^2\alpha - a^2]}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2[a^2(\sec^2\alpha - 1) + b^2\sec^2\alpha]}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2[a^2(\sec^2\alpha - 1) + b^2\sec^2\alpha]}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2(a^2\tan^2\alpha + b^2\sec^2\alpha)}{(a^2\tan^2\alpha + b^2\sec^2\alpha)} = b^2$

30. (C) $\cos^2 53\frac{1}{2} - \cos^2 36\frac{1}{2}$

$\Rightarrow \cos^2 53\frac{1}{2} - \left(90 - 53\frac{1}{2}\right)$

$\Rightarrow \cos^2 53\frac{1}{2} - \sin^2 53\frac{1}{2}$

$\Rightarrow \cos\left(2 \times 53\frac{1}{2}\right) \Rightarrow \cos(107)$

$\Rightarrow \cos(90 + 17) = -\sin 17$

31. (A) Given that X = $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 11 \\ 5 & 20 \end{bmatrix}$ and

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

AX = B

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 11 \\ 5 & 20 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+b & -3a+4b \\ 2c+d & -3c+4d \end{bmatrix} = \begin{bmatrix} 0 & 11 \\ 5 & 20 \end{bmatrix}$

On comparing

$2a + b = 0$ and $-3a + 4b = 11$

On solving $a = -1$ and $b = 2$

Now, $2c + d = 5$ and $-3c + 4d = 20$

On solving $c = 0$ and $d = 5$

$\therefore$ A = $\begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$

32. (C) $\left[\frac{\cos\frac{\pi}{3} - i\left(1 - \sin\frac{\pi}{3}\right)}{\cos\frac{\pi}{3} + i\left(1 - \sin\frac{\pi}{3}\right)}\right]^2$

$\Rightarrow \left[\frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - i\left[1 - \cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right)\right]}{\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) + \left[1 - \cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right)\right]}\right]^2$

$\Rightarrow \left[\frac{\sin\frac{\pi}{6} - i\left[1 - \cos\frac{\pi}{6}\right]}{\sin\frac{\pi}{6} + i\left(1 - \cos\frac{\pi}{6}\right)}\right]^2$

$\Rightarrow \left[\frac{2\sin\frac{\pi}{12}.\cos\frac{\pi}{12} - i \times 2\cos^2\frac{\pi}{12}}{2\sin\frac{\pi}{12}.\cos\frac{\pi}{12} + i \times 2\cos^2\frac{\pi}{12}}\right]^2$

$\Rightarrow \left[\frac{2\cos\frac{\pi}{12}}{2\cos\frac{\pi}{12}}\right]^2\left[\frac{\cos\frac{\pi}{12} - i\cos\frac{\pi}{12}}{\cos\frac{\pi}{12} + i\cos\frac{\pi}{12}}\right]^2$

$\Rightarrow \left[\frac{\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}}{\sin\frac{\pi}{12} + i\cos\frac{\pi}{12}}\right]^2$

$\Rightarrow \left[\frac{\left(\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}\right)\left(\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}\right)}{\left(\sin\frac{\pi}{12} + i\cos\frac{\pi}{12}\right)\left(\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}\right)}\right]^2$

$\Rightarrow \left[\frac{\left(\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}\right)^2}{\sin^2\frac{\pi}{12} - i^2\cos^2\frac{\pi}{12}}\right]^2$

$\Rightarrow \frac{\left(\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}\right)^4}{1}$

$\Rightarrow \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{12}\right) - i\sin\left(\frac{\pi}{2} - \frac{\pi}{12}\right)\right]^4$

$\Rightarrow \left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)^4$

$\Rightarrow \cos\left(4 \times \frac{5\pi}{12}\right) - i\sin\left(4 \times \frac{5\pi}{12}\right)$

$\Rightarrow \cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}$

$\Rightarrow \cos\left(2\pi - \frac{\pi}{3}\right) + i\sin\left(2\pi - \frac{\pi}{3}\right)$

$\Rightarrow \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\Rightarrow \frac{1}{2} + \frac{i\sqrt{3}}{2} = \frac{1 + i\sqrt{3}}{2}$

33. (A) $[x^3 + 1] = (x + 1)(x^2 - x + 1)$

$\Rightarrow [x^3 + 1] = (x + 1)(x + \omega)(x + \omega^2)$

34. (D) Let $y = \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$ ...(i)

and $z = \sin^{-1}x \Rightarrow x = \sin z$

On putting $x = \sin z$ in eq(i)

$\Rightarrow y = \tan^{-1}\left(\frac{1-\sqrt{1-\sin^2 z}}{\sin z}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{1-\cos z}{\sin z}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{2\sin^2\frac{z}{2}}{2\sin\frac{z}{2}.\cos\frac{z}{2}}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\sin\frac{z}{2}}{\cos\frac{z}{2}}\right)$

$\Rightarrow y = \tan^{-1}\left(\tan\frac{z}{2}\right)$

$\Rightarrow y = \frac{z}{2}$

On differentiating both side w.r.t.'z'

$\Rightarrow \frac{dy}{dz} = \frac{1}{2}$

35. (B) $x = 7 + 7^{1/3} + 7^{1/3}$

$\Rightarrow x - 7 = 7^{1/3} + 7^{2/3}$ ...(i)

$\Rightarrow (x - 7)^3 = (7^{1/3} + 7^{2/3})^3$

$\Rightarrow x^3 - 243 - 3 \times x \times 7(x - 7) = 7 + 7^2 + 3 \times 7^{1/3} \times 7^{2/3}(7^{1/3} + 7^{2/3})$

$\Rightarrow x^3 - 243 - 21x^2 + 147x = 56 + 21(x - 7)$ [from eq(i)]

$\Rightarrow x^3 - 243 - 21x^2 + 147x = 56 + 21x - 147$

$\Rightarrow x^3 - 21x^2 + 126x = 152$

36. (D) $I = \int_0^{\pi} |\cos x|\, dx$

$I = 2\int_0^{\pi/2} \cos x\, dx$

$I = 2[\sin x]_0^{\pi/2}$

$I = 2\left[\sin\frac{\pi}{2} - \sin 0\right]$

$I = 2[1 - 0]$

$I = 2$

37. (B) $y = \sin^{-1}(e^{x\log x}) \Rightarrow y = \sin^{-1}(e^{\log x^x})$

$\Rightarrow y = \sin^{-1}(x^x)$ ...(i)

Let $x^x = z$

taking log both side

$x\log x = \log z$

On differentiating both side w.r.t. 'x'

$\Rightarrow x \times \frac{1}{x} + \log x = \frac{1}{z}\frac{dz}{dx}$

$\Rightarrow \frac{dz}{dx} = x^x(1 + \log x)$ ...(ii)

from eq(i)

$y = \sin^{-1}z$

On differentiaiting both side w.r.t.'z'

$\frac{dy}{dz} = \frac{1}{\sqrt{1-z^2}} = \frac{1}{\sqrt{1-x^{2x}}}$

Now, $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^{2x}}} \times x^x(1 + \log x)$

$\Rightarrow \frac{dy}{dx} = \frac{x^x(1+\log x)}{\sqrt{1-x^{2x}}}$

38. (A) In ΔABC, $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - \hat{k}$

$\overrightarrow{BC} = 2\hat{i} + 2\hat{j} + 5\hat{k}$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & -1 \\ 2 & 2 & 5 \end{vmatrix}$

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \hat{i}(20+2) - \hat{j}(10+2) + \hat{k}(4-8)$

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = 22\hat{i} + 12\hat{j} - 4\hat{k}$

Area of ΔABC $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{(22)^2 + (-12)^2 + (-4)^2}$

$= \frac{1}{2}\sqrt{484 + 144 + 16} = \frac{1}{2}\sqrt{644}$

$= \frac{1}{2} \times 2\sqrt{161} = \sqrt{161}$ sq.unit

39. (A) In the expansion of $\left(x^4 - \frac{1}{x^2}\right)^{13}$

$T_r = T_{(r-1)+1} = {}^{13}C_{r-1}(x^4)^{14-r}\left(\frac{-1}{x^2}\right)^{r-1}$

$T_r = {}^{13}C_{r-1}(-1)^{r-1}x^{58-6r}$

Now, $58 - 6r = -2$

$\Rightarrow 6r = 60 \Rightarrow r = 10$

40. (D) $I = \int_{-1}^{1} \frac{|x|}{x} dx$

$I = \int_{-1}^{0} \frac{-x}{x} dx + \int_{0}^{1} \frac{x}{x} dx$

$I = -\int_{-1}^{0} 1.dx + \int_{0}^{1} 1.dx$

$I = -[x]_{-1}^{0} + [x]_{0}^{1}$

$I = -[0 + 1] + [1 - 0]$

$I = -1 + 1 = 0$

41. (B) $I = \int e^{\frac{x^2-1}{x}} dx + \int \frac{x^2+1}{x} e^{\frac{x^2+1}{x}} dx$

$\Rightarrow I = e^{\frac{x^2-1}{x}} \int 1.dx - \int \left\{ \frac{d}{dx}\left(e^{\frac{x^2-1}{x}}\right) . \int 1.dx \right\} dx$

$+ \int \frac{x^2+1}{x} e^{\frac{x^2-1}{x}} dx$

$\Rightarrow I = e^{\frac{x^2-1}{x}} .x - \int e^{\frac{x^2-1}{x}} \frac{x.(2x-0)-(x^2-1)}{x^2} .x\ dx$

$+ \int \frac{x^2+1}{x} e^{\frac{x^2-1}{x}} dx + c$

$\Rightarrow I = x.e^{\frac{x^2-1}{x}} - \int e^{\frac{x^2-1}{x}} . \frac{x^2+1}{x} dx + \int \frac{x^2-1}{x} . e^{\frac{x^2-1}{x}} dx + c$

$\Rightarrow I = x.e^{\frac{x^2-1}{x}} + c$

42. (C) Given that $A = \begin{bmatrix} 3 & 2 \\ -2 & 4 \end{bmatrix}$

Now, $A^2 - 7A + 16 I_2 = O$

$\Rightarrow A^{-1}(A^2 - 7A + 16 I_2) = A^{-1}O$

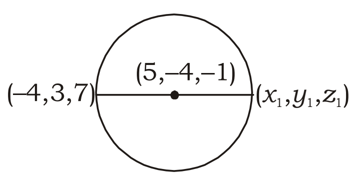
$\Rightarrow A - 7I + 16A^{-1} = O$

$\Rightarrow 16A^{-1} = -A + 7I$

$\Rightarrow 16A^{-1} = -\begin{bmatrix} 3 & 2 \\ -2 & 4 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 16A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{16}\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$

43. (D)



Equation of sphere

$x^2 + y^2 + z^2 - 10x + 8y + 2z = 0$

Centre (5, –4, –1)

Let other end point of a sphere = (x_1, y_1, z_1)

Now, $5 = \frac{-4 + x_1}{2} \Rightarrow x_1 = 14$

$-4 = \frac{3 + y_1}{2} \Rightarrow y_1 = -11$

$-1 = \frac{7 + z_1}{2} \Rightarrow z_1 = -9$

Other end point of a sphere = (14,–11, –9)

44. (A) A.T.Q,

$2a = 4 \times 2b \Rightarrow a = 4b$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{b^2}{16b^2}} \Rightarrow e = \frac{\sqrt{15}}{4}$

45. (C) Class size = Difference between two consecutive class marks = 9.5 – 8 = 1.5

46. (C) Shaded region is $\{(A \cap C) \cup (B \cap C)\} - (A \cap B \cap C)$

47. (A) 1 1 0 1 1

$1\times2^0 = 1$
$1\times2^1 = 2$
$0\times2^2 = 0$
$1\times2^3 = 8$
$1\times2^4 = 16$
27

0 . 0 1

$0 = 0\times2^{-1}$
$\frac{1}{4} = 1\times2^{-2}$
0.25

$(11011)_2 = (27)_{10}$, $(0.01)_2 = (0.25)_{10}$

Hence $(11011.01)_2 = (27.25)_{10}$

48. (C)

49. (A)

50. (B) Line $\frac{2x-1}{8} = \frac{y+5}{-2} = \frac{z-3}{4}$

$\Rightarrow \frac{2\left(x - \frac{1}{2}\right)}{8} = \frac{y+5}{-2} = \frac{z-3}{4}$

$\Rightarrow \frac{x - \frac{1}{2}}{4} = \frac{y+5}{-2} = \frac{2-3}{4}$

Direction cosines

$= \left\langle \frac{4}{\sqrt{4^2+(-2)^2+4^2}}, \frac{-2}{\sqrt{4^2+(-2)^2+4^2}}, \frac{4}{\sqrt{4^2+(-2)^2+4^2}} \right\rangle$

$= \left\langle \frac{4}{6}, \frac{-2}{6}, \frac{4}{6} \right\rangle = \left\langle \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right\rangle$

51. (C) A.T.Q.

$\frac{2b^2}{a} = \frac{1}{2} \times 2b \Rightarrow \frac{2b^2}{a} = b$

$\Rightarrow 2b = a$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{b^2}{4b^2}}$

$\Rightarrow e = \sqrt{1 - \frac{1}{4}}$

$\Rightarrow e = \sqrt{\frac{3}{4}} \Rightarrow e = \frac{\sqrt{3}}{2}$

52. (B) $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Now, $(\vec{a} - 2\vec{b}) \times (2\vec{a} + \vec{b})$

$\Rightarrow 2(\vec{a} \times \vec{a}) - 4(\vec{b} \times \vec{a}) + (\vec{a} \times \vec{b}) - 2(\vec{b} \times \vec{b})$

$\Rightarrow 0 + 4(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{b}) - 0 = 5(\vec{a} \times \vec{b})$

53. (C) $z = \frac{4-3i}{3+4i} - \frac{3+4i}{4-3i}$

$z = \frac{4-3i}{3+4i} \times \frac{3-4i}{3-4i} - \frac{3+4i}{4-3i} \times \frac{4+3i}{4+3i}$

$z = \frac{12-9i-16i+12i^2}{9-16i^2} - \frac{12+16i+9i+12i^2}{16-9i^2}$

$z = \frac{12-25i-12}{9+16} - \frac{12+25i-12}{16+9}$

$z = \frac{-25i}{25} - \frac{25i}{25}$

$z = -i - i = -2i$

54. (B) We know that

curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Area $= \frac{a^2}{6}$

Now, $\sqrt{x} + \sqrt{y} = 2 \Rightarrow \sqrt{x} + \sqrt{y} = \sqrt{4}$

The required area $= \frac{4^2}{6} = \frac{8}{3}$ sq. unit

55. (C) $\begin{vmatrix} a^2+b^2 & a+b & \lambda \\ b^2+c^2 & b+c & \lambda \\ c^2+a^2 & c+a & \lambda \end{vmatrix} = (a-b)(b-c)(c-a)$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ c^2-a^2 & c-a & 0 \\ c^2-b^2 & c-b & 0 \end{vmatrix} = (a-b)(b-c)(c-a)$

$\Rightarrow (c-a)(c-b) \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ c+a & 1 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$(c-b)(c-a)$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ c+a & 1 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ a-b & 0 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$\Rightarrow (a-b) \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ 1 & 0 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -(a-b)$

$\Rightarrow \begin{vmatrix} a^2+b^2 & a+b & \lambda \\ 1 & 0 & 0 \\ c+b & 1 & 0 \end{vmatrix} = -1$

$\Rightarrow (a^2 + b^2) \times 0 - (a + b) \times 0 + \lambda(1 - 0) = -1$

$\Rightarrow \lambda = -1$

56. (B) In ΔABC, $\frac{1}{b+c} + \frac{1}{a+c} = \frac{3}{a+b+c}$

$\Rightarrow \frac{a+b+2c}{(b+c)(a+c)} = \frac{3}{a+b+c}$

$\Rightarrow a^2 + ab + 2ac + ab + b^2 + 2bc + ac + bc + 2c^2$

$= 3ab + 3bc + 3ca + 3c^2$

$\Rightarrow a^2 + b^2 - c^2 = ab$

$\Rightarrow \frac{a^2+b^2-c^2}{ab} = 1$

$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$

$\Rightarrow \cos C = \cos\frac{\pi}{3} \Rightarrow C = \frac{\pi}{3}$

57. (B) $\frac{a\omega^7 + b\omega^9 + c\omega^{14}}{b\omega^{12} + a\omega^{10} + c\omega^{11}}$

$\Rightarrow \frac{a\omega^{3\times2+1} + b(\omega^3)^3 + c\omega^{3\times4+2}}{b(\omega^3)^4 + a\omega^{3\times3+1} + c\omega^{3\times3+2}}$

$\Rightarrow \frac{a\omega + b + c\omega^2}{b + a\omega + c\omega^2}$

$\Rightarrow \frac{b + a\omega + c\omega^2}{b + a\omega + c\omega^2} = 1$

58. (B) $n(S) = {}^{12}C_3 = 220$

$n(E) = {}^3C_1 \times {}^4C_2 \times {}^5C_0 + {}^3C_1 \times {}^4C_1 \times {}^5C_1 + {}^3C_1 \times {}^4C_0 \times {}^5C_2 + {}^3C_2 \times {}^4C_1 \times {}^5C_0 + {}^3C_2 \times {}^4C_0 \times {}^5C_1 + {}^3C_3 \times {}^4C_0 \times {}^5C_0$

$n(E) = 3\times6\times1 + 3\times4\times5 + 3\times1\times10 + 3\times4\times1 + 3\times1\times5 + 1\times1\times1$

$n(E) = 18 + 60 + 30 + 12 + 15 + 1 = 136$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$= \frac{136}{220} = \frac{34}{55}$

59. (B) $r^{1/3} + \frac{1}{r^{1/3}} = 4$

$\Rightarrow \left(r^{1/3} + \frac{1}{r^{1/3}}\right)^3 = 4^3$

$\Rightarrow r + \frac{1}{r} + 3r^{1/3} \times \frac{1}{r^{1/3}}\left(r^{1/3} + \frac{1}{r^{1/3}}\right) = 64$

$\Rightarrow r + \frac{1}{r} + 3\times4 = 64$

$\Rightarrow r + \frac{1}{r} = 64 - 12 = 52$

60. (D) $I = \int_2^3 \frac{\log x}{x}\, dx$

Let $\log x = t$ when $x \to 2,\ t \to \log 2$

$\Rightarrow \frac{1}{x}\, dx = dt$ $x \to 3,\ t \to \log 3$

$I = \int_{\log 2}^{\log 3} t\, dt$

$I = \left[\frac{t^2}{2}\right]_{\log 2}^{\log 3}$

$I = \frac{1}{2}[(\log 3)^2 - (\log 2)^2]$

61. (C) $I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x.\cos x}\, dx$ ...(i)

Prop.IV $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right).\cos\left(\frac{\pi}{2} - x\right)}\, dx$

$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \cos x.\sin x}\, dx$ ...(ii)

from eq(i) and eq(ii)

$I + I = \int_0^{\pi/2} \left(\frac{\cos x - \sin x}{1 + \sin x.\cos x} + \frac{\sin x - \cos x}{1 + \cos x.\sin x}\right) dx$

$2I = \int_0^{\pi/2} 0\, dx$

$I = 0$

62. (A) $x\sqrt{1+y} + y\sqrt{1+y} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+y}$

On squaring

$\Rightarrow x^2(1 + y) = y^2(1 + x)$

$\Rightarrow x^2 + x^2y = y^2 + xy^2$

$\Rightarrow x^2 + y^2 = xy^2 - x^2y$

$\Rightarrow (x - y)(x + y) = -xy(x - y)$

$\Rightarrow x + y = -xy$

$\Rightarrow y + xy = -x$

$\Rightarrow y(1 + x) = -x$

$\Rightarrow y = -\frac{x}{1+x}$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = -\frac{(1+x).1 - x.1}{(1+x)^2}$

$\Rightarrow \frac{dy}{dx} = -\frac{1 + x - x}{(1+x)^2}$

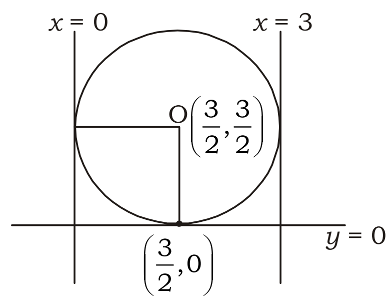
$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

63. (A) cos336 + cos156 + cos234 + cos54

$\Rightarrow \cos(360-24) + \cos(180-24) + \cos(270-36) + \cos(90-36)$

$\Rightarrow$ cos24 – cos24 – sin36 + sin36 = 0

64. (B)



Centre $O = \left(\frac{3}{2}, \frac{3}{2}\right)$ and Radius $= \frac{3}{2}$

Equation of circle

$\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$

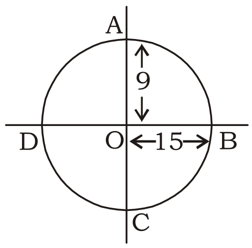
$\Rightarrow x^2 + \frac{9}{4} - 2 \times x \times \frac{3}{2} + y^2 + \frac{9}{4} - 2 \times y \times \frac{3}{2} = \frac{9}{4}$

$\Rightarrow x^2 - 3x + y^2 + \frac{9}{4} - 3y = 0$

$\Rightarrow 4x^2 - 12x + 4y^2 + 9 - 12y = 0$

$\Rightarrow 4x^2 + 4y^2 - 12x - 12y + 9 = 0$

65. (A)



Given that $e = \frac{4}{5}$

and $2ae = 24$

$\Rightarrow 2a \times \frac{4}{5} = 24 \Rightarrow a = 15$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 225\left(1 - \frac{16}{25}\right)$

$\Rightarrow b^2 = 225 \times \frac{9}{25} \Rightarrow b = 9$

Area of AOB $= \frac{1}{2} \times$ OA $\times$ OB

$= \frac{1}{2} \times 15 \times 9$

Area of ABCD = 4 × Area of DAOB

$= 4 \times \frac{1}{2} \times 15 \times 9 = 270$ sq. unit

66. (D) In the expansion of $\left(\sqrt{x} + \frac{1}{4\sqrt{x}}\right)^8$

$T_{r+1} = {}^8C_r\,(\sqrt{x})^{8-r}\left(\frac{1}{4\sqrt{x}}\right)^r$

$T_{r+1} = {}^8C_r\left(\frac{1}{4}\right)^r x^{\frac{8-2r}{2}}$

here $\frac{8-2r}{2} = 1$

$\Rightarrow 8 - 2r = 2 \Rightarrow r = 3$

coefficient of $x = {}^8C_3\left(\frac{1}{4}\right)^3$

$= \frac{8!}{3!5!} \times \frac{1}{64}$

$= 56 \times \frac{1}{64} = \frac{7}{8}$

67. (C) $S_n = n^2 + 3n + 1$

$S_{n-1} = (n-1)^2 + 3(n-1) + 1$

$S_{n-1} = n^2 + 1 - 2n + 3n - 3 + 1$

$S_{n-1} = n^2 + n - 1$

Now, $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = (n^2 + 3n + 1) - (n^2 + n - 1)$

$\Rightarrow T_n = n^2 + 3n + 1 - n^2 - n + 1$

$\Rightarrow T_n = 2n + 2 = 2(n + 1)$

68. (B) $I = \int_0^{\pi/2} \sin 2x.\log\tan x\,dx$ ...(i)

Prop.IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right).\log\tan\left(\frac{\pi}{2} - x\right)dx$

$I = \int_0^{\pi/2} \sin(\pi - x).\log\cot x\,dx$

$I = \int_0^{\pi/2} \sin 2x.\log\cot x\,dx$ ...(ii)

from eq(i) and eq(ii)

$I + I = \int_0^{\pi/2} (\sin 2x.\log\tan x + \sin 2x.\log\cot x)\,dx$

$2I = \int_0^{\pi/2} \sin 2x[\log\tan x + \log\cot x)\,dx$

$2I = \int_0^{\pi/2} \sin 2x.\log(\tan x.\cot x)\,dx$

$2I = \int_0^{\pi/2} \sin 2x.\log(1)\,dx$

$2I = \int_0^{\pi/2} 0\,dx \Rightarrow I = 0$

69. (C) Differential equation

$\frac{dy}{dx} = xy + \frac{1}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = xy\left(\frac{dy}{dx}\right) + 1$

Order = 1 and Degree = 2

70. (C) Let points (h, k) is equidistant from the points (a, b) and $(-a, -b)$.

then $\sqrt{(h-a)^2 + (k-b)^2} = \sqrt{(h+a)^2 + (k+b)^2}$

On squaring

$\Rightarrow (h - a)^2 + (k - b)^2 = (h + a)^2 + (k + b)^2$

$\Rightarrow h^2 + a^2 - 2ha + k^2 + b^2 - 2kb = h^2 + a^2 + 2ha + k^2 + b^2 + 2kb$

$\Rightarrow -2ha - 2kb = 2ha + 2kb$

$\Rightarrow 4ha + 4kb = 0$

$\Rightarrow 4(ah + kb) = 0 \Rightarrow ah + kb = 0$

Locus of a point

$ax + by = 0 \Rightarrow ax = -by$

71. (B) Let two numbers = a and b

A.T.Q,

$\frac{a+b}{2} = 9 \Rightarrow a + b = 18$ ...(i)

and $\sqrt{ab} = 16 \Rightarrow ab = 256$ ...(ii)

Now, H.M. = $\frac{a+b}{2ab}$

$\Rightarrow$ H.M. = $\frac{18}{2\times 56}$

$\Rightarrow$ H.M. = $\frac{9}{256}$

72. (C) **Statement I**

We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$

$\Rightarrow 1 + \cos 2\alpha + 1 + \cos 2\beta + 1 + \cos 2\gamma = 2$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Statement I is correct.

Statement II

from eq(i)

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow 3 - 1 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Statement II is correct.

73. (B) $3^x - 3^y = 3^{x+y}$ ...(i)

On differentiating both side w.r.t.'x'

$\Rightarrow 3^x \log 3 - 3^y \log 3 \frac{dy}{dx} = 3^{x+y}.\log 3.\frac{dy}{dx}$

$\Rightarrow 3^x - 3^y \frac{dy}{dx} = 3^{x+y}.\frac{dy}{dx}$

$\Rightarrow 3^x = 3^y \frac{dy}{dx} + 3^{x+y}\frac{dy}{dx}$

$\Rightarrow 3^x = \frac{dy}{dx}(3^y + 3^x - 3^y)$ [from eq(i)]

$\Rightarrow 3^x = 3^x.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = 1$

74. (B) Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 5\hat{i} + 3\hat{j} + 2\hat{k}$

from option B

Let $\vec{c} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

Now, $\vec{a}.\vec{c} = (2\hat{i} + 3\hat{j} - \hat{k}).\frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

$\Rightarrow \vec{a}.\vec{c} = 2 - 3 + 1 = 0$

and $\vec{b}.\vec{c} = (5\hat{i} + 3\hat{j} + 2\hat{k}).\frac{(\hat{i} - \hat{j} - \hat{k})}{\sqrt{3}}$

$\Rightarrow \vec{b}.\vec{c} = 5 - 3 - 2 = 0$

Hence verctor $\frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$ is perpendicular to the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $5\hat{i} + 3\hat{j} + 2\hat{k}$.

75. (B) $\text{cosec}^2\theta + 5 = 3\sqrt{3}\cot\theta$

$\Rightarrow 1 + \cot^2\theta + 5 = 3\sqrt{3}\cot\theta$

$\Rightarrow \cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$

$\Rightarrow \cot^2\theta - 2\sqrt{3}\cot\theta - \sqrt{3}\cot\theta + 6 = 0$

$\Rightarrow \cot\theta(\cot\theta - 2\sqrt{3}) - \sqrt{3}(\cot\theta - 2\sqrt{3}) = 0$

$\Rightarrow (\cot\theta - 2\sqrt{3})(\cot\theta - \sqrt{3}) = 0$

$\Rightarrow \cot\theta - \sqrt{3} = 0$

$\Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \cot\theta = \cot\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$

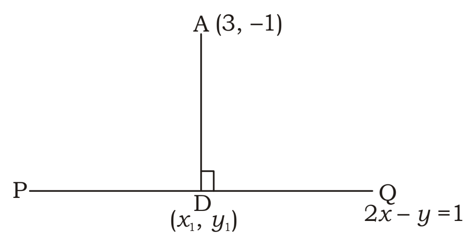
76. (C) $[(A \cap B) \cup C]'$

$\Rightarrow (A \cap B)' \cap C'$ $[\because (X \cup Y)' = X' \cap Y']$

$\Rightarrow A' \cup B' \cap C'$ $[\because (X \cap Y)' = X' \cup Y']$

77. (B)

78. (C)



line $2x - y = 1$

$\Rightarrow 2x - 1 = y$ ...(i)

Slope of line PQ = 2

Slope of line AD = $\frac{-1}{2}$

A.T.Q,

$\frac{y_1 + 1}{x_1 - 3} = \frac{-1}{2}$

$\Rightarrow 2y_1 + 2 = -x_1 + 3$

$\Rightarrow x_1 + 2y_1 = 1$ ...(ii)

Point D lie on the line (i)

$2x_1 - 1 = y_1$ ...(iii)

from eq(ii) and eq(iii)

$x_1 = \frac{3}{5}$ and $y_1 = \frac{1}{5}$

Hence foot of perpendicular = $\left(\frac{3}{5}, \frac{1}{5}\right)$

79. (C) Given that $e = \frac{2}{\sqrt{3}}$

and $ae = 3$

$\Rightarrow a \times \frac{2}{\sqrt{3}} = 3 \Rightarrow a = \frac{3\sqrt{3}}{2}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 \left(1 - \frac{4}{9}\right)$

$\Rightarrow b^2 = \frac{27}{4} \times \frac{5}{9} \Rightarrow b^2 = \frac{15}{4}$

Equation of an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{27/4} + \frac{y^2}{15/4} = 1$

$\Rightarrow \frac{4x^2}{27} + \frac{4y^2}{15} = 1$

$\Rightarrow \frac{20x^2 + 36y^2}{135} = 1$

$\Rightarrow 20x^2 + 36y^2 = 135$

80. (B) $\int_1^2 \{k^2 + (1-k)x + 2x^3\}dx \le 10$

$\Rightarrow \left[k^2 x + (1-k)\frac{x^2}{2} + \frac{2x^4}{4}\right]_1^2 \le 10$

$\Rightarrow (2k^2 + (1 - k) \times 2 + 8)$

$-\left(k^2 + (1-k) \times \frac{1}{2} + \frac{1}{2}\right) \le 10$

$\Rightarrow k^2 - \frac{3k}{2} + 9 \le 10$

$\Rightarrow 2k^2 - 3k + 18 = 20$

$\Rightarrow 2k^2 - 3k - 2 \le 0$

$\Rightarrow (2k + 1)(k - 2) \le 0$

Hence $\frac{-1}{2} \le k \le 2$

81. (B) Given that $\cos\theta = \sin^2\theta$

Now, $\sin^2\theta(1 + \sin^2\theta) = \cos\theta(1 + \sin\theta)$

$\Rightarrow \sin^2\theta(1 + \sin^2\theta) = \cos\theta + \cos^2\theta$

$\Rightarrow \sin^2\theta(1 + \sin^2\theta) = \cos\theta + 1 - \sin^2\theta$

$\Rightarrow \sin^2\theta(1 + \sin^2\theta) = \cos\theta + 1 - \cos\theta$

$\Rightarrow \sin^2\theta(1 + \sin^2\theta) = 1$

82. (C) $I = \int \frac{x}{\sin^2 x.\cos^2 x} dx$

$I = \int \frac{x(\sin^2 x + \cos^2 x)}{\sin^2 x.\cos^2 x} dx$

$I = \int (x.\sec^2 x + x.\text{cosec}^2 x)dx$

$I = x\int \sec^2 x\, dx - \int \left\{\frac{d}{dx}(x).\int \sec^2 x\, dx\right\}dx$

$+ x\int \text{cosec}^2 x\, dx - \int \left\{\frac{d}{dx}(x).\int \text{cosec}^2 x\, dx\right\}dx$

$I = x.\tan x - \int 1.\tan x\, dx + x(-\cot x)$

$-\int 1.(-\cot x)dx$

$I = x.\tan x - \log\sec x - x\cot x + \log\sin x + c$

$I = x(\tan x - \cot x) - \log(\sin x.\cos x) + c$

$I = x(\tan x - \cot x) - \log\left(\frac{2\sin x.\cos x}{2}\right) + c$

$I = x(\tan x - \cot x) - \log(\sin 2x) + \log 2 + c$

$I = x(\tan x - \cot x) - \log\sin 2x + C$

83. (A) Equation of circle

$x^2 + y^2 = 3 \Rightarrow x^2 + y^2 = (\sqrt{3})^2$

Area of circle $= \pi r^2$

$= \pi \times (\sqrt{3})^2 = 3\pi$ sq. unit

84. (D) Determinant $\begin{vmatrix} 6 & 3 & 4 \\ 2 & 3 & 9 \\ 8 & -1 & 5 \end{vmatrix}$

Minor of 4 = $\begin{vmatrix} 2 & 3 \\ 8 & -1 \end{vmatrix}$

$= -2 - 24 = -26$

85. (A) Equation $2ax^2 - 5bx + 3c = 0$

Let roots $= 3\alpha, 4\alpha$

Now, $3\alpha + 4\alpha = \frac{-(-5b)}{2a}$

$\Rightarrow 7\alpha = \frac{5b}{2a} \Rightarrow \alpha = \frac{5b}{14a}$ ...(i)

and $3\alpha.4\alpha = \frac{3c}{2a} \Rightarrow 12\alpha^2 = \frac{3c}{2a}$

$\Rightarrow \alpha^2 = \frac{c}{8a}$

$\Rightarrow \left(\frac{5b}{14a}\right)^2 = \frac{c}{8a}$ [from eq(i)]

$\Rightarrow \frac{25b^2}{196a^2} = \frac{c}{8a}$ 50 $\Rightarrow b^2 = 49ac$

86. (C) $y = (1-x^{1/16})(1+x^{1/8})(1+x^{1/4})(1+x^{1/16})$
$y = (1+x^{1/4})(1+x^{1/8})(1+x^{1/16})(1-x^{1/16})$
$y = (1+x^{1/4})(1+x^{1/8})(1-x^{1/8})$
$y = (1+x^{1/4})(1-x^{1/4})$
$y = (1-x^{1/2})$
On differentiating both side w.r.t 'x'
$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$$

87. (A) $\frac{1+\cos(B-C).\cos A}{1+\cos(B-A).\cos C}$
$$\Rightarrow \frac{1+\cos(B-C).\cos[180-(B+C)]}{1+\cos(B-A).\cos[180-(B+A)]}$$
$$\Rightarrow \frac{1-\cos(B-C).\cos(B+C)}{1-\cos(B-A).\cos(B+A)}$$
$$\Rightarrow \frac{1-\cos^2 B+\sin^2 C}{1-\cos^2 B+\sin^2 A}$$
$$\Rightarrow \frac{\sin^2 B+\sin^2 C}{\sin^2 B+\sin^2 A}$$
$$\Rightarrow \frac{b^2+c^2}{b^2+a^2} \quad \text{[by Sine Rule]}$$

88. (C) $\lim_{x\to 0} \frac{1-(\cos x)^{1/3}}{1-(\cos x)^{2/3}}$ $\left[\frac{0}{0}\right]$ form
by L-Hospital's Rule
$$\Rightarrow \lim_{x\to 0} \frac{-\frac{1}{3}(\cos x)^{-2/3}.(-\sin x)}{-\frac{2}{3}(\cos x)^{-1/3}(-\sin x)}$$
$$\Rightarrow \lim_{x\to 0} \frac{(\cos x)^{1/3}}{2(\cos x)^{2/3}}$$
$$\Rightarrow \frac{(1)^{1/3}}{2(1)^{2/3}} = \frac{1}{2}$$

89. (C) $P(16, 11) = k.C(16, 5)$
$$\Rightarrow \frac{16!}{(16-11)!} = k \times \frac{16!}{5!(16-5)!}$$
$$\Rightarrow \frac{1}{5!} = k \times \frac{1}{5!\times 11!}$$
$\Rightarrow k = 11!$

90. (B) Differential equation
$$\sin\left(\frac{dy}{dx}\right) = x$$
$$\Rightarrow \frac{dy}{dx} = \sin^{-1}x$$
On integrating
$$\Rightarrow \int dy = \int \sin^{-1} x\, dx$$
$$\Rightarrow y = \sin^{-1}x\int 1.dx - \int\left\{\frac{d}{dx}(\sin^{-1}x).\int 1.dx\right\} dx$$
$$\Rightarrow y = (\sin^{-1}x).x - \int \frac{1}{\sqrt{1-x^2}} \times x\, dx$$
$$\Rightarrow y = x.\sin^{-1}x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\, dx$$
$$\Rightarrow y = x.\sin^{-1}x + \frac{1}{2}\,\frac{(1-x^2)^{1/2}}{1/2} + c$$
$$\Rightarrow y = x.\sin^{-1}x + \sqrt{1-x^2} + c$$

91. (B) $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+\omega \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$
On comparing
$x + y = 4$ ...(i)
$x - y = 0$ ...(ii)
$2x + z = 7$ ...(iii)
$2z + w = 10$ ...(iv)
On solving eq(i) and eq(ii)
$x = 2,\ y = 2$
from eq(ii)
$2 \times 2 + z = 7 \Rightarrow z = 3$
from eq(iv)
$2 \times 3 + w = 10 \Rightarrow w = 4$
Now, $x + y + z + w = 2 + 2 + 3 + 4 = 11$

92. (A)

93. (B) 5 points out of 13 are in the same line, then, The required no. of triangle
$= {}^{13}C_3 - {}^{6}C_3$
$$= \frac{13!}{3!10!} - \frac{6!}{3!3!}$$
$= 286 - 20 = 266$

94. (C) We know that
$$1^c = \left(\frac{180}{\pi}\right)^\circ$$
$$1^c = \left(\frac{180\times 7}{22}\right)^\circ$$
$$1^c = \left(\frac{630}{22}\right)^\circ = 57°16'\ 22''$$

95. (D) $\alpha = \frac{1-\sqrt{3}i}{2} = -\omega$

Now, $1 + \alpha^2 + \alpha^4 + \alpha^6 + \alpha^8$

$\Rightarrow 1 + (-\omega)^2 + (-\omega)^4 + (-\omega)^6 + (-\omega)^8$

$\Rightarrow 1 + \omega^2 + \omega^4 + \omega^6 + \omega^8$

$\Rightarrow 1 + \omega^2 + \omega + 1 + \omega^2$ $[\because \omega^3 = 1]$

$\Rightarrow 0 + 1 + \omega^2$

$\Rightarrow -\omega = \alpha$ $[\because 1 + \omega + \omega^2 = 1]$

96. (A) $\begin{vmatrix} a^2 & bc & 1/a \\ b^2 & ca & 1/b \\ c^2 & ab & 1/c \end{vmatrix}$

$\Rightarrow \frac{abc}{abc}\begin{vmatrix} a^2 & bc & 1/a \\ b^2 & ca & 1/b \\ c^2 & ab & 1/c \end{vmatrix}$

$\Rightarrow \frac{1}{abc}\begin{vmatrix} a^3 & abc & 1 \\ b^3 & abc & 1 \\ c^3 & abc & 1 \end{vmatrix}$

$\Rightarrow \frac{abc}{abc}\begin{vmatrix} a^3 & 1 & 1 \\ b^3 & 1 & 1 \\ c^3 & 1 & 1 \end{vmatrix}$

$\Rightarrow 0$ [$\because$ Two columns are identical.]

97. (C) Let $y = \log_x x$ and $z = e^{x^2}$

$\Rightarrow y = 1$ and $\frac{dz}{dx} = 2x.e^{x^2}$

$\Rightarrow \frac{dy}{dx} = 0$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz} = 0$

98. (C)

99. (B) We know that

Minimum value of $\left(ax^2 + \frac{b}{x^2}\right) = 2\sqrt{ab}$

so minimum value of $(8\tan^2\theta + 32\cot^2\theta)$

$= 2\sqrt{8 \times 32} = 32$

100. (A) $\begin{bmatrix} 2 & 2 & -x \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 5 & -3 & 2 \\ 6 & -4 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix} = [0]$

$\Rightarrow \begin{bmatrix} 2 & 2 & -x \end{bmatrix}\begin{bmatrix} 1\times3+2\times4+0\times2 \\ 5\times3+(-3)\times4+2\times(-2) \\ 6\times3+(-4)\times4+(-1)\times(-2) \end{bmatrix} = [0]$

$\Rightarrow \begin{bmatrix} 2 & 2 & -x \end{bmatrix}\begin{bmatrix} 11 \\ -1 \\ 4 \end{bmatrix} = [0]$

$\Rightarrow [2\times11 + 2\times(-1) + (-x)\times4] = [0]$

$\Rightarrow [20 - 4x] = [0]$

$\Rightarrow 20 - 4x = 0 \Rightarrow x = 5$

101. (C) $y = \left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x.\tan^2 x}\right)\cot 3x$

$y = \left[\frac{(\tan 2x - \tan x)(\tan 2x + \tan x)}{(1 - \tan 2x.\tan x)(1 + \tan 2x.\tan x)}\right]\cot 3x$

$y = \left(\frac{\tan 2x - \tan x}{1 + \tan 2x.\tan x}\right)\left(\frac{\tan 2x + \tan x}{1 + \tan 2x.\tan x}\right)\cot 3x$

$y = \tan(2x - x).\tan(2x + x).\cot 3x$

$y = \tan x.\tan 3x.\times\frac{1}{\tan 3x}$

$y = \tan x$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \sec^2 x$

102. (B) (sinB + sinC + sinA)(sinB + sinC – sinA) = 3sinB. sinC

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow (b + c + a)(b + c - a) = 3bc$

$\Rightarrow (b + c)^2 - a^2 = 3bc$

$\Rightarrow b^2 + c^2 + 2bc - a^2 = 3bc$

$\Rightarrow b^2 + c^2 - a^2 = bc$

$\Rightarrow \frac{b^2 + c^2 - a^2}{bc} = 1$

$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2}$

$\Rightarrow \cos A = \cos\frac{\pi}{3} \Rightarrow A = \frac{\pi}{3}$

103. (C) $\log_{ax} x = \frac{1}{\log_x ax}$

$\Rightarrow \log_{ax} x = \frac{1}{\log_x a + \log_x x}$

$\Rightarrow \log_{ax} x = \frac{1}{\log_x a + 1}$

Now, a, b, c are in G.P.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in A.P.

$\Rightarrow 1 + \log_x a, 1 + \log_x b, 1 + \log_x c$ are in A.P.

$\Rightarrow \frac{1}{1 + \log_x a}, \frac{1}{1 + \log_x b}, \frac{1}{1 + \log_x c}$ are in H.P.

$\Rightarrow \log_{ax} x, \log_{bx} x, \log_{cx} x$ are in H.P.

104. (B) $\log_4[\log_4(\log_4 x)] = \log_4 4$

On comparing

$\Rightarrow \log_4(\log_4 x) = 4$

$\Rightarrow \log_4(\log_4 x) = 4\log_4 4$

$\Rightarrow \log_4(\log_4 x) = \log_4 4^4$

On comparing

$\Rightarrow \log_4 x = 4^4$

$\Rightarrow \log_4 x = 256 \Rightarrow x = 4^{256}$

105. (C)

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

$$\begin{array}{r} 0.75 \\ \times 2 \\ \hline \boxed{1}.50 \\ \times 2 \\ \hline \boxed{1}.00 \end{array}$$

$(0.75)_{10} = 0.11$

$(37)_{10} = (100101)_2$

Hence $(37.75)_{10} = (100101.11)_2$

106. (A)

107. (B) Line $\frac{x}{3} + \frac{y}{6} = 1$

$\Rightarrow \frac{2x+y}{6} = 1$

$\Rightarrow 2x + y = 6$

Slope of line = –2

Slope of perpendicular line $= \frac{-1}{-2} = \frac{1}{2}$

108. (B) One year = 365 years

= 52 weeks and 1 day

S = {Mon, Tue, Wed, Thu, Fri, Sat, Sun}

$n(\text{S}) = 7$

E = {Sun, Mon}; $n(\text{E}) = 2$

The required Probability $\text{P(E)} = \frac{n(\text{E})}{n(\text{S})} = \frac{2}{7}$

109. (B) Data 2, 3, 6, 8, 11, 12, 13, 5; $n = 8$

$\sum_{i=0}^{n} x_i = 2 + 3 + 6 + 8 + 11 + 12 + 13 + 5 = 60$

$\sum_{i=0}^{n} x_i^2 = 2^2 + 3^2 + 6^2 + 8^2 + 11^2 + 12^2 + 13^2 + 5^2$

= 572

$$\text{S.D.}(\sigma) = \sqrt{\frac{\sum_{i=0}^{n} x_i^2}{n} - \left(\frac{\sum_{i=0}^{n} x_i}{n}\right)^2}$$

$$\text{S.D.}(\sigma) = \sqrt{\frac{572}{8} - \left(\frac{60}{8}\right)^2}$$

$$\text{S.D.}(\sigma) = \sqrt{\frac{572}{8} - \frac{225}{4}}$$

$$\text{S.D.}(\sigma) = \sqrt{\frac{572 - 450}{8}}$$

$$\text{S.D.}(\sigma) = \sqrt{\frac{122}{8}} = \sqrt{\frac{61}{4}}$$

Now, Variance = $(\text{S.D.})^2$

$\Rightarrow \text{Variance} = \left(\sqrt{\frac{61}{4}}\right)^2 = \frac{61}{4}$

110. (C) Let two numbers are a and b.

A.T.Q.,

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{5}{4}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$$

By Componendo and Dividendo Rule

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$$

$$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{9}{1}$$

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3}{1}$$

By Componendo and Dividendo rule

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{4}{2} \Rightarrow \sqrt{\frac{a}{b}} = \frac{2}{1}$$

$$\Rightarrow \frac{a}{b} = \frac{4}{1}$$

Hence $a : b = 4 : 1$

111. (D) Equation of line

$5x + 9y = 9$

Equation of line which is prependicular to the given line

$9x - 5y = \text{C}$ ...(i)

Mid-point of two points (–2, 3) and (– 4, –8)

$= \left(\frac{-2-4}{2}, \frac{3-8}{2}\right) = \left(-3, \frac{-5}{2}\right)$

Eq(i) passes through the point $\left(-3, \frac{-5}{2}\right)$

$9(-3) - 5\left(\frac{-5}{2}\right) = \text{C}$

$\Rightarrow -27 + \frac{25}{2} = \text{C} \Rightarrow \text{C} = \frac{-29}{2}$

from eq(i)

$\Rightarrow 9x - 5y = \frac{-29}{2}$

$\Rightarrow 18x - 10y + 29 = 0$

112. (B) $I = \int \frac{x^4+x+1}{x^2+1} dx$

$I = \int \left(x^2 - 1 + \frac{x+2}{x^2+1} \right) dx$

$I = \int x^2.dx - \int 1.dx + \frac{1}{2}\int \frac{2x}{x^2+1} dx + 2\int \frac{1}{1+x^2} dx$

$I = \frac{x^3}{3} - x + \frac{1}{2}\log(x^2+1) + 2\tan^{-1}x + C$

113. (A) A. M. $\geq$ G. M. $\geq$ H. M.

114. (C) $\frac{\log_{27} 3 \times \log_{16} 2}{\log_{64} 4}$

$\Rightarrow \frac{\frac{1}{\log_3 27} \times \frac{1}{\log_2 16}}{\frac{1}{\log_4 64}}$

$\Rightarrow \frac{\frac{1}{3\log_3 3} \times \frac{1}{4\log_2 2}}{\frac{1}{3\log_4 4}} \Rightarrow \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{1}{3}} = \frac{1}{4}$

115. (B)

Class	x	f	$f \times x$
0-3	1.5	5	7.5
3-6	4.5	6	27.0
6-9	7.5	12	90.0
9-12	10.5	15	157.5
12-15	13.5	18	243.0
15-18	16.5	4	66.0

$\sum f = 60, \ \sum f \times x = 591$

Mean $= \frac{\Sigma f \times x}{\Sigma f}$

Mean $= \frac{591}{60} = 9.85$

116. (D) $f(x) = \frac{3}{5}x + \frac{1}{2}$

Let $f(x) = y$

$\Rightarrow y = \frac{3}{5}x + \frac{1}{2} \Rightarrow y - \frac{1}{2} = \frac{3}{5}x$

$\Rightarrow \frac{2y-1}{2} = \frac{3x}{5} \Rightarrow x = \frac{5}{3}\left(\frac{2y-1}{2}\right)$

$\Rightarrow x = \frac{5}{6}(2y-1)$

$\Rightarrow f^{-1}(y) = \frac{5}{6}(2y-1)$

$\Rightarrow f^{-1}(x) = \frac{5}{6}(2x-1)$

117. (B) We know that

$\tan^{-1}A = \sin^{-1}\frac{A}{\sqrt{1+A^2}}$

Now, $\tan^{-1}\left(\frac{1}{x}\right) = \sin^{-1}\left(\frac{\frac{1}{x}}{\sqrt{1+\left(\frac{1}{x}\right)^2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \sin^{-1}\left(\frac{\frac{1}{x}}{\frac{\sqrt{x^2+1}}{x}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \sin^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)$

118. (B) $I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Let $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

$I = \int 2e^t \, dt$

$I = 2e^t + c$

$I = 2e^{\sqrt{x}} + c$

119. (D) $I = \int e^x(\sin x + \cos x)\, dx$

$I = e^x.\sin x + c$

$\left[\because \int e^x [f(x) + f'(x)] dx = e^x.f(x) + c\right]$

120. (A) $f(x) = \begin{cases} ax+7, & x \leq 2 \\ x^2-1, & x > 2 \end{cases}$ is continuous at $x = 2$, then

$\Rightarrow \lim_{x \to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x \to 2} (x^2 - 1) = a \times 2 + 7$

$\Rightarrow 2^2 - 1 = 2a + 7$

$\Rightarrow 3 = 2a + 7$

$\Rightarrow 2a = -4 \Rightarrow a = -2$

NDA (MATHS) MOCK TEST - 158 (Answer Key)

1. (C)	21. (B)	41. (B)	61. (C)	81. (B)	101. (C)
2. (A)	22. (A)	42. (C)	62. (A)	82. (C)	102. (A)
3. (B)	23. (C)	43. (D)	63. (A)	83. (A)	103. (C)
4. (A)	24. (B)	44. (A)	64. (B)	84. (D)	104. (B)
5. (B)	25. (D)	45. (C)	65. (A)	85. (A)	105. (C)
6. (C)	26. (A)	46. (C)	66. (D)	86. (C)	106. (A)
7. (C)	27. (B)	47. (A)	67. (C)	87. (A)	107. (B)
8. (D)	28. (B)	48. (C)	68. (B)	88. (C)	108. (B)
9. (B)	29. (C)	49. (A)	69. (C)	89. (C)	109. (B)
10. (C)	30. (C)	50. (B)	70. (C)	90. (B)	110. (C)
11. (C)	31. (A)	51. (C)	71. (B)	91. (B)	111. (D)
12. (B)	32. (C)	52. (B)	72. (C)	92. (A)	112. (B)
13. (B)	33. (A)	53. (C)	73. (B)	93. (B)	113. (A)
14. (C)	34. (D)	54. (B)	74. (B)	94. (C)	114. (C)
15. (C)	35. (B)	55. (C)	75. (B)	95. (D)	115. (B)
16. (B)	36. (D)	56. (B)	76. (C)	96. (A)	116. (D)
17. (B)	37. (B)	57. (B)	77. (B)	97. (C)	117. (B)
18. (A)	38. (A)	58. (B)	78. (C)	98. (C)	118. (B)
19. (B)	39. (A)	59. (B)	79. (C)	99. (B)	119. (D)
20. (D)	40. (D)	60. (D)	80. (B)	100. (A)	120. (A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777