KD Campus

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PLOT NO. 2 SSI, OPP METRO PILLAR 150, GT KARNAL ROAD, JAHANGIRPURI, DELHI: 110033

NDA MATHS MOCK TEST - 152 (SOLUTION)

1. (B) Let point = (h, k, l)

$$\sqrt{(h+2)^2+(k-3)^2+(l+1)^2} = \sqrt{(h+1)^2+(k-2)^2+(l-4)^2}$$

$\Rightarrow h^2 + 4 + 4h + k^2 + 9 - 6k + l^2 + 1 + 2l = h^2 + 1 + 2h + k^2 + 4 - 4k + l^2 + 16 - 8l$

$\Rightarrow 4h - 6k + 2l + 14 = 2h - 4k - 8l + 21$

$\Rightarrow 2h - 2k + 10l = 7$

Locus of a point

$2x - 2y + 10z = 7$

2. (B) $I = \int \frac{dx}{x(x^4-1)}$

$I = \int \frac{x^3 dx}{x^4(x^4-1)}$

Let $x^4 = t$

$\Rightarrow 4x^3\, dx = dt \Rightarrow x^3\, dx = \frac{1}{4} dt$

$I = \frac{1}{4}\int \frac{dt}{t(t-1)}$

$I = \frac{1}{4}\int \left(\frac{1}{t-1} - \frac{1}{t}\right) dx$

$I = \frac{1}{4}\left[\log(t-1) - \log t\right] + c$

$I = \frac{1}{4}\log\left(\frac{t-1}{t}\right) + c$

$I = \frac{1}{4}\log\left(\frac{x^4-1}{x^4}\right) + c$

3. (C)

4. (B) We know that

${}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots\ldots. + {}^nC_n = (1+x)^n$

On putting $x = 1$

${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots. + {}^nC_n = (1+1)^n = 2^n$

5. (B) $\frac{1}{i^{n+3}} + \frac{1}{i^{n+2}} + \frac{1}{i^{n+1}} + \frac{1}{i^n}$

$\Rightarrow i^{-n-3} + i^{-n-2} + i^{-n-1} + i^{-n}$

$\Rightarrow i^{-n}(i^{-3} + i^{-2} + i^{-1} + i^0)$

$\Rightarrow i^{-n}[(i^3)^{-1} + (i^2)^{-1} + i^{-1} + 1]$

$\Rightarrow i^{-n}[(-i)^{-1} + (-1)^{-1} + i^{-1} + 1]$

$\Rightarrow i^{-n}(-i^{-1} - 1 + i^{-1} + 1) = 0$

6. (B) $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$

$y = \sin x$

$\frac{dy}{dx} = \cos x$

7. (C) $\lim_{x\to\pi/4} \frac{(1-\cot x)(1+\sin 2x)}{(1+\tan x)(\pi-4x)}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to\pi/4} \frac{(1-\cot x)\times 2\cos 2x + (1+\sin 2x)(\mathrm{cosec}^2 x)}{(1+\tan x)(-4)+(\pi-4x)(\sec^2 x)}$

$\Rightarrow \frac{\left(1-\cot\frac{\pi}{4}\right)\times 2\cos\frac{\pi}{2} + \left(1+\sin\frac{\pi}{2}\right).\mathrm{cosec}^2\frac{\pi}{4}}{-4\left(1+\tan\frac{\pi}{4}\right)+0}$

$\Rightarrow \frac{0+2\times 2}{-4\times 2} = \frac{-1}{2}$

8. (C) A×(B – C) = A×B + C×A

$\Rightarrow$ A×B – A×C = A×B + C×A

$\Rightarrow$ A×B + C×A = A×B + C×A

Hence option (C) is correct.

9. (C) $S_n = 2n^2 + 3n + 4$

$S_{n-1} = 2(n-1)^2 + 3(n-1) + 4$

$S_{n-1} = 2n^2 - n + 3$

Now, $T_n = S_n - S_{n-1}$

$T_n = 2n^2 + 3n + 4 - 2n^2 + n - 3$

$T_n = 4n + 1$

$T_7 = 4 \times 7 + 1 = 29$

10. (B) $I = \int_0^4 |2x-3|\, dx$

$I = \left|\int_0^{3/2}(2x-3)\, dx\right| + \left|\int_{3/2}^4 (2x-3)\, dx\right|$

$I = \left|\left[2\times\frac{x^2}{2} - 3x\right]_0^{3/2}\right| + \left|\left[2\times\frac{x^2}{2} - 3x\right]_{3/2}^4\right|$

$I = \left|\left(\frac{9}{4} - \frac{9}{2}\right) - 0\right| + \left|(16-12) - \left(\frac{9}{4} - \frac{9}{2}\right)\right|$

$I = \frac{9}{4} + 4 + \frac{9}{4} = \frac{17}{2}$

11. (A) Given that

$a = 3,\ b = \dfrac{9}{2}$

$f(x) = x^2 + x - 1$

$f'(x) = 2x + 1$

$f'(c) = 2c + 1$

$f(a) \Rightarrow f(3) = 11,\ f(b) \Rightarrow f\left(\dfrac{9}{2}\right) = \dfrac{95}{4}$

Now, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow 2c + 1 = \dfrac{\frac{95}{4} - 11}{\frac{9}{2} - 3}$

$\Rightarrow 2c + 1 = \dfrac{\frac{51}{4}}{3/2}$

$\Rightarrow 2c + 1 = \dfrac{17}{2} \Rightarrow c = \dfrac{15}{4}$

12. (C)

13. (B) Sphere $x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$

$u = -4,\ v = 1,\ w = 3,\ d = 1$

$r = \sqrt{u^2 + v^2 + w^2 - 1}$

$r = \sqrt{(-4)^2 + 1^2 + 3^2 - 1} = 5$

Diameter = $2r$ = 10 unit

14. (C) Two circles

$x^2 + y^2 + 2x - 4y + 6 = 0$

and $x^2 + y^2 - 3x + 6y + \lambda = 0$

Condition of orthogonality

$2gg' + 2ff' = c + c'$

$\Rightarrow 2 \times 1 \times \left(\dfrac{-3}{2}\right) + 2 \times (-2) \times 3 = 6 + \lambda$

$\Rightarrow -3 - 12 = 6 + \lambda \Rightarrow \lambda = -21$

15. (C) Given that $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

Now, $[(\vec{a} + 2\vec{b}) \times (\vec{b} - 3\vec{a})].\vec{a}$

$\Rightarrow [(\vec{a} \times \vec{b}) + 2(\vec{b} \times \vec{b}) - 3(\vec{a} \times \vec{a}) - 6(\vec{b} \times \vec{a})].\vec{a}$

$\Rightarrow [(\vec{a} \times \vec{b}) + 6(\vec{a} \times \vec{b})].\vec{a}$

$\Rightarrow 7(\vec{a} \times \vec{b}).\vec{a} = 0$

16. (D) Digits 0, 1, 2, 3, 4, 5, 6

4	6	5	4	3

= 4×6×5×4×3 =1440

↓ (3, 4, 5, 6)

17. (C) Ratio of angles = 4 : 3 : 5

Let angles = $4x, 3x, 5x$

$4x + 3x + 5x = 180$

$\Rightarrow 12x = 180 \Rightarrow x = 15$

Angles = 60, 45, 75

Sine Rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\Rightarrow \dfrac{a}{\sin 60} = \dfrac{b}{\sin 45} = \dfrac{c}{\sin 75}$

$\Rightarrow \dfrac{a}{\frac{\sqrt{3}}{2}} = \dfrac{b}{\frac{1}{\sqrt{2}}} = \dfrac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

$\Rightarrow \dfrac{a}{\sqrt{6}} = \dfrac{b}{2} = \dfrac{c}{\sqrt{3}+1}$

Hence $a : b : c = \sqrt{6} : 2 : \sqrt{3} + 1$

18. (B) $\dfrac{d}{dx}(\text{cosec}hx) = -\text{cosec}hx.\coth hx$

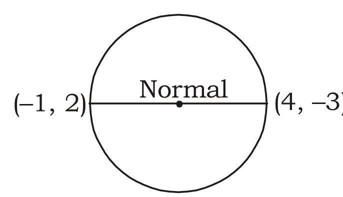
19. (C) $\begin{vmatrix} 1 & -1 & -3 & 5 \\ 4 & 2 & -1 & 6 \\ -6 & -4 & 3 & 1 \\ -2 & 0 & 1 & 7 \end{vmatrix}$

Minor of element 2 = $\begin{vmatrix} 1 & -3 & 5 \\ -6 & 3 & 1 \\ -2 & 1 & 7 \end{vmatrix}$

$= 1(21 - 1) + 3(-42 + 2) + 5(-6 + 6)$

$= 20 - 120 = -100$

20. (A)



Equation of circle

$(x + 1)(x - 4) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 - 3x - 4 + y^2 + y - 6 = 0$

$\Rightarrow x^2 + y^2 - 3x + y = 10$

21. (A) $f(x) = \begin{cases} 3x^2 - 6x + 1, & 0 < x \le 2 \\ 3x - 5, & 2 < x \le 3 \\ 15 - x, & 3 < x \le 4 \end{cases}$

At $x = 2$

L.H.L. = $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (3x^2 - 6x + 1)$

$= 3 \times 2^2 - 6 \times 2 + 1 = 1$

R.H.L. = $\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (3x - 5)$

$= 3 \times 2 - 5 = 1$

L.H.L. = R.H.L.

$f(x)$ is continuous at $x = 2$.

22. (C) $\begin{vmatrix} \frac{x^2+z^2}{y} & y & y \\ x & \frac{y^2+z^2}{x} & x \\ z & z & \frac{x^2+y^2}{z} \end{vmatrix} = kxyz$

On putting $x = y = z = 2$ (any number expect '0')

$\Rightarrow \begin{vmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{vmatrix} = k \times 8$

$\Rightarrow 4(16 - 4) - 2(8 - 4) + 2(4 - 8) = 8 \times k$
$\Rightarrow 48 - 8 - 8 = 8 \times k$
$\Rightarrow 32 = 8 \times k \Rightarrow k = 4$

23. (C) $I = \int_0^1 \frac{x^8}{\sqrt{1-x^6}}\, dx$

$I = \int_0^1 \frac{x^6 . x^2}{\sqrt{1-(x^3)^2}}\, dx$

Let $x^3 = \sin\theta$ when $x = 0,\ \theta = 0$

$\Rightarrow 3x^2\, dx = \cos\theta\, d\theta$ $x = 1,\ \theta = \frac{\pi}{2}$

$\Rightarrow x^2\, dx = \frac{1}{3}\cos\theta\, d\theta$

$I = \int_0^{\pi/2} \frac{1}{3} \frac{\sin^2\theta . \cos\theta}{\sqrt{1-\sin^2\theta}}\, d\theta$

$I = \frac{1}{3} \int_0^{\pi/2} \frac{\sin^2\theta . \cos\theta}{\cos\theta}\, d\theta$

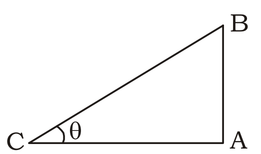
$I = \frac{1}{3} \int_0^{\pi/2} \sin^2\theta\, d\theta$

$I = \frac{1}{3} \int_0^{\pi/2} \frac{1-\cos 2\theta}{2}\, d\theta$

$I = \frac{1}{6}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$I = \frac{1}{6}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{12}$

24. (B)



Let angle of elevation = θ

AB = h, AC = $\sqrt{3}\, h$

In ΔABC:-

$\tan\theta = \frac{AB}{AC}$

$\Rightarrow \tan\theta = \frac{h}{\sqrt{3}h}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$

Angle of elevation = $\frac{\pi}{6}$

25. (C) Plane $3x + 4y - 5z + 11 = 0$ and point $(-1, 2, 4)$

Distance = $\left|\frac{3\times(-1)+4\times 2-5\times 4}{\sqrt{3^2+4^2+(-5)^2}}\right|$

$= \frac{15}{5\sqrt{2}} = \frac{3}{\sqrt{2}}$

26. (B) tanA + 2tan2A + 4 tan4A + 8 cot8A

⇒ cotA – (cotA– tanA) + 2 tan2A + 4 tan4A + 8 cot8A

We know that

cotA – tanA = 2 cot2A

⇒ cotA – 2 cot2A + 2 tan2A + 4 tan4A + 8 cot8A

⇒ cotA – 2(cot2A – tan2A)+ 4tan4A + 8cot8A

⇒ cotA – 2×2 cot4A + 4 tan4A + 8 cot8A

⇒ cotA – 4(cot4A – tan4A) + 8 tan8A

⇒ cotA – 4×2cot8A + 8 cot8A = cotA

27. (B) When θ = 180°

$M = \frac{60}{11}(H \pm 6)$ where $+\ H < 6$

$-\ H > 6$

H = 7(between 7 and 8 o'clock)

$M = \frac{60}{11}(7 - 6)$

$M = \frac{60}{11} \times 1 = 6\frac{5}{11}$

Hence time = $7 : 5\frac{5}{11}$

28. (C) $\sin\frac{\pi}{12} < \tan\frac{\pi}{12} < \cos\frac{\pi}{12}$

29. (B) 1 1 0 1 1

$1\times2^0 = 1$
$1\times2^1 = 2$
$0\times2^2 = 0$
$1\times2^3 = 8$
$1\times2^4 = 16$
27

0 . 0 1

$0 = 0\times2^{-1}$
$\frac{1}{4} = 1\times2^{-2}$
$\frac{1}{4} = 0.25$

Hence $(11011.01)_2 = (27.25)_{10}$

30. (B) The required Probability $= \frac{{}^6C_3 \times {}^4C_1}{{}^{10}C_4}$

$= \frac{20\times4}{10\times3\times7} = \frac{8}{21}$

31. (D) Let $\vec{a} = -\lambda\hat{i} + 2\hat{j} + (1-3\lambda)\hat{k}$ and $\vec{b} = 6\hat{i} + -\lambda\hat{j} - 4\hat{k}$

$\vec{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}.\vec{b} = 0$

$\Rightarrow -\lambda\times6 + 2(-\lambda) + (1-3\lambda)\times(-4) = 0$
$\Rightarrow -6\lambda - 2\lambda - 4 + 12\lambda = 0$
$\Rightarrow 4\lambda = 4 = \lambda = 1$

32. (B) Let $\vec{a} = \frac{1}{2}\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \frac{1}{2}\hat{j} + \hat{k}$

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

$\cos\theta = \frac{\frac{1}{2}\times1 + 1\times\frac{1}{2} + 1\times1}{\sqrt{\left(\frac{1}{2}\right)^2 + 1^2 + 1^2}\sqrt{1^2 + \left(\frac{1}{2}\right)^2 + 1^2}}$

$\cos\theta = \frac{2\times4}{9} \Rightarrow \theta = \cos^{-1}\left(\frac{8}{9}\right)$

33. (D) Equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ ...(i)

$px^2 + qx + r = 0$

$\alpha - h + \beta - h = \frac{-q}{p}$

$\Rightarrow \frac{-b}{a} - 2h = \frac{-q}{p}$

$\Rightarrow 2h = \frac{-b}{a} + \frac{q}{p}$

$\Rightarrow h = \frac{1}{2}\left[\frac{q}{p} - \frac{b}{a}\right]$

34. (B)

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      1001
101) 101111 (
     101
       111
       101
        10
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Quotient = $(1001)_2$ and Remainder = $(10)_2$

35. (B) $x = 1 + \left(\frac{y}{5}\right) + \left(\frac{y}{5}\right)^2 + \left(\frac{y}{5}\right)^3 + ,....$ where $|y| < 5$

$\Rightarrow x = \frac{1}{1-\frac{y}{5}}$

$\Rightarrow x = \frac{5}{5-y}$

$\Rightarrow 5x - xy = 5 \Rightarrow y = \frac{5x-5}{x}$

36. (B) $\sin(-1140) = -\sin(1140)$

$\Rightarrow \sin(-1140) = -\sin(3\times360 + 60)$

$\Rightarrow \sin(-1140) = -\sin60 = \frac{-\sqrt{3}}{2}$

37. (A) A' = cofactor of A

|A'| = |co-factor of A|

$|A'| = (A)^{3-1}$ [$\because$ order = 3]

$|A'| = A^2$

38. (C) $\lim_{x\to0}\frac{\sin x + \tan x}{x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to0}\frac{\cos x + \sec^2 x}{1}$

$\Rightarrow \cos0 + \sec^2 0$

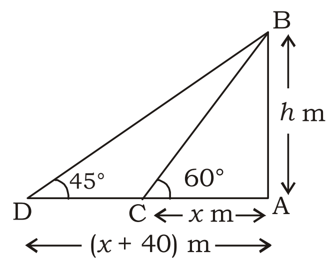
$\Rightarrow 1 + 1 = 2$

39. (C) $AA^T = I$

$\Rightarrow |AA^T| = 1$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm1$

40. (B)



Let height of the house (AB) = h m

AC = x m

A.T.Q,

AD = $(x + 40)$ m

In ΔABC:-

$\tan60° = \frac{AB}{AC}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$

In ΔABD:-

$\tan 45° = \frac{AB}{AD}$

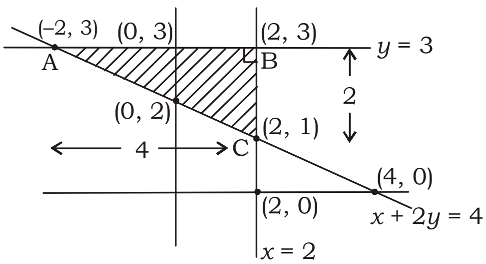
$\Rightarrow 1 = \frac{h}{x+40} \Rightarrow x + 40 = h$

$\Rightarrow \frac{h}{\sqrt{3}} + 40 = h \Rightarrow h\left(1-\frac{1}{\sqrt{3}}\right) = 40$

$\Rightarrow h = \frac{40\sqrt{3}}{\sqrt{3}-1} \Rightarrow h = 20(3+\sqrt{3})$ m

Height of the house = $20(3+\sqrt{3})$ m

41. (A)



lines $x = 2$, $y = 3$ and $x + 2y = 4$
A.T.Q,
AB = 4, BC = 2

Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 4 \times 2 = 4$ sq.unit

42. (D) $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $|\vec{a}+\vec{b}| = 2\sqrt{3}$

Now, $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left[|\vec{a}|^2 + |\vec{b}|^2\right]$

$\Rightarrow (2\sqrt{3})^2 + |\vec{a}-\vec{b}|^2 = 2\times\left[(\sqrt{3})^2 + (2)^2\right]$

$\Rightarrow 12 + |\vec{a}-\vec{b}|^2 = 2[3+4]$

$\Rightarrow 12 + |\vec{a}-\vec{b}|^2 = 14$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 2 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{2}$

43. (D) $\left(\frac{d^2y}{dx^2}\right)^{3/4} + 2\frac{d^3y}{dx^3} + \frac{dy}{dx} = 7$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^{3/4} = 7 - 2\frac{d^3y}{dx^3} - \frac{dy}{dx}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^{3} = \left(7 - 2\frac{d^3y}{dx^3} - \frac{dy}{dx}\right)^4$

Order = 3, Degree = 4

44. (B) $\frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$

$\Rightarrow \frac{\tan x + \sec x - (\sec^2 x - \tan^2 x)}{\tan x - \sec x + 1}$

$\Rightarrow \frac{\tan x + \sec x - (\sec x + \tan x)(\sec x - \tan x)}{\tan x - \sec x + 1}$

$\Rightarrow \frac{(\tan x + \sec x)[1 - (\sec x - \tan x)]}{\tan x - \sec x + 1}$

$\Rightarrow \frac{(\tan x + \sec x)(\tan x - \sec x + 1)}{\tan x - \sec x + 1}$

$\Rightarrow \tan x + \sec x$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = \frac{1 + \sin x}{\cos x}$

45. (C) $f(x) = x^2 - 6x$ $\quad x \in [0, 8]$
$f'(x) = 2x - 6$
$f''(x) = 2$ ...(i)
for minima and maxima
$f'(x) = 0$
$\Rightarrow 2x - 6 = 0 \Rightarrow x = 3$
from eq(i)
$f''(3) = 2$(minima)
Function $f(x)$ attains minimum value at $x = 3$.

46. (A) $I = \int_{-\pi/4}^{\pi/4} \frac{\cos x}{\sin^5 x}\, dx$

$I = 0$ $\quad$ [$\because f(x)$ is an odd function.]

47. (B) $f(x) = \begin{cases} x - 2, & x \le 5 \\ 3x - \lambda, & x > 5 \end{cases}$ is continuous

at $x = 5$, then

$\lim_{x\to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5)$

Now, $\lim_{x\to 5^+} f(x) = f(5)$

$\Rightarrow \lim_{h\to 0} f(5+h) = 5 - 2$

$\Rightarrow \lim_{h\to 0} 3(5+h) - \lambda = 3$

$\Rightarrow 15 - \lambda = 3 \Rightarrow \lambda = 12$

48. (C) Marks of students
30, 35, 36, 32, 31, 38, 40, 42

Mean = $\frac{30+35+36+32+31+38+40+42}{8}$

Mean = $\frac{284}{8} = 35.5$

The required number of students = 4

49. (B) $I = \int \tan^{-1}(\sec 2x - \tan 2x)\, dx$

$$I = \int \tan^{-1}\left(\frac{1}{\cos 2x} - \frac{\sin 2x}{\cos 2x}\right) dx$$

$$I = \int \tan^{-1}\left[\frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{\sin\left(\frac{\pi}{2}-2x\right)}\right] dx$$

$$I = \int \tan^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\sin\left(\frac{\pi}{4}-x\right).\cos\left(\frac{\pi}{4}-x\right)}\right] dx$$

$$I = \int \tan^{-1}\left[\tan\left(\frac{\pi}{4}-x\right)\right] dx$$

$$I = \int \left(\frac{\pi}{4}-x\right) dx$$

$$I = \frac{\pi}{4}x - \frac{x^2}{2} + c$$

50. (C) $\sqrt{\frac{1+\sqrt{15}i}{1-\sqrt{15}i}} = \sqrt{\frac{1+\sqrt{15}i}{1-\sqrt{15}i} \times \frac{1+\sqrt{15}i}{1+\sqrt{15}i}}$

$$\sqrt{\frac{1+\sqrt{15}i}{1-\sqrt{15}i}} = \sqrt{\frac{(1+\sqrt{15}i)^2}{1-15i^2}}$$

$$\sqrt{\frac{1+\sqrt{15}i}{1-\sqrt{15}i}} = \sqrt{\frac{(1+\sqrt{15}i)^2}{1+15}}$$

$$\sqrt{\frac{1+\sqrt{15}i}{1-\sqrt{15}i}} = \frac{1+\sqrt{15}i}{4}$$

51. (B) Equations $3x + ky = 5$ and $8y - 6x + 10 = 0 \Rightarrow -6x + 8y = -10$ will have infinitely many solutions,

then $\frac{3}{-6} = \frac{k}{8} \Rightarrow k = -4$

52. (B) Area $= \int_0^3 (e^x - e^{-x})\, dx$

Area $= \left[e^x + e^{-x}\right]_0^3$

Area $= (e^3 + e^{-3}) - (e^0 + e^0)$

Area $= (e^3 + e^{-3} - 2)$ sq. unit

53. (A) A = {0, 1, 2, 3, 4, 5, 6}, $n = 7$

No. of proper subsets of A = $2^n - 1 = 2^7 - 1$ = 128 − 1 = 127

54. (B)

55. (B) The required Probability = 0

56. (B) $\sum_{r=1}^{4} C(21+r, 3) + C(22, 4)$

⇒ C(22, 3) + C(23, 3) + C(24, 3), C(25, 3) + C(22, 4)

⇒ C(22, 3) + C(22, 4) + C(23, 3) + C(24, 3) + C(25, 3)

We know that

$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

⇒ C(23, 4) + C(23, 3) + C(24, 3) + C(25, 3)

⇒ C(24, 4) + C(24, 3) + C(25, 3)

⇒ C(25, 4) + C(25, 3) = C(26, 4)

57. (C) $2(l + b) = 48 \Rightarrow l + b = 24$

Area (A) = lb

A = $l \times (24 - l)$

A = $24l - l^2$

$$\frac{dA}{dl} = 24 - 2l$$

$$\frac{d^2A}{dl^2} = -2 \text{ (maxima)}$$

For maxima and minima

$$\frac{dA}{dl} = 0$$

$\Rightarrow 24 - 2l = 0 \Rightarrow l = 12$

Maximum area = 12×12

= 144 sq. cm

58. (C) Data 53, 52, 54, 53, 53, 52, 54, 53, 52, 54

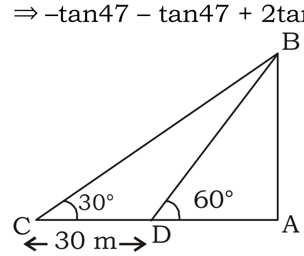
Mode = 53

59. (D) tan133 − tan227 + 2 cot43

$\Rightarrow \tan(180-47) - \tan(180+47) + 2\cot(90-47)$

⇒ −tan47 − tan47 + 2tan 47 = 0

60. (A)



Let height of the house = h m

In ΔABD:-

$$\tan 60° = \frac{AB}{AD}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AD} \quad AD = \frac{h}{\sqrt{3}}$$

In ΔABC:-

$$\tan 30° = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{\frac{h}{\sqrt{3}} + 30}$$

$$\Rightarrow \frac{h}{\sqrt{3}} + 30 = h\sqrt{3}$$

$$\Rightarrow h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 30 \Rightarrow h = 15\sqrt{3}$$

Hence height of the house = $15\sqrt{3}$ m

61. (A) No. of ways = $^8P_5$

$$= \frac{8!}{(8-5)!} = \frac{8!}{3!} = 6720$$

62. (C) $n(S) = 6\times6 = 36$

E = {(6, 4), (5, 5), (4, 6)} ; $n(E) = 3$

The required Probability $= \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

63. (D) Parabola $3y^2 = 2x \Rightarrow y^2 = \frac{2}{3}x$

$4a = \frac{2}{3} \Rightarrow a = \frac{1}{6}$

Equation of directrix

$x = -a$

$\Rightarrow x = \frac{-1}{6} \Rightarrow 6x + 1 = 0$

64. (B) We know that

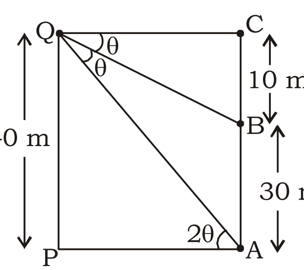
adj(adj A) = $|A|^{n-2}$.A [$\because$ Order = n]

Given that $|A| = 2$, $n = 4$

Now, adj(adj A) = $|A|^{n-2}$.A

$\Rightarrow$ adj(adj A) = 2^{4-2}.A = 4A

65. (A)



Let $\angle CQB = \theta$ and $\angle BQA = \theta$

then $\angle CQA = 2\theta$

In ΔCQB:-

$\tan\theta = \frac{BC}{QC}$

$\Rightarrow \tan\theta = \frac{10}{QC}$ …(i)

In ΔAPQ:-

$\tan2\theta = \frac{PQ}{AP}$

$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{40}{QC}$

$\Rightarrow \frac{2\times\frac{10}{QC}}{1-\frac{100}{QC^2}} = \frac{40}{QC}$ [from eq(i)]

$\Rightarrow \frac{1}{1-\frac{100}{QC^2}} = 2$

$\Rightarrow 1 - \frac{100}{QC^2} = \frac{1}{2} \Rightarrow QC = 10\sqrt{2}$

Hence distance of the observer from the top of the flag staff = $10\sqrt{2}$ m

66. (C) Series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{10}+$……

$\Rightarrow \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+\ldots$

Next term $= 1 - \frac{1}{32} = \frac{31}{32}$

67. (C) $3^{x+2} = 72 + 3^x$

$\Rightarrow 3^x.9 = 72 + 3^x$

$\Rightarrow 3^x(9-1) = 72$

$\Rightarrow 3^x \times 8 = 72$

$\Rightarrow 3^x = 9 \Rightarrow x = 2$

68. (D) Line $2x + 5y = 3$

slope of line $m_1 = \frac{-2}{5}$

slope of perpendicular line $(m_2) = \frac{-1}{m_1}$

$m_2 = \frac{-1\times5}{-2} = \frac{5}{2}$

69. (C) $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx$ …(i)

Prop.IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$I = \int_0^{\pi/2} \frac{\sin^2 x}{\cos x + \sin x}dx$ …(ii)

from eq(i) and eq(ii)

$$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x}dx$$

$$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$$

$$2I = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\frac{1}{\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x}\,dx$$

$$2I = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\frac{\pi}{4}.\sin x+\cos x.\sin\frac{\pi}{4}}\,dx$$

$$2I = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\frac{1}{\sin\left(x+\frac{\pi}{4}\right)}\,dx$$

$$2I = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\text{cosec}\left(x+\frac{\pi}{4}\right)dx$$

$$2I = \frac{1}{\sqrt{2}}\left[\log\left|\text{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$$

$$2I = \frac{1}{\sqrt{2}}\left[\log\left|\text{cosec}\left(\frac{\pi}{2}+\frac{\pi}{4}\right)-\cot\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right|-\log\left|\text{cosec}\frac{\pi}{4}-\cot\frac{\pi}{4}\right|\right]$$

$$I = \frac{1}{2\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|-\log\left|\sqrt{2}-1\right|\right]$$

$$I = \frac{1}{2\sqrt{2}}\left[\log\left|\sqrt{2}+1\right|-\log\left|\sqrt{2}-1\right|\right]$$

$$I = \frac{1}{2\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

$$I = \frac{1}{2\sqrt{2}}\log|3+2\sqrt{2}|$$

70. (C) $f(x) = \begin{vmatrix} x & a & a \\ a & x & b \\ b & b & x \end{vmatrix}$

$f(x) = x(x^2 - b^2) - a(ax - b^2) + a(ab - bx)$
$f(x) = x^3 - xb^2 - a^2x + ab^2 + a^2b - abx$
$f'(x) = 3x^2 - b^2 - a^2 - ab$
$f''(x) = 6x$

71. (D) The required Probability $= \frac{4}{52} = \frac{1}{13}$

72. (C) Given that $n = 127!$

Now, $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\ldots+\frac{1}{\log_{127} n}$

$\Rightarrow \log_n 2 + \log_n 3 + \log_n 4 + \ldots\ldots + \log_n 127$
$\Rightarrow \log_n(2\times3\times4\ldots\ldots\times127)$
$\Rightarrow \log_n 127! = \log_n n = 1$

73. (A) In ΔABC,

Now, $\frac{1+\cos(B-C).\cos A}{1+\cos(B-A).\cos C}$

$$\Rightarrow \frac{1+\cos(B-C).\cos[180-(B+C)]}{1+\cos(B-A).\cos[180-(A+B)]}$$

$$\Rightarrow \frac{1-\cos(B-C).\cos(B+C)}{1-\cos(B-A).\cos(B+A)}$$

$$\Rightarrow \frac{1-\cos^2 B+\sin^2 C}{1-\cos^2 B+\sin^2 A}$$

$$\Rightarrow \frac{\sin^2 B+\sin^2 C}{\sin^2 B+\sin^2 A} \Rightarrow \frac{b^2+c^2}{b^2+a^2}$$

74. (C) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$\Rightarrow \cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi + \pi + \pi$
Here, $\cos^{-1}x = \pi \Rightarrow x = \cos\pi = -1$
similarly $y = z = -1$
Now, $x + y + z = -1-1-1 = -3$

75. (D) $4\cot^{-1}5 = 2[2\cot^{-1}5]$

$$= 2\left[2\tan^{-1}\frac{1}{5}\right]$$

$$= 2\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right)$$

$$= 2\tan^{-1}\frac{5}{12}$$

$$= \tan^{-1}\left(\frac{2\times\frac{5}{12}}{1-\frac{25}{144}}\right) = \tan^{-1}\frac{120}{119}$$

Now, $4\cot^{-1}5 - \cot^{-1}239$

$$\Rightarrow \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}\right)$$

$$\Rightarrow \left(\frac{239\times120-119}{239\times119+120}\right)$$

$$\Rightarrow \left(\frac{239(119+1)-119}{239\times119+120}\right)$$

$$\Rightarrow \left(\frac{239\times119+120}{239\times119+120}\right) = \tan^{-1}1 = \frac{\pi}{4}$$

76. (B) Let $y = \log\left[e^x\left(\frac{x-2}{x+2}\right)\right]$

On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = \frac{1}{e^x\left(\frac{x-2}{x+2}\right)}\left[e^x\left(\frac{x-2}{x+2}\right) + e^x\left(\frac{4}{(x+2)^2}\right)\right]$$

$$\frac{dy}{dx} = \frac{x+2}{x-2}\left[\frac{(x-2)(x+2)+4}{(x+2)^2}\right]$$

$$\frac{dy}{dx} = \frac{x+2}{x-2} \times \frac{x^2}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{x^2}{x^2-4}$$

77. (C) Given that $\cot\alpha = 2m+1 \Rightarrow \tan\alpha = \frac{1}{2m+1}$

and $\cot\beta = \frac{m+1}{m} \Rightarrow \tan\beta = \frac{m}{m+1}$

Now, $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$$\Rightarrow \tan(\alpha+\beta) = \frac{\frac{1}{2m+1} + \frac{m}{m+1}}{1 - \frac{1}{2m+1} \times \frac{m}{m+1}}$$

$$\Rightarrow \tan(\alpha+\beta) = \frac{\frac{m+1+2m^2+m}{(2m+1)(m+1)}}{\frac{2m^2+m+2m+1-m}{(2m+1)(m+1)}}$$

$$\Rightarrow \tan(\alpha+\beta) = \frac{2m^2+2m+1}{2m^2+2m+1}$$

$$\Rightarrow \tan(\alpha+\beta) = 1 \Rightarrow \alpha+\beta = \frac{\pi}{4}$$

78. (C) sin36.sin72.sin108.sin 144

$\Rightarrow$ sin36.sin(90–18).sin(90+18).sin(180–36)

$\Rightarrow$ sin36.cos18.cos18.sin36

$\Rightarrow \sin^2 36.\cos^2 18$

We know that

$$\sin 36 = \frac{\sqrt{10-2\sqrt{5}}}{4}, \cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\Rightarrow \frac{10-2\sqrt{5}}{10} \times \frac{10+2\sqrt{5}}{16}$$

$$\Rightarrow \frac{100-20}{16\times16} = \frac{80}{16\times16} = \frac{5}{16}$$

79. (D) $2\cos\theta = x + \frac{1}{x}$

On squaring

$$\Rightarrow 4\cos^2\theta = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 4\cos^2\theta - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2(2\cos^2\theta - 1)$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2\cos 2\theta$$

$$\Rightarrow 2\left(x^2 + \frac{1}{x^2}\right) = 4\cos 2\theta$$

80. (C) Given that $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \lambda\hat{i} + \hat{j} - 2\hat{k}$

and $\vec{c} = -\hat{i} + \hat{j} + 3\hat{k}$ are coplaner,

then $\begin{vmatrix} 2 & 1 & -2 \\ \lambda & 1 & -2 \\ -1 & 1 & 3 \end{vmatrix} = 0$

$\Rightarrow 2(3+2) - 1(3\lambda - 2) - 2(\lambda + 1) = 0$

$\Rightarrow 10 - 3\lambda + 2 - 2\lambda - 2 = 0$

$\Rightarrow -5\lambda + 10 = 0 \Rightarrow \lambda = 2$

81. (B) tan(–330) + cot(–750)

$\Rightarrow$ –tan330 – cot 750

$\Rightarrow$ –tan(360 – 30) – cot(720 +30)

$\Rightarrow$ tan30 – cot30

$$= \frac{1}{\sqrt{3}} - \sqrt{3} = \frac{-2}{\sqrt{3}}$$

82. (C) The required Probability $= \frac{5!\times 3!}{7!}$

$$= \frac{5!\times 6}{7\times 6\times 5!} = \frac{1}{7}$$

83. (C) $A = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \Rightarrow A^2 = A$

Hence matrix A is an idempotent matrix.

84. (B) Adjacent sides of the parallelogram

$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$

Area of parallelogram = $|\vec{a} \times \vec{b}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{vmatrix}$$

$= |\hat{i}(1-4) - \hat{j}(-2-12) + \hat{k}(2+3)|$

$= |-3\hat{i} + 14\hat{j} + 5\hat{k}|$

$= \sqrt{(-3)^2 + 14^2 + 5^2} = \sqrt{230}$ sq.unit

85. (B)

86. (C) 1

87. (D) The required no. of triangle = ${}^{14}C_3 - {}^{6}C_3$ = 364 – 20 = 344

88. (B) $\vec{a} = a\hat{i} - \hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - b\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + c\hat{k}$ are mutually orthogonal,

then $\vec{a}.\vec{b} = 0$

$2a + b - 2 = 0 \Rightarrow 2a + b = 2$ …(i)

and $\vec{b}.\vec{c} = 0$

$2 + b + c = 0 \Rightarrow b + c = -2$ …(ii)

and $\vec{a}.\vec{c} = 0$

$a + 1 - 2c = 0 \Rightarrow a - 2c = -1$ …(iii)

On solving eq(i), (ii) and (iii)

$a = 3$, $b = -4$ and $c = 2$

89. (C) $I_n = \int_{\pi/6}^{\pi/4} \tan^n x\, dx$

$I_n = \int_{\pi/6}^{\pi/4} \tan^{n-2} x . \tan^2 x\, dx$

$I_n = \int_{\pi/6}^{\pi/4} (\tan x)^{n-2} (\sec^2 x - 1)\, dx$

$I_n = \int_{\pi/6}^{\pi/4} (\tan x)^{n-2}.\sec^2 x\, dx - \int_{\pi/6}^{\pi/4} \tan^{n-2} x\, dx$

$I_n = \int_{\pi/6}^{\pi/4} (\tan x)^{n-2}.\sec^2 x\, dx - I_{n-2}$

$I_n + I_{n-2} = \int_{\pi/6}^{\pi/4} (\tan x)^{n-2}.\sec^2 x\, dx$

Let $\tan x = t$ when $x = \frac{\pi}{6}$, $t = \frac{1}{\sqrt{3}}$

$\Rightarrow \sec^2 x\, dx = dt$ $x = \frac{\pi}{4}$, $t = 1$

$I_n + I_{n-2} = \int_{1/\sqrt{3}}^{1} t^{n-2} dt$

$I_n + I_{n-2} = \left[\frac{t^{n-2+1}}{n-2+1}\right]_{1/\sqrt{3}}^{1}$

$I_n + I_{n-2} = \frac{1}{n-1}\left[1 - \left(\frac{1}{\sqrt{3}}\right)^{n-1}\right]$

$n = 3$

$I_3 + I_1 = \frac{1}{3-1}\left[1 - \left(\frac{1}{\sqrt{3}}\right)^2\right]$

$I_3 + I_1 = \frac{1}{2}\left[1 - \frac{1}{3}\right]$

$I_3 + I_1 = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

90. (D)

91. (C) series 1.2 + 2.3 + 3.4 +……+ $n(n + 1)$

$T_n = n(n+1)$

$T_n = n^2 + n$

Now, $S_n = \sum T_n$

$\Rightarrow S_n = \sum (n^2 + n)$

$\Rightarrow S_n = \sum n^2 + \sum n$

$\Rightarrow S_n = \frac{n}{6}(n+1)(2n+1) + \frac{n(n+1)}{2}$

$\Rightarrow S_n = \frac{n(n+1)}{6}[2n + 1 + 3]$

$\Rightarrow S_n = \frac{n(n+1)}{6} \times (2n+4)$

$\Rightarrow S_n = \frac{n(n+1)(n+2)}{3}$

92. (A) $\begin{vmatrix} x+2 & x+3 & x+5 \\ x+7 & x+9 & x+12 \\ x+14 & x+17 & x+21 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} x+2 & x+3 & x+5 \\ 5 & 6 & 7 \\ 12 & 14 & 16 \end{vmatrix}$

$R_3 \to R_3 - 2R_2$

$\Rightarrow \begin{vmatrix} x+2 & x+3 & x+5 \\ 5 & 6 & 7 \\ 2 & 2 & 2 \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} x+2 & 1 & 3 \\ 5 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix}$

$\Rightarrow (x+2)\times 0 - 1(0-4) + 3(-2)$

$= 4 - 6 = -2$

93. (C) The required no. of hand shakes in party $= {}^{12}C_2 = 66$

94. (B) $\theta = \left|\frac{11M - 60H}{2}\right|$

Time = 4 : 10

$\theta = \left|\frac{11\times10 - 60\times4}{2}\right|$

$\theta = 65°$

$\theta = 65\times\frac{\pi}{180} = \frac{13\pi}{36}$

95. (B) We know that

Mode = 3 Median – 2 Mean

Mode = 3×17 – 2 × 23

Mode = 51 – 46 = 5

96. (D) $y = e^x(a\cos x + b\sin x)$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$

$\Rightarrow \frac{dy}{dx} = y + e^x(-a\sin x + b\cos x)$ ...(i)

Again, differentiating

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-a\sin x + b\cos x) + e^x(-a\cos x - b\sin x)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-a\sin x + b\cos x) - e^x(a\cos x + b\sin x)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - y$

$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

97. (B) Curve $y^2 = 5x$

$\Rightarrow 2y\frac{dy}{dx} = 5$

$\Rightarrow \frac{dy}{dx} = \frac{5}{2y}$ ...(i)

Slope of tangent at point (2,–3) $= \frac{5}{2\times-3} = \frac{-5}{6}$

Slope of normal $= \frac{-1}{-516} = \frac{6}{5}$

Equation of normal at point (2, –3)

$y + 3 = \frac{6}{5}(x - 2)$

$\Rightarrow 6x - 5y = 27$ ...(ii)

from eq(i)

$\frac{dy}{dx} = \frac{5}{2y}$

Slope of tangent at point (2, 3) $= \frac{5}{2\times3} = \frac{5}{6}$

Slope of normal $= \frac{-1}{5/6} = \frac{-6}{5}$

Equation of normal at point (2, 3)

$y - 3 = \frac{-6}{5}(x - 2)$

$\Rightarrow 6x + 5y = 27$ ...(iii)

from eq(ii) and eq(iii)

$x = \frac{9}{2}$ and $y = 0$

Hence intersection point $\left(\frac{9}{2}, 0\right)$.

98. (B) $I = \int a^x.e^x\,dx$

$I = a^x\int.e^x\,dx - \int\left\{\frac{d}{dx}(a^x).\int e^x\,dx\right\}dx$

$I = a^x.e^x - \int a^x.\log a.e^x\,dx + c$

$I = a^x.e^x - \log a\int a^x.e^x\,dx + c$

$I = a^x.e^x - I.\log a + c$

$I(1 + \log a) = a^x.e^x + c$

$I = \frac{a^x.e^x}{1+\log a} + c$

99. (C) Mean of first 13 natural numbers

$= \frac{13\times14}{2\times13} = 7$

$\sum\left(x-\bar{x}\right)^2 = (1-7)^2 + (2-7)^2 + (3-7)^2 + (4-7)^2 + (5-7)^2 + (6-7)^2 + (7-7)^2 + (8-7)^2 + (9-7)^2 + (10-7)^2 + (11-7)^2 + (12-7)^2 + (13-7)^2$

$\sum\left(x-\bar{x}\right)^2 = 36 + 25 + 16 + 9 + 4 + 1 + 0 + 1 + 4 + 16 + 25 + 36$

$\sum\left(x-\bar{x}\right)^2 = 182$

S.D. $= \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}} = \sqrt{\frac{182}{13}} = 3.74$

100. (A) Let $z = \frac{1-i}{1+2i} \Rightarrow z = \frac{1-i}{1+2i}\times\frac{1-2i}{1-2i}$

$\Rightarrow z = \frac{1-i-2i+2i^2}{1-4i^2} \Rightarrow z = \frac{-1-3i}{5}$

Amplitute of $z = \tan^{-1}\left(\frac{-3/5}{-1/5}\right)$

$= \tan^{-1}(3)$

101. (C) $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}(\hat{i}\times\hat{j})$

$\Rightarrow \hat{i}.\hat{i} + \hat{j}.\hat{j} + \hat{k}.\hat{k}$

$\Rightarrow 1 + 1 + 1 = 3$

102. (C) A = {1, 2, 4} , B = {2, 3, 5} and C = {1, 3, 5}
(A×B) = {(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (4, 2), (4, 3), (4, 5)}
(B×C) = {(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}
Now, (A×B) – (B×C) = {(1, 2), (1, 3), (1, 5), (2, 2), (4, 2), (4, 3), (4, 5)}

103. (D) Let $y = \sqrt{6+5\sqrt{6+5\sqrt{6+....\infty}}}$

$\Rightarrow y = \sqrt{6+5y}$

$\Rightarrow y^2 = 6 + 5y$

$\Rightarrow y^2 - 5y - 6 = 0$

$\Rightarrow (y - 6)(y + 1) = 0$

$\Rightarrow y = 6, -1$

Hence $\sqrt{6+5\sqrt{6+5\sqrt{6+....\infty}}} = 6$

104. (C) Hyperbola

$9x^2 - 4y^2 + 36x + 8y = 14$

$\Rightarrow 9(x^2 + 4x) - 4(y^2 - 2y) = 14$

$\Rightarrow 9(x + 2)^2 - 36 - 4(y - 1)^2 - 4 = 14$

$\Rightarrow 9(x + 2)^2 - 4(y - 1)^2 = 54$

$\Rightarrow \frac{(x+2)^2}{6} - \frac{(y-1)^2}{27/2} = 1$

$h = -2,\ k = 1,\ a = \sqrt{6},\ b = \frac{3\sqrt{3}}{\sqrt{2}}$

Equation of asymptotes

$y = \pm \frac{b}{a}(x - h) + k$

$\Rightarrow y = \pm \frac{3\sqrt{3}}{\sqrt{2}\times\sqrt{6}}(x + 2) + 1$

$\Rightarrow y = \pm \frac{3}{2}(x + 2) + 1$

$\Rightarrow 2y = \pm 3(x + 2) + 2$

$3x - 2y + 8 = 0,\ 3x + 2y + 4 = 0$

105. (C) We know that

$3x = x + 2x$

$\Rightarrow \tan 3x = \tan(x + 2x)$

$\Rightarrow \tan 3x = \frac{\tan x + \tan 2x}{1 - \tan x.\tan 2x}$

$\Rightarrow \tan 3x - \tan x.\tan 2x.\tan 3x = \tan x + \tan 2x$

$\Rightarrow \tan x + \tan 2x - \tan 3x = -\tan x.\tan 2x.\tan 3x$

106. (B) $\log_{10}\left\{99 + \sqrt{x^2 - 3x + 3}\right\} = 2$

$\Rightarrow 99 + \sqrt{x^2 - 3x + 3} = 100$

$\Rightarrow \sqrt{x^2 - 3x + 3} = 1$

$\Rightarrow x^2 - 3x + 3 = 1$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 2, 1$

107. (C) Let $y = \cos^{-1}(\sin x^2)$

On differentiating both side w.r.t. 'x'

$y = \frac{-1}{\sqrt{1-(\sin x^2)^2}} \times (\cos x^2) \times 2x$

$y = \frac{-2x.\cos x^2}{\cos x^2} \Rightarrow y = -2x$

108. (D) Equation of line

$ax\tan\alpha + by\sec\alpha = ab$

perpendicular distance from point $(0, \sqrt{a^2+b^2})$

$d_1 = \left|\frac{0 + b\sec\alpha\left(\sqrt{a^2+b^2}\right) - ab}{\sqrt{(a\tan\alpha)^2 + (b\sec\alpha)^2}}\right|$

$d_1 = \left|\frac{b\left(\sqrt{a^2+b^2}\right)\sec\alpha - ab}{\sqrt{a^2\tan^2\alpha + b^2\sec^2\alpha}}\right|$

Similarly, perpendicular distance from point $(0, -\sqrt{a^2+b^2})$

$d_2 = \left|\frac{0 + b\sec\alpha(-\sqrt{a^2+b^2}) - ab}{\sqrt{(a\tan\alpha)^2 + (b\sec\alpha)^2}}\right|$

$d_2 = \left|\frac{b(\sqrt{a^2+b^2})\sec\alpha + ab}{\sqrt{a^2\tan^2\alpha + b^2\sec^2\alpha}}\right|$

Now, $d_1 \times d_2 = \frac{(b\sec\alpha\sqrt{a^2+b^2})^2 - (ab)^2}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2(a^2+b^2)\sec^2\alpha - a^2b^2}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2(a^2\sec^2\alpha + b^2\sec^2\alpha - a^2)}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2[a^2(\sec^2\alpha - 1) + b^2\sec^2\alpha]}{a^2\tan^2\alpha + b^2\sec^2\alpha}$

$\Rightarrow d_1 \times d_2 = \frac{b^2(a^2\tan^2\alpha + b^2\sec^2\alpha)}{a^2\tan^2\alpha + b^2\sec^2\alpha} = b^2$

109. (A)

110. (D) $\phi = \{\}$

111. (B) $\alpha = \frac{1+\sqrt{3}i}{2} = -\omega^2$

Now, $1 + \alpha^2 + \alpha^4 + \alpha^6 + \alpha^8$

$\Rightarrow 1 + (-\omega^2)^2 + (-\omega^2)^4 + (-\omega^2)^6 + (-\omega^2)^8$

$\Rightarrow 1 + \omega^4 + \omega^8 + \omega^{12} + \omega^{15}$

$\Rightarrow 1 + \omega + \omega^2 + 1 + \omega$

$\Rightarrow 0 + (-\omega^2)$ $[\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow \alpha$

112. (C)

113. (D) In the expansion of $\left(x^2 - \frac{1}{2x^{1/2}}\right)^9$

$$T_{r+1} = {}^9C_r (x^2)^{9-r}\left(\frac{-1}{2x^{1/2}}\right)^r$$

$$T_{r+1} = {}^9C_r\left(\frac{-1}{2}\right)^r x^{\frac{36-5r}{2}}$$

here, $\frac{36-5r}{2} = 3$

$\Rightarrow 36 - 5r = 6 \Rightarrow r = 6$

The coefficient of $x^3 = {}^9C_6\left(\frac{-1}{2}\right)^6$

$$= \frac{12\times 7}{64} = \frac{21}{16}$$

114. (B) We know that
sin20° < sin45° and cos20° > cos 45°
sin20° < cos45° < cos20°
then sin20° < cos20°
cos20° – sin20° > 0
Hence (cos20 – sin20) is positive but less than 1.

115. (B) $x = a\sin\theta.\cos\theta$

$$\frac{dx}{d\theta} = a\sin\theta.(-\sin\theta) + a\cos\theta.\cos\theta$$

$$\frac{dx}{d\theta} = a(\cos^2\theta - \sin^2\theta)$$

and $y = a\theta.\cos\theta$

$$\Rightarrow \frac{dy}{d\theta} = a\theta.(-\sin\theta) + a\cos\theta.\ 1$$

$$\Rightarrow \frac{dy}{d\theta} = a(\cos\theta - \theta.\sin\theta)$$

Now, $\frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx}$

$$\Rightarrow \frac{dy}{dx} = a(\cos\theta - \theta.\sin\theta)\times\frac{1}{a(\cos^2\theta - \sin^2\theta)}$$

$$\Rightarrow \frac{dy}{d\theta} = \frac{\cos\theta - \theta.\sin\theta}{\cos 2\theta}$$

116. (B) Word "EAGERNESS"

Total no. of arrangements = $\frac{9!}{3!2!}$

the total no. of arrangements, when 'S's come together = $\frac{8!}{3!}$

The total no. of arrangements, when 'S's do not come together = $\frac{9!}{3!2!} - \frac{8!}{3!}$

$$= \frac{7\times 8!}{2\times 3!}$$

The required Probability = $\dfrac{\frac{7\times 8!}{2\times 3!}}{\frac{9!}{3!2!}} = \frac{7}{9}$

117. (B) $\begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 1 & 1 & c \end{vmatrix}$

$\Rightarrow c(c^2 - 1) - 1(c - 1)$
$\Rightarrow c^3 - c - c + 1$
$\Rightarrow c^3 - 2c + 1$
$\Rightarrow 8\sin^3\theta - 4\sin\theta + 1$ $[\because c = 2\sin\theta]$

118. (C) $(\cos\theta - \sin\theta)(\sin\theta - \text{cosec}\theta)(\cot\theta + \tan\theta)$

$$\Rightarrow \left(\cos\theta - \frac{1}{\cos\theta}\right)\left(\sin\theta - \frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$\Rightarrow \frac{\cos^2\theta - 1}{\cos\theta}\times\frac{\sin^2\theta - 1}{\sin\theta}\times\frac{\sin^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}$$

$$\Rightarrow \frac{-\sin^2\theta}{\cos\theta}\times\frac{-\cos^2\theta}{\sin\theta}\times\frac{1}{\sin\theta.\cos\theta} = 1$$

119. (B) $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6}$$

$$\Rightarrow 2\cos^{-1}x = \frac{\pi}{3}$$

$$\Rightarrow \cos^{-1}x = \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$$

120. (D) $y = A\sin 2t + B\cos 2t$ ...(i)
On differentiating both side w.r.t. 't'

$$\frac{dy}{dt} = 2A\cos 2t - 2B\sin 2t$$

Again, differentiating

$$\frac{d^2y}{dt^2} = -4A\sin 2t - 4B\cos 2t$$

$$\frac{d^2y}{dt^2} = -4(A\sin 2t + B\cos 2t)$$

$$\frac{d^2y}{dt^2} = -4y \quad \text{[from eq(i)]}$$

NDA (MATHS) MOCK TEST - 152 (Answer Key)

1. (B)	21. (A)	41. (A)	61. (A)	81. (B)	101. (C)
2. (B)	22. (C)	42. (D)	62. (C)	82. (C)	102. (C)
3. (C)	23. (C)	43. (B)	63. (D)	83. (C)	103. (D)
4. (B)	24. (B)	44. (B)	64. (B)	84. (B)	104. (C)
5. (B)	25. (C)	45. (C)	65. (A)	85. (B)	105. (C)
6. (B)	26. (B)	46. (A)	66. (C)	86. (C)	106. (B)
7. (C)	27. (B)	47. (B)	67. (C)	87. (D)	107. (C)
8. (C)	28. (C)	48. (C)	68. (D)	88. (B)	108. (D)
9. (C)	29. (B)	49. (B)	69. (C)	89. (C)	109. (A)
10. (B)	30. (B)	50. (C)	70. (C)	90. (D)	110. (D)
11. (A)	31. (D)	51. (B)	71. (D)	91. (C)	111. (B)
12. (C)	32. (B)	52. (B)	72. (C)	92. (A)	112. (C)
13. (B)	33. (D)	53. (A)	73. (A)	93. (C)	113. (D)
14. (C)	34. (B)	54. (B)	74. (C)	94. (B)	114. (B)
15. (C)	35. (B)	55. (B)	75. (D)	95. (B)	115. (B)
16. (D)	36. (B)	56. (B)	76. (B)	96. (D)	116. (B)
17. (C)	37. (A)	57. (C)	77. (C)	97. (B)	117. (B)
18. (B)	38. (C)	58. (C)	78. (C)	98. (B)	118. (C)
19. (C)	39. (C)	59. (D)	79. (D)	99. (C)	119. (B)
20. (A)	40. (B)	60. (A)	80. (C)	100. (A)	120. (D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777