## SSC JE CONVENTIONAL 2017

## GENERAL ENGINEERING (MECHANICAL)

1. (a) Define the following:
(i) Reversible and Irreverasible process
(ii) External and Internal irreversibility
(iii) Intensive and Extensive properties

## Solution :

(i) A reversible process is defined as a process that can be reversed without leaving any trace on the surroundings. That is, both the system and the sorroundings are returned to their initial states at the end of the reverse process. This is possible only if the net heat and net work exchange between the system and the surroundings is zero for the combined (original and reverse) process.
A reversible process is carried out infinitely slowly. every states of path are eqilibriuim with surrounding.


But in Irreversible process every states carried by system are nonequilibrium. All spontanceous process are irreversible in nature.
(ii) A process is called internally reversible if no irreversibilities occur within the boundaries of the system during the process. During an internally reversible process, a system proceeds through a series of equilibrium state, and when the process is reversed, the system passes through exactly the same equilibrium states while returing to its initial state. The quasi-equilibrium process is an example of an internally reversible process.
A process is called externally reversible if no irreversibilities occur out side the system boundaries during the process. Heat transfer between a reservior and a system is an externally reversible process if the outer surface of the system is at the temperature of the reservoir.
A process is callled totally reversible, or simply reversible, if it involves no irreversibilities within the system or its surroundings.
(iii) Any characteristic of a system is called a property. Properties are considered to be either intensive or extensive. Intensive properties are those that are independent of the mass of a system, such as temperature, pressure, and density. Extensive properties are those whose values depend on the size-or extent-of the system. Total mass, total volume, and total momentum are some examples of extensive properties.
(b) Describe the following:
(i) Clausius Statement
(ii) Kelvin-Planck Statement
(iii) Perpetual motion machine of the second kind

## Solution :

Clausius states that,
(i) It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.

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(ii) Kelvin-Planck Statement

It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.
The Kelvin-Planck statement can also be expressed as no heat engine can have a thermal efficiency of 100 percent or as for a power plant to operate, the working fluid must exchange heat with the environment as well as the furnace.
(iii) A engine interacts with a single reservoir produce net work in a complete cycle by exchanging heat, such type of engine have $100 \%$ efficiency called PMM2.

(c) Volume of $0.1 \mathrm{~m}^{3}$ of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8 bar. It is then cooled at constant volume and further expanded isothermally so as the reach the condition from where it started. Determine:
(i) Pressure at the end of constant volume cooling.
(ii) Change in internal energy during constant volume process
(iii) Net work done and heat transferred during the cycle.

Take $\mathrm{c}_{\mathrm{p}}=\mathbf{1 4 . 3} \mathbf{k J} / \mathrm{kg} \mathrm{K}$ and $\mathrm{c}_{\mathrm{v}}=\mathbf{1 0 . 2} \mathbf{k J} / \mathrm{kg} \mathrm{K}$.
Solution: $\quad \mathrm{V}_{1}^{\mathrm{p}}=0.1 \mathrm{~m}^{3}$

$\mathrm{T}_{1}=300 \mathrm{~K}$
$\mathrm{P}_{1}=1 \mathrm{bar}$
$\mathrm{P}_{2}=8 \mathrm{bar}$



Solution :
(i)

$$
\begin{aligned}
& \begin{aligned}
& \gamma=\frac{\mathrm{C}_{\mathrm{P}}}{\mathrm{C}_{\mathrm{v}}}=\frac{14.3}{10.2}=1.4 \\
& \mathrm{R}=\mathrm{C}_{\mathrm{P}}-\mathrm{C}_{\mathrm{V}}=14.3-10.2=4.1 \\
& \text { For process } 1-2
\end{aligned} \\
& \mathrm{~T}_{2}=\left(\frac{\mathrm{P}_{2}}{\mathrm{P}_{1}}\right)^{\frac{\gamma-1}{\gamma}} \cdot \mathrm{~T}_{1} \\
& =(8)^{\frac{0.4}{1.4} \times 300} \\
& \mathrm{~T}_{2}=543.43 \mathrm{~K} \\
& \text { For process } 2-3,
\end{aligned}
$$

$\frac{\mathrm{P}_{3}}{\mathrm{P}_{2}}=\frac{\mathrm{T}_{3}}{\mathrm{~T}_{2}}$
$P_{3}=8 \times \frac{300}{543.43}$
$\mathrm{P}_{3}=4.416 \mathrm{bar}$
(ii) For process 2-3
$\mathrm{dU}=\mathrm{mc}_{\mathrm{v}} \mathrm{dT}$
For calculating mass of gas
$\mathrm{P}_{1} \mathrm{~V}_{1}=\mathrm{mR} \mathrm{T} \mathrm{T}_{1}$
$\mathrm{m}=\frac{1.01325 \times 10^{5} \times 0.1}{4.1 \times 10^{3} \times 300}$
$\mathrm{m}=0.00823 \mathrm{~kg}$
Now $\int_{2}^{3} d \mathrm{U}=\mathrm{m} . \mathrm{C}_{\mathrm{v}} \int_{2}^{3} d \mathrm{~T}$
$\mathrm{U}_{3}-\mathrm{U}_{2}=0.00823 \times 10.2\left[\mathrm{~T}_{3}-\mathrm{T}_{2}\right]$
$=0.084025$ [300-543.43]
$\mathrm{U}_{3}-\mathrm{U}_{2}=-20.454 \mathrm{~kJ}$
(iii) $\sum \mathrm{W}=\mathrm{W}_{1-2}+\mathrm{W}_{2-3}+\mathrm{W}_{3-1}$

$$
\mathrm{W}_{1-2}=\mathrm{Q}_{1-2}-\left(\mathrm{U}_{2}-\mathrm{U}_{1}\right)
$$

$\because$ process $1-2$ is isentropic -

$$
\text { so } \mathrm{Q}_{1-2}=0
$$

$\therefore \mathrm{W}_{1-2}=-\left(\mathrm{U}_{2}-\mathrm{U}_{1}\right)=-\mathrm{mC}_{\mathrm{v}}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)$

$$
=-0.00823 \times 10.2(543.43-300)
$$

$\mathrm{W}_{1-2}=-20.454 \mathrm{~kJ}$
Process 2-3

$$
W_{2-3}=0 \quad\{\because d V=0\}
$$

Process 3-1

$$
\begin{aligned}
\mathrm{W}_{3-1} & =\mathrm{P}_{1} \mathrm{~V}_{1} \ln \left(\frac{\mathrm{P}_{3}}{\mathrm{P}_{1}}\right) \\
& =101.325 \times 0.1 \ln \left(\frac{4.416}{1}\right) \\
& =15.05 \mathrm{~kJ}
\end{aligned}
$$

So $\sum \mathrm{W}=-20.454+0+15.05$
$W_{\text {net }}=-5.4048 \mathrm{~kJ}$
and $\sum \mathrm{Q}=\mathrm{Q}_{1-2}+\mathrm{Q}_{2-3}+\mathrm{Q}_{3-1}$
For process 1-2,

$$
Q_{1-2}=0
$$

For process 2-3,

$$
\mathrm{Q}_{2-3}=\left(\mathrm{U}_{3}-\mathrm{U}_{2}\right)
$$

$$
=-20.454
$$

Process 3-1
$\mathrm{Q}_{3-1}=\mathrm{W}_{3-1}$

$$
=15.05 \mathrm{~kJ}
$$

So $\sum \mathrm{Q}=0-20.454+15.05$
$\mathrm{Q}_{\text {net }}=-5.404 \mathrm{~kJ}$

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Alternative
From1st law of thermodynamics
$\sum \mathrm{Q}=\sum \mathrm{W}$
So $Q_{\text {net }}=-5.404 \mathrm{~kJ}$
(d) A reversible heat engine operates between two reservoirs at temperatures $700^{\circ} \mathrm{C}$ and $50^{\circ} \mathrm{C}$. the engine drives a reversible refrigerator which operates between reservoirs at temperatures of $50^{\circ} \mathrm{C}$ and $-25^{\circ} \mathrm{C}$. The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ .
(i) Calculate the heat transfer to the refirgerant and the net heat transfer to the reservoir at $50^{\circ} \mathrm{C}$;
(ii) Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 percent of their maximum possible values.
Solution :


According to questions the given engine and refrigerator are reversible. for Engine;
(i) $\frac{\mathrm{Q}_{2}}{\mathrm{Q}_{1}}=\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}$
$\mathrm{Q}_{2}=2500 \times \frac{323}{973}$
$\mathrm{Q}_{2}=829.90 \mathrm{~kJ}$
and $\mathrm{W}_{\mathrm{E}}=\mathrm{Q}_{1}-\mathrm{Q}_{2}$

$$
=2500-829.90
$$

$\mathrm{W}_{\mathrm{E}}=1670.09 \mathrm{~kJ}$
For Refrigerator;
$\mathrm{W}_{\mathrm{R}}=\mathrm{W}_{\mathrm{E}}-400$
$\begin{aligned} & =1670-400 \\ W_{\mathrm{B}} & =1270 \mathrm{~kJ}\end{aligned}$
-
and $\frac{\mathrm{Q}_{3}}{\mathrm{Q}_{4}}=\frac{248}{323}$
$\mathrm{Q}_{4}-\mathrm{Q}_{3}=\mathrm{W}_{\mathrm{R}}$
from eq (i) and (ii)
$\mathrm{Q}_{4}-\frac{248}{323} \mathrm{Q}_{4}=1270$
$\mathrm{Q}_{4}=5469.46 \mathrm{~kJ}$

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and $\mathrm{Q}_{3}=5469.46-1270$

$$
\mathrm{Q}_{3}=4199.46 \mathrm{~kJ}
$$

So Heat transfer to Refrigerant
$\mathrm{Q}_{3}=4199.46 \mathrm{~kJ}$
and Heat transfer to the
Reservior of $50^{\circ} \mathrm{C}=\mathrm{Q}_{4}+\mathrm{Q}_{2}$

$$
\begin{aligned}
& =5469.46+829.9 \\
& =6299.36 \mathrm{~kJ}
\end{aligned}
$$

(ii) Now,
$\eta_{\mathrm{E}}=0.45 \eta_{\max }$
$=0.45\left[1-\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}\right]$
$=0.45\left[1-\frac{323}{973}\right]$
$\eta_{\mathrm{E}}=0.3006$
$\frac{\mathrm{W}_{e}}{\mathrm{Q}_{1}}=0.3006$
$\mathrm{W}_{\mathrm{E}}=0.3006 \times 2500=751.54 \mathrm{~kJ}$
and $\mathrm{Q}_{2}=\mathrm{Q}_{1}-\mathrm{W}_{\mathrm{E}}=2500-751.54=1748.46 \mathrm{~kJ}$
so $\quad W_{R}=W_{E}-400$
$=751.54-400$
$\mathrm{W}_{\mathrm{R}}=351.54 \mathrm{~kJ}$
and $\mathrm{COP}=0.45(\mathrm{COP})_{\max }$
$=0.45\left[\frac{\mathrm{~T}_{3}}{\mathrm{~T}_{2}-\mathrm{T}_{3}}\right]$
$=0.45\left[\frac{248}{323-248}\right]$
$\mathrm{COP}=1.488$
also $\frac{\mathrm{Q}_{3}}{\mathrm{~W}_{\mathrm{R}}}=1.488$
$\mathrm{Q}_{3}=1.488 \times 351.54$
$\mathrm{Q}_{3}=523.09 \mathrm{~kJ}$
$Q_{4}=Q_{3}+W_{R}=523.09+351.54=874.63 \mathrm{~kJ}$
so Heat transfer to refrigerant $\mathrm{Q}_{3}=523.09 \mathrm{~kJ}$

Heat transfer to the reservoir of $50^{\circ} \mathrm{C} \quad \mathrm{Q}_{2}+\mathrm{Q}_{4}=1748.46+874.63=2623.08 \mathrm{~kJ}$
(2) (a) Give the comparisons between Otto cycle, Diesel cycel and Dual cycle. Solution:

The three cycles can be compared on the basis of either the same compression ratio or the same maximum pressure and temperature.
The comparison of Otto, Diesel, and Dual cycles for the same compression ratio and heat rejection.

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$$
\begin{array}{ll}
1-2-6-5 & \text { Otto cycle } \\
1-2-7-5 & \text { Diesel cycle } \\
1-2-34-5 & \text { Dual cycle }
\end{array}
$$

For the same $Q_{2}$, the higher the $Q_{1}$, the higher is the cycle efficiency. In the T-s diagram, the area under 2-6 represents $Q_{1}$ for the Otto cycle, the area under 2-7 represents $Q_{1}$ for the Diesel cycle, and the area under 2-3-4 represents $Q_{1}$ for the Dual cycle. Therefore, for the same comression ratio and $\mathrm{Q}_{2}$.

$$
\eta_{\text {otto }}>\eta_{\text {Dual }}>\eta_{\text {Diesel }}
$$

Now the comparison of the three air standard cycles for the same maximum pressure and temperature, the heat rejection being also the same.

1-6-4-5
Otto cycle
1-7-4-5 Diesel cycle
1-2-3-4-5 Dual cycle
$\mathrm{Q}_{1}$ is represented by the area under 6-4 for the Otto cycle, by the are under 7-4 for the Diesel cycle and by the area under 2-3-4 for the Dual cycle in the T-s plot, $Q_{2}$ being the same.

$$
\therefore \eta_{\text {otto }}<\eta_{\text {Dual }}<\eta_{\text {Diesel }}
$$

(b) An air standard Otto cycle is to be designed according to the following specifications. Pressure at the start of the compression process $=101 \mathrm{kPa}$; Temperature at the start of the compression process $=300 \mathrm{~K}$; Compression ratio $=8$; Maximum pressure in the cycle $=8.0 \mathrm{MPa}$. Find
(i) the net work output per unit mass of air
(ii) cycle efficiency
(iii) MEP

Solution:
Given data
$\mathrm{P}_{1}=101 \mathrm{kPa}$
$\mathrm{T}_{1}=300 \mathrm{~K}$
$r=8$
$\mathrm{P}_{\text {max }}=\mathrm{P}_{3}=8 \mathrm{MPa}=8000 \mathrm{kPa}$

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Process 1-2

$$
\begin{aligned}
\mathrm{T}_{2} & =r^{\gamma-1} \mathrm{~T}_{1} \\
& =8^{0.4} \times 300 \\
\mathrm{~T}_{2} & =689.21 \mathrm{~K} \\
\text { and } \mathrm{P}_{2} & =r^{\gamma} \cdot \mathrm{P}_{1} \\
& =8^{1.4} \times 101=1856.29 \mathrm{kPa}
\end{aligned}
$$

Process 2-3
(Volume is constant)
$\mathrm{T}_{3}=\mathrm{P}_{3} \cdot \frac{\mathrm{~T}_{2}}{\mathrm{P}_{2}}$
$=\frac{8000 \times 689.21}{1856.29}$
$\mathrm{T}_{3}=2970.25 \mathrm{~K}$
So $\quad Q_{1}=C_{v}\left(T_{3}-T_{2}\right)$
$=0.718(2970.25-689.21)$
$\mathrm{Q}_{1}=1637.78 \mathrm{~kJ} / \mathrm{kg}$
and $\eta=1-\frac{1}{8^{0.4}}$

$$
=0.5647
$$

also $\frac{\mathrm{W}}{\mathrm{Q}_{1}}=0.5647$

$$
\begin{aligned}
& \mathrm{W}=1637.78 \times 0.5647 \\
& \mathrm{~W}=924.89 \mathrm{~kJ}
\end{aligned}
$$

(ii) $\quad \eta=0.5647$
or $\quad \eta=56.47 \%$
(iii) $\mathrm{P}_{\mathrm{m}}=\frac{\mathrm{W}}{\mathrm{V}_{\mathrm{S}}}$
and

$$
=V_{1}\left[1-\frac{1}{r}\right]
$$

$$
=\frac{\mathrm{RT}_{1}}{\mathrm{P}_{1}}\left[1-\frac{1}{r}\right]
$$

$\mathrm{V}_{\mathrm{s}}=\frac{0.287 \times 300}{101}\left[1-\frac{1}{8}\right]=0.746 \mathrm{~m}^{3}$

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So $\quad P_{m}=\frac{924.89}{0.746}$
$\mathrm{P}_{\mathrm{m}}=1239.93 \mathrm{kPa}$
or
$\mathrm{P}_{\mathrm{m}}^{\mathrm{m}}=1.239 \mathrm{MPa}$
(c) Explain the effect of Super heating and Sub-cooling on vapour compression refrigeration cycle.
Solution:
Superheating of the suction vapour is advisable in practive because it ensures complete vaporization of the liquid in the evaporator before it enters the compressor.
The effect of superheating of the vapour as follows:
(i) Increase in specific volume of suction vapour from $v_{1}$ to $v^{\prime}{ }_{1}$. Thus increase in volumetric efficiency.
(ii) Increase in refrigerating effect.
(iii) Increase in specific work.



Subcooling- It is possible to reduce the temperature of the liquid refrigerant to within a few degrees of the temperature of the water entering the condenser in some condenser designs by installing a subcooler, between the condenser and the expansion valve. The effect of subcooling of the liquid.
It will be seen that subcooling reduces flashing of the liquid during expansion and increases the refrigerating effect. Consequently, the piston displacement and horsepower per ton are reduced for all refrigerants.


(d) An air standard Brayton cycle has air entering the compressor at 100 kPa and $27^{\circ} \mathrm{C}$. The pressure ratio is 10 and the maximum allowable temperature in the cycle is 1350
K. Determine
(i) temperatures at salient points of the cycle
(ii) compressor and turbine work per unit mass of air
(iii) net work output and work ratio
(iv) thermal efficiency of the cycle
(v) specific air consumption in $\mathrm{kg} / \mathrm{kWh}$
(vi) Improvement in the thermal efficiency of the cycle if a regenerator with $\mathbf{1 0 0 \%}$ effectiveness is incorporated in the cycle.

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## Solution

Given Data
$\mathrm{P}_{1}=100 \mathrm{kPa}$
$\mathrm{T}_{1}=27^{\circ} \mathrm{C}=300 \mathrm{~K}$
$\mathrm{r}_{\mathrm{p}}=10$
$\mathrm{T}_{\max }=\mathrm{T}_{3}=1350 \mathrm{~K}$


(i)

$$
\begin{aligned}
\mathrm{T}_{2} & =\left(r_{P}\right)^{\frac{\gamma-1}{\gamma}} \cdot \mathrm{~T}_{1} \\
& =10^{0.4 / 1.4} \times 300 \\
& =579.2 \mathrm{~K} \\
\mathrm{~T}_{4} & =\left(\frac{1}{r_{P}}\right)^{\frac{\gamma-1}{\gamma}} \mathrm{~T}_{3} \\
& =(0.1)^{\frac{0.4}{1.4}} \times 1350
\end{aligned}
$$

$\mathrm{T}_{4}=699.22 \mathrm{~K}$
So the temperatures at all points
$\mathrm{T}_{1}=300 \mathrm{~K}$
$\mathrm{T}_{2}=579.2 \mathrm{~K}$
$\mathrm{T}_{3}=1350 \mathrm{~K}$
$\mathrm{T}_{4}=699.22 \mathrm{~K}$
(ii) $\quad \mathrm{W}_{\mathrm{c}}=\mathrm{C}_{\mathrm{p}}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)$
$=1.005(579.2-300)$
$\mathrm{W}_{\mathrm{c}}=280.596 \mathrm{~kJ} / \mathrm{kg}$
and $\mathrm{W}_{\mathrm{T}}=\mathrm{C}_{\mathrm{P}}\left(\mathrm{T}_{3}-\mathrm{T}_{4}\right)$
$=1.005$ (1350-699.22)
$\mathrm{W}_{\mathrm{T}}=654.0339 \mathrm{~kJ} / \mathrm{kg}$
(iii)
$\mathrm{W}_{\text {net }}=\mathrm{W}_{\mathrm{T}}-\mathrm{W}_{\mathrm{C}}$
$=654.034-280.596$
$\mathrm{W}_{\text {net }}=373.46 \mathrm{~kJ} / \mathrm{kg}$
and work ratio $=\frac{W_{\text {net }}}{W_{\mathrm{T}}}$

$$
\begin{aligned}
& =\frac{373.46}{654.034} \\
& =0.5710
\end{aligned}
$$

(iv) $\quad \eta=\frac{W_{\text {net }}}{Q_{1}}$
$Q_{1}=\mathrm{C}_{\mathrm{P}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)$
$=1.005$ (1350-579.2)
$Q_{1}=774.654$

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So $\quad \eta=\frac{373.46}{774.65}=0.482$
or $\quad \eta=48.2 \%$
(v)
S.A.C $=\frac{3600}{W_{\text {net }}}$
$=3600 / 373.46$
Specific air consumption $=9.64 \mathrm{~kg} / \mathrm{kWh}$

(vi)
$\varepsilon=\frac{T_{5}-T_{2}}{T_{4}-T_{2}}=1$
So $T_{5}=T_{4}=699.22 \mathrm{~K}$
Now, $\mathrm{Q}_{1}=\mathrm{C}_{\mathrm{P}}\left(\mathrm{T}_{3}-\mathrm{T}_{5}\right)$

$$
=1.005(1350-699.22)
$$

$$
\mathrm{Q}_{1}=654.0339 \mathrm{~kJ} / \mathrm{kg}
$$

and $\mathrm{W}_{\text {net }}$ remains unchanged
$\mathrm{W}_{\text {net }}=373.46 \mathrm{~kJ} / \mathrm{kg}$
So $\quad \eta^{\prime}=\frac{373.46}{654.0339}$
$=0.5710$
or $\quad \eta^{\prime}=57.1 \%$
$\Delta \eta=\eta^{\prime}-\eta$
$=0.571-0.482$
$=0.089$
or $\quad \Delta \eta=8.9 \%$
By using regenerator efficiency of plant increases by $8.9 \%$
3. (a) Define density, specific volume, weight density, specific gravity and Bulk Modulus. Solution:

Density ( $\rho$ ): It is defined as the ratio of mass per unit volume. It is also known as mass density. Bascially represents number of molecules ocupying a given volume.
Mathematically- $\quad \rho=\frac{\text { mass }}{\text { volume }}$
SI unit is $\frac{\mathrm{kg}}{\mathrm{m}^{3}}$.
Specific volume: It is defined as reciprocal of density or we can say that volume occupied by unit mass of fluid.

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Specific volume $=\frac{1}{\text { Density }}=\frac{\text { Volume }}{\text { mass }}$.
SI unit is $\frac{m^{3}}{\mathrm{~kg}}$
It used for gases basically.
Weight Density ( $\gamma$ ): It is defined as the ratio of weight of the fluid per unit volume of the fluid. Also known as specific weight and is a variable quantity in terms of location.
$\gamma=\frac{\text { weight of the fluid }}{\text { Volume of the fluid }}=\left(\frac{M g}{V}\right)=\rho g$
SI unit $=\frac{N}{m^{3}}$
Specific Gravity (S): It is defined as the ratio of weight density of a fluid to the of weight density of standard fluid.
$\mathrm{S}=\frac{\text { Weight density of a fluid }}{\text { Weight density of standard fluid }}$

- For liquids standard fluid is water and for gases air is taken as a standard fluid.
- It is a dimensionless quantity.

Bulk modulus (K): It is defined as the ratio of Hydrostatic stress to the volumetric strain.
$\mathrm{K}=\frac{\text { Hydrostatic stress }}{\text { Volumetric strain }}=\frac{-d p}{\left(\frac{d V}{V}\right)}$
$\mathrm{K}=-\mathrm{V} \frac{d P}{d V}$ or $\mathrm{K}=\rho \frac{d P}{d \rho}$

$$
\left\{\because-\frac{d v}{v}=\frac{d \rho}{\rho}\right\}
$$

It has same unit as pressure or stress ( $\mathrm{N} / \mathrm{m}^{2}$ ).
(b) A ship weighing 4000 tons and having an area of $465 \mathrm{~m}^{2}$ at water line submerging to a depth of 4.5 m in sea water with a density or $1024 \mathrm{~kg} / \mathrm{m}^{3}$ moves to fresh water. Determine the depth of subergence in fresh water. Assume that the sides are vertical at the water line.

## Solution:

Given data
$\mathrm{W}=\mathrm{Mg}=40 \times 10^{3} \times 9.81$
$\mathrm{A}=465 \mathrm{~m}^{2}$
Depth $=4.5 \mathrm{~m}$
$\rho_{\text {sea water }}=1024 \mathrm{Kg} / \mathrm{m}^{3}$
Orignally the weight of the ship is equal to weight of sea water displaced
Volume of sea water displaced $=\frac{M g}{\rho_{\text {SeaWater }} \times g}$

$$
=\frac{4000 \times 10^{3} \times 9.81}{1024 \times 9.81}=3906.25 \mathrm{~m}^{3}
$$

To support the same weight volume of fresh water displaced $=\frac{M g}{\rho_{\text {Fresh Water }} \times g}$

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$$
\begin{aligned}
& =\frac{4000 \times 10^{3} \times 9.81}{1000 \times 9.81} \\
& =4000 \mathrm{~m}^{3}
\end{aligned}
$$

Volume covered in fresh water $=4000-3906.25=93.75 \mathrm{~m}^{3}$
Area at this level $=465 \mathrm{~m}^{2}$
Depth $=\frac{93.75}{465}=0.2 \mathrm{~m}$
So Total Depth of submergence in fresh water $=4.5+0.2$

$$
=4.7 \mathrm{~m}
$$

(c) What is cavitation? How does it affect the performance of hydraulic machines?

## Solution:

When a liquid flows into a region where it's pressure is reduced to vapour pressure it starts boiling and bubbles are formed. These bubbles are carried along the flowing fluid to a region of high pressure where they get collapsed.
These bubbles can also get collapsed where they were reformed due to increase in pressure. When a Bubble of vapour collapses a cavity is formed and the surrounding liquid rush to fill it. The process of formation of vapour Bubbles and their collapsing is called cavitation.
The phenemnon of cavitation is alwasy accompanied by noise and vibrations.
When the cavitation occur the flow is desturbed flow passage is damaged and efficiency is decreased.
In turbines the chances of cavitation is high at the outlet because of pressure at the outlet is less then at the inlet.
In pumps the chances of cavity formation is high at the inlet because of the vaccum pressure at the inlet.
A parameter called cavitation number $(\sigma)$ is used to measure the occurance of cavitation. Mathematically it is given as
$\sigma=\frac{P-P_{V}}{\left(\rho V^{2} / 2\right)}$
where, $\mathrm{P}=$ Absolute pressure
$\mathrm{V}=$ Velocity of fluid
$\mathrm{P}_{\mathrm{V}}=$ Vapour pressure
(d) The following details refer to a centrifugal pump:

Outer diameter: 30 cm , Eye diameter: 15 cm , Blade angle at inlet: 30 ${ }^{\circ}$, Blade angle at outlet: $\mathbf{2 5}^{\circ}$, Speed 1450 rpm . The flow velocity remains constant. The whirl at inlet is zero.
Determine the work done per kg. If the manometric efficiency is $\mathbf{8 2 \%}$, determine the working head. If width at outlet is $\mathbf{2} \mathbf{m}$, determine the power $\eta_{o}=\mathbf{7 6 \%}$

## Solution:

Given data
$\mathrm{D}_{2}=0.3 \mathrm{~m}$
$\mathrm{D}_{1}=0.15 \mathrm{~m}$
$\theta=30^{\circ}$
$\phi=25^{\circ}$
$\mathrm{N}=1450 \mathrm{rpm}$
$\mathrm{V}_{f 1}=\mathrm{V}_{f 2}$
$B_{2}=2 \mathrm{~m}$
$\eta_{O}=0.76$

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$\mathrm{u}_{1}=\frac{\pi \mathrm{D}_{1} \mathrm{~N}}{60}=\frac{3.14 \times 0.15 \times 1450}{60}=11.38 \mathrm{~m} /$
$\mathrm{u}_{2}=\frac{\pi \mathrm{D}_{2} \mathrm{~N}}{60}=\frac{3.14 \times 0.3 \times 1450}{60}=22.77 \mathrm{~m} / \mathrm{s}$

## From Inlet Velocity Triangle

$\tan 30=\frac{V_{f 1}}{u_{1}} \Rightarrow V_{f 1}=\tan 30 u_{1}=11.39 \times \tan 30$

From outlet Velocity Triangle
$V_{f 1}=V_{f_{2}}=6.57 \mathrm{~m} / \mathrm{s}$
$\mathrm{V}_{W_{2}}=\mathrm{u}_{2}-\frac{V_{f_{2}}}{\tan 25}=22.77-\frac{6.57}{\tan 25}=8.67 \mathrm{~m} / \mathrm{s}$
Work done

$\eta_{\text {mainometric }}=0.82=\frac{g H_{m}}{197.4}$
$\mathrm{H}_{\mathrm{m}}=16.50 \mathrm{~m}$
Flow Rate $=\pi \mathrm{D}_{2} \mathrm{~B}_{2} \mathrm{~V}_{f 2}=\pi \times 0.3 \times 2 \times 6.57$

$$
=12.384 \mathrm{~m}^{3} / \mathrm{s}
$$

$\eta_{o}=\frac{\text { Water power }}{\text { Shaft power }}=\frac{\rho g Q H_{m}}{\text { Shaft power }}$
$0.76=\frac{10^{3} \times 9.81 \times 12.384 \times 16.50}{\text { Shaft power }}$
Shaft power $=2637.58 \mathrm{~kW}$

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4. (a) Write short notes on the following:
(i) Statinless steel
(ii) High speed steel
(iii) High carbon steel

## Solution:

Stainless steels are characterized primarily by their corrosion resistance, high strength and ducitility, and high chromium content. They are called statinless because in the presence of oxygen (air) they develop a thin, hard adherent film of chromium oxide that protects the metal from corrosion. In addition to chromium, other alloying elements in stainless steels typicaly are nickel, molybdenum, copper, titanium, silicon, manganese, columbium, aluminum, nitroge, and sulfur. The higher the carbon content is, the lower is the corrosion resistance of stainless steels.
Typical applications include cutlery, kitchen equipment, health care and surgical equipment, and In the petroleum industries.
(ii) High-speed steels (HSS) are the most highly alloyed tool and die steels.It maintain their hardness and strength at elevated operating temperatures.
There are two basic types of high-speed steels: the molybdenum type ( M series) and the tungsten type ( T series).
The M-series steels contain up to about $10 \%$ molybdenum, with chromium, vanadium, tungsten, and cobalt as other alloying elements. The-Tseries steels contain $12 \%$ to $18 \%$ tungsten, with chromium, vanadium, and cobalt as other alloying elements. The M-series steels generally have higher abrasion resistance than the T-series steels, undergo less distortion in heat treatment, and are less expensive.
(iii) High-carbon steel has more then $0.60 \%$ carbon. It is generally used for parts requiring strength, hardness, and wear resistance, such as cutting tools, cable, music wire, springs, and cutlery. After being manufactured into shapes, the parts are usually heat treated and tempered. Ther higher the carbon content of the steel, the higher is its hardness, strength, and wear resistance after heat treatment.
(b) With the help of figure, describe the Shielded Metal Arc Welding process.

Solution:


Shielded metal-arc welding (SMAW) is one of the oldest, simplest, and most versatile joining processes. The electric arc is generated by touching the tip of a coated electrode against the workpiece and then withdrawing it quickly to a distance sufficient to maintain the arc. The heat generated melts a portion of the tip of the electrode of its coating and of the base metal in the immediate area of the arc. A weld forms after the molten metal, a mixture of the base metal (workpiece), the electrode metal, and substances from the coating on the electrode, solidifies in the weld area. The electrode coating deoxidizes the weld area and provides a shielding gas to protect it from oxygen in the environment.
The current used for the formation of arc my be DC or AC. DC is preferred because of the steady arc it produces. The polarity of the DC current, that is, the direction of current flow, can be important; its selection depends on such factors as type of electrode, the metals to be welded, and the depth of the heated zone.
In Straight polarity, the workpiece is positive and the electrode negative; it is preferred for sheet metals.

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In reverse polarity, the electrode is positive, and deeper weld penetration is possible.
The SMAW process is commonly used in general construction, in shipbuilding, on pipelines, and for maintenance work, because the equipment is portable and can be easily maintained.
(c) Explain the different operations performed in grinding machine.

## Solution:

The basic types of grinding operations surface, cylindrical, internal, and centerless grinding

## (i) Surface Grinding-

Surface grinding involves grinding flat surfaces and is one of the most common grinding operations.
A straight wheel is mounted on the horizontal spindle of the grinder. Traverse grinding occurs as the table reciprocates longitudinally and feeds laterally after each stroke. In plunge grinding, the wheel is moved radially into the workpiece, as it is when grinding a groove.


## (ii) Cylindrical Grinding

In cylindrical grinding, also called center-type grinding external cylindrical surfaces and shoulders of the workpiece are ground.
The roatating cylindrical workpiece reciprocates laterally along its axis. In grinders used for large and long workpieces, the grinding wheel reciprocates; called a roll grinder. The workpiece in cylindrical grinding is held between centers or in a chuck. For straight cylindrical surfaces, the axes of rotation of the wheel and workpiece are parallel.


## (iii) Internal Grinding

In internal grinding a small wheel is used to grind the inside diameter of the part, such as to bushings and bearing races. The workpiece is held in a rotating chuck and the wheel rotates at $30,000 \mathrm{rpm}$ or higher. Internal profiles can also be ground with profile dressed wheels that move radially into the workpiece.

Workpiece


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(iv) Centerless Grinding

Centerless grinding is a high-production process for continuously grinding cylindrical surfaces in which the workpiece is supported not by centers (hence the term "centerless").

(d) Mention the differences between shaper and planer machine tools.

## Solution:

Planing is a relatively simple cutting operation by which flat surfaces, as well as various cross sections with grooves and notches, are produced along the lenght of the workpiece. Planing is usually done on large workpieces.
In a planer, the workpiece is mounted on a table that travels along a straight path. A horizontal cross-rail, which can be moved vertically along the ways in the column, is equipped with one or more tool heads.
Because of the reciprocating motion of the workpiece, elapsed noncutting time during the return stroke is significant.Cutting speed in planer can range up to $120 \mathrm{~m} / \mathrm{min}$.
Shaping is used to machine parts; it is much like planing, except that the parts are smaller and Cutting by shaping is basically the same as by planing. In a horizontal shaper, the tool travels along a straight path, and the workpiece is stationary. The cutting tool is attached to the tool head, which is mounted on the ram.
The ram has a reciprocating motion, and in most machines, cutting is done during the forward movement of the ram.
5. (a) Give the classification of kinematic pairs.

## Solution:

The kinematic pairs may be classified according to the following considerations; according to the type of relative motion between the elements:
(i) Sliding pair: When the two elements of a pair are connected in such a way that one can only slide relative to the other. Example-piston and cylinder, cross-head and guides of a reciprocatings team engine.
(ii) Turning pair: When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link. Example- Shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine.
(iii) Rolling pair: When the two elements of a pair are connected in such a way that one rolls over another fixed link. Example-Ball and roller bearings.
(iv) Screw pair: When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads. Example-Lead screw of a lathe with nut, and bolt with a nut.
(v) Spherical pair: When the two elements of a pair are connected in such a way that one element turns or swivels about the other fixed element, the pair formed is called a spherical pair. Example-Ball and socket joint, attachment of a car mirror, pen stand etc.
According to the type of contact between the elements:
(i) Lower pair: When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other. It will be seen that sliding pair turning pairs and screw pairs form lower pairs.

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(ii) Higher pair: When the two elements of a pair have a line or point contact whenrelative motion takes place and the motion between the two elements is partly turning and partly sliding.
Example- A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower.

## According to the type of closure:

(i) Self closed pair: When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.
(ii) Force-closed pair: When the two elements of a pairs are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.
(b) An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm . A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Calculate the speed of the dynamo shaft, when (i) there is no slip, and (ii) there is a slip of $2 \%$ at each drive.

## Solution:

Given data
$\mathrm{N}_{1}=150 \mathrm{rpm}$
$\mathrm{D}_{1}=750 \mathrm{~mm}$
$\mathrm{D}_{2}=450 \mathrm{~mm}$
$\mathrm{D}_{3}=900 \mathrm{~mm}$
$\mathrm{D}_{4}=150 \mathrm{~mm}$

When there is no slip


Velocity Ratio $=\frac{N_{4}}{N_{1}}=\frac{D_{1} \times D_{3}}{D_{2} \times D_{4}}$

$$
\begin{aligned}
=\frac{\mathrm{N}_{4}}{150} & =\frac{750 \times 900}{450 \times 150} \\
\frac{\mathrm{~N}_{4}}{150} & =10 \\
\mathrm{~N}_{4} & =1500 \mathrm{rpm}
\end{aligned}
$$

(ii) When slip is $2 \%$ on each Drive.

> Velocity Ratio $=\frac{N_{4}}{N_{1}}=\frac{D_{1} \times D_{3}}{D_{2} \times D_{4}}\left(1-\frac{S}{100}\right)$ $S=$ total slip $=2+2=4 \% \quad$ \{because of two drives\}
$\frac{\mathrm{N}_{4}}{150}=\frac{750 \times 900}{450 \times 150}\left(1-\frac{4}{100}\right)$
$\frac{\mathrm{N}_{4}}{150}=9.6$
$\mathrm{N}_{4}=1440 \mathrm{rpm}$
(c) Mention the comparison between involute and cycloidal gears.

## Solution:

## Cycloidal Profile Teeth:

In this type, the faces of the teeth are epicycloids and the flanks are the hypocycloids. A cycloid is the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.

## Involute Profile Teeth:

An involute is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle. Also, it is the path traced out by the end of a piece of taut cord being unwound from the circumference of a circle. The circle on which the straight line rolls or from which the cord is unwound is known as the base circle.

## Cycloidal Teeth

(a) Pressure angle varies from maximum at the beginning of engagement, reduces to zero at the pitch point and again increases to maximum at the end of engagement resulting in less smooth running of the gears.
(b) It involves double curve for the teeth, epicycloid and hypocycloid. This complicates the manufacture.
(c) Exact centre-distance is required to transmit a constant velocity ratio.
(d) Phenomenon of interference does not occur at all.
(e) The teeth the spreadig flanks and thus are stronger.
(f) In this, a convex flank always has contact with a concave face resulting in less wear.
(g) These are costlier.

## Involute Teeth

Pressure angle is constant throughout the engagement of teeth. This results in smooth running of the gears.

It involves single curve for the teeth resulting in simplicity of manufacturing and of tools.
A little variation in centre distance does not affect the velocity ratio.

Interference can occur if the condition of minimum number of teeth on a gear is not followed.
The teeth have radial flanks and thus are weaker as compared to the cycloidal form for the same pitch.
Two convex surfaces are in contact and thus there is more wear. These are cheaper.
(d) Explain the term height of the governor. Derive an expression for the height in the case of a Watt governor.

## Solution:

## Height of Governor-

It is the vertical distance from the centre of the governor balls to a point where the axes of the upper arm intersect on the spindle axis it is demoted by "h".

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(a)

Let. $m=$ mass of each ball
$h=$ height of governor
$\mathrm{w}=$ weight of each ball (= mg)
$\omega$ = angular velocity of the balls, arms and the sleeve
$\mathrm{T}=$ tension in the arm
$r$ = radial distance of ball-centre from spindle-axis
Assuming the links to be massless and neglecting the friction of the sleeve, the mass m at $A$ is in static equilibrium under the action of

- Weight w (= mg)
- Centrifugal force $\operatorname{mr} \omega^{2}$
- If the sleeve is masless and also friction in neglected, the lower links will be tension free.
The equilibrium of the mass provides

$$
\begin{aligned}
& \mathrm{T} \cos \theta=\mathrm{mg} \text { and } \mathrm{T} \sin \theta=\mathrm{mr} \omega^{2} \\
& \tan \theta=\frac{m r \omega^{2}}{m g}=\frac{r \omega^{2}}{g} \\
& \frac{r}{h}=\frac{r \omega^{2}}{g} \\
& \left.h=\frac{g}{\omega^{2}}=\frac{2 \pi / N}{60}\right)^{2}=\left(\frac{60}{2 \pi}\right)^{2} \times \frac{9.81}{\mathrm{~N}^{2}} \\
& h=\frac{895}{\mathrm{~N}^{2}} \mathrm{~m} \\
& h=\frac{895000}{\mathrm{~N}^{2}} \mathrm{~m}
\end{aligned}
$$

Thus, the height of a Watt governor is inversely proportional to the square of the speed.
6. (a) Three forces of $2 P, 3 P$ and $4 P$ act along the three sides of an equilateral triangle of side 100 mm taken in order. Find the magnitude and position of the resultant force.
Solution:


We take 2 P is acting at X -direction, then take it as reference.

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$\sum \mathrm{F}_{x}=2 \mathrm{P} \cos 0^{\circ}+3 \mathrm{P} \cos 120^{\circ}+4 \cos 240^{\circ}$.
$=2 P-\frac{3 P}{2}-2 P=-\frac{3 P}{2}$
and $\sum \mathrm{F}_{y}=2 \mathrm{P} \sin 0^{\circ}+3 \mathrm{P} \sin 120^{\circ}+4 \mathrm{P} \sin 240^{\circ}$.

$$
\begin{aligned}
& =0+\left(3 P \times \frac{\sqrt{3}}{2}\right)+4 \mathrm{P} \times\left(-\frac{\sqrt{3}}{2}\right) \\
& =-\frac{\sqrt{3} \mathrm{P}}{2}
\end{aligned}
$$

Resultant force $\mathrm{R}=\sqrt{\left(\sum F_{x}\right)^{2}+\left(\sum F_{y}\right)^{2}}$

$$
\begin{aligned}
& =\sqrt{\left(-\frac{3 \mathrm{P}}{2}\right)^{2}+\left(-\frac{\sqrt{3}}{2} \mathrm{P}\right)^{2}} \\
& =\left(\frac{\mathrm{P}}{2}\right) \sqrt{9+3} \\
& =\frac{\mathrm{P}}{2} \cdot 2 \sqrt{3}=\sqrt{3} \mathrm{P}
\end{aligned}
$$

Direction of the resultant force with x -axis.
$\tan \theta_{R}=\frac{\Sigma F_{y}}{\Sigma F_{x}}$

$$
\begin{aligned}
& =\frac{\left(-\frac{\sqrt{3}}{2} P\right)}{\left(-\frac{3 P}{2}\right)}=\frac{1}{\sqrt{3}} \\
& \theta_{R}=30^{\circ}, 210^{\circ}
\end{aligned}
$$

(b) A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3 . Find the magnitude of the force, which can move the body, while acting at an angle of $25^{\circ}$ with the horizontal.
Solution:


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Free body diagram


Condition for equilibrium of the body.
$\Sigma F_{y}=0$
$\mathrm{N}+\mathrm{F} \sin 25^{\circ}=300$
$\mathrm{N}=300-\mathrm{F} \sin 25^{\circ}$
and
$\Sigma F_{x}=0$
$\mathrm{F} \cos 25^{\circ}-\mu N=0$
From equation (i)
$F \cos 25^{\circ}-0.3 \times\left(300-F \sin 25^{\circ}\right)=0$
$\mathrm{F}\left(\cos 25^{\circ}+0.3 \times \sin 25^{\circ}\right)=90$
$\mathrm{F}=\frac{90}{1.033}=87.12 \mathrm{~N}$.
(c) Derive the expression for the shear stress in a circular shaft subjected to torsion.

## Solution:

A solid cylindrical shaft of radius R and length $l$ subjected to a couple or a twisting moment $T$ at one end, while its other end is held or fixed by the application of a balancing couple of the same magnitude.


Let $A B$ be a line on the surface of the shaft and parallel to the axis of the shaft before the deformation of the shaft. AS an effect of torsion this line, after the deformation of the shaft, takes the form AC.
The angle $\mathrm{CAB}=\phi$ represents the shear strain of the shaft material at the surface. This angle being small, we have

$$
\begin{aligned}
\mathrm{BC} & =l \phi \\
\phi & =\frac{B C}{l}
\end{aligned}
$$

Let the angle $B O C$ be the angular movement of the radius $O B$ due to the strain in the length of the shaft. Let $B O C=\theta$.
Let $\tau$ be the shear stress intensity at the surface of the shaft.
We know,

$$
\tau=\phi G
$$

Where, $G$ is Modulus of rigidity of the shaft material

$$
\tau=\left(\frac{B C}{l}\right) G
$$

But

$$
B C=R \theta
$$

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$$
\begin{aligned}
& \tau=\frac{\mathrm{R} \theta}{l} \mathrm{G} . \\
& \frac{\tau_{r}}{r}=\frac{\tau}{R}=\frac{G \theta}{l}
\end{aligned}
$$

The shear stress in torsion is linear function of radius of the shaft.
(d) Derive the expression for circumferential stress in a thin cylindrical vessel. Solution:

A thin cylindrical shell whose internal diameter is $d$, the thicknes of the shell being $t$. Let the length of the shell be $l$. Let the shell be subjected to an internal pressure of intensity P.


Let us consider a longitudinal section XX through the axis, dividing the shell into two halves A and B. Now let us consider two elementary strips subtending an angle $d \theta$ at the centre at an angle $\theta$ on either side of the vertical through the centre.
Normal force on each strip $d p_{n}=p r d \theta l$, where $r=$ radius of the shell.
The resultant of the two normal forces on the two elemental strips $=d P=2 p r l d \theta \cos \theta$ acting vertically, i.e. normal to XX.
Total force normal to XX on one side of $\mathrm{XX}=\mathrm{P}$

$$
\begin{align*}
\mathrm{P} & =\int_{0}^{\pi / 2} 2 p r l \cos \theta d \theta \\
& =2 p r l=p d l \tag{i}
\end{align*}
$$

p is intensity of radial pressure and $d l$ is projected area
Let $\sigma_{c}$ intensity of tensile stress induced in the metal across the section XX.
Resisting force $=\sigma_{c} \times 2 l t$
equating to equation (i)
$\sigma_{c} \times 2 l t=p d l$

$$
=\frac{p d}{2 t}
$$

This is the circumferential stress developed in the thin cylindrical pressure vessel due to internal pressure ' $p$ '.

