NDA (MATHS) MOCK TEST - 39 (SOLUTION)

1. (B) The binary number is

$$= 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1$$
$$\downarrow\ \ \downarrow\ \ \downarrow\ \downarrow\ \downarrow\ \downarrow$$
$$2^7\ \ 2^5\ \ 2^3\ 2^2\ 2^1\ 2^0$$

decimal number = 125 + 32 + 8 + 4 + 2 + 1 = 175

2. (B) $z = \left[\frac{\sqrt{3}}{2}+\frac{i}{2}\right]^5 + \left[\frac{\sqrt{3}}{2}-\frac{i}{2}\right]^5$

$= \left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6 + \left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^5$

$\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + \cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}$

$2\cos\frac{5\pi}{6}$ = Re (z)

3. (A) Let H be the harmonic mean of two numbers.

$\therefore$ G = H + 1.6 and A = H + 1.6 + 2 = H + 3.6

We know that, $AH = G^2$

$(H + 3.6) H = (H + 1.6)^2$

$\Rightarrow H^2 + 3.6 H = H^2 + 2.56 + 3.2 H$

$\Rightarrow H = \frac{2.56}{0.4} = 6.4$

$\therefore$ A = 6.4 + 3.6 = 10 and G = 6.4 + 1.6 = 8

Let two numbers are a and b.

$\therefore$ a + b = 20 .... (i)

and ab = 64 .... (ii)

We know that,

$(a - b)^2 = (a + b)^2 - 4ab = 400 - 256 = 144$

$\Rightarrow$ a – b = 12

On solving Eqs. (i) and (iii), we get

a = 16 and b = 4

4. (D) Let $\frac{1}{x} = u,\ \frac{1}{y} = v$

$\therefore a_1 u + b_1 v = c_1$ and $a_2 u + b_2 v = c_2$

Using the method of cross multiplication.

$$\frac{u}{b_1c_2 - b_2c_1} = \frac{v}{c_2a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{\frac{1}{x}}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{\frac{1}{y}}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\Rightarrow \frac{\frac{1}{x}}{\Delta_2} = \frac{\frac{1}{y}}{\Delta_3} = \frac{-1}{\Delta_1}$$

$\therefore \frac{1}{x} = \frac{\Delta_2}{\Delta_1}$ and $\frac{1}{y} = -\frac{\Delta_3}{\Delta_1}$

$\Rightarrow x = -\frac{\Delta_1}{\Delta_2}$ and $y = -\frac{\Delta_1}{\Delta_3}$

5. (D) The given equation of curve is

$\sqrt{x} + \sqrt{y} = \sqrt{a}$ $(x, y, \geq 0)$

$\Rightarrow \sqrt{y} = \sqrt{a} - \sqrt{x}$

$\Rightarrow (\sqrt{y})^2 = (\sqrt{a} - \sqrt{x})^2$

$\Rightarrow y = (\sqrt{a} - \sqrt{x})^2$

At $x = 0$, $\sqrt{y} = \sqrt{a} \Rightarrow y = a$

At $y = 0$, $\sqrt{x} = \sqrt{a} \Rightarrow x = a$

So, curve cuts the axes at (a, 0) and (0, a) respectively.

$\therefore$ Required area $= \int_0^a y\,dx = \int_0^a \left(\sqrt{a}-\sqrt{x}\right)^2 dx$

$= \int_0^a \left(a + x - 2\sqrt{a}\sqrt{x}\right)dx$

$= \left[ax + \frac{x^2}{2} - \frac{4}{3}\sqrt{a}(x)^{3/2}\right]_0^a$

$= a^2 + \frac{a^2}{2} - \frac{4}{3}\sqrt{a}\ : a^{3/2}$

$\frac{3a^2}{2} - \frac{4}{3}a^2 = \frac{(9-8)}{6}a^2 = \frac{a^2}{6}$

6. (B) Given that, $f : N \to N$ and $f(x) = x + 1$, for $x \in N$, if

$x_1, x_2 \in N$, then $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$

i.e, $f(x)$ is one-one.

Range of $f(x) \in N - \{1\}$

$\therefore$ Range $\subseteq$ Codomain

So, $f(x)$ is into functions.

Hence, f is one-one but not onto.

7. (A)
8. (B)
9. (A) We have, $f'(a) = 2a^2$, $f'(b) = 2ab$ and $f'(c) = 2ac$

$\therefore 2b = a + c \Rightarrow 2a \cdot 2b = 2a \cdot a + 2a \cdot c$

$\Rightarrow 2(2ab) = 2a^2 + 2ac \Rightarrow 2f'(b) = f'(a) + f'(c)$

Hence, $f'(a)$, $f'(b)$ and $f'(c)$ are in AP.

10. (D)
11. (B) $(b - x)^2 - (a - + b - c)\,x + (a - b) = 0$

$(b - c)\,x^2 - (b - c)\,x - (a - b)\,x + (a - b) = 0$

$(b - c)\,x\,[x - 1] - (a - b)\,[x - 1] = 0$

$[(a-c)x-(a-b)][x-1]=0$

$x=\dfrac{a-b}{b-c}, 1$

12. (A) Given, $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)\,dx = \int_0^1 (a+bx+cx^2)\,dx$$

$$= \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1 = a + \frac{b}{2} + \frac{c}{3} \quad \text{...... (i)}$$

Here, $f(0) = a$, $f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

and $f(1) = a + b + c$

Now, $\dfrac{f(0) + 4f\left(\frac{1}{2}\right) + f(1)}{6}$

$$= \frac{a + 4\left(a + \frac{b}{2} + \frac{c}{4}\right) + a + b + c}{6} = a + \frac{b}{2} + \frac{c}{3} \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$\int_0^1 f(x)\,dx = \frac{f(0) + 4f\left(\frac{1}{2}\right) + f(1)}{6}$$

13. (B) Given that,
Mean of 20 observations = 15
∴ Sum of 20 observations = 20 × 15 = 300
∴ Sum of actual (correct) observations
= 300 – (3 + 6) + (8 + 4)
= 300 – 9 + 12 = 303
∴ Correct mean = 15.15

14. (B) Given quadratic equation is
$ax^2 + bx + b = 0$
Let (α, β) be the roots of given equations.

$\therefore \alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{b}{a}$

Now, we have

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{b}{a}} = \frac{-b}{a} \times \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$$

$$= -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}} = 0$$

15. (B) Take a, b and c common from R_1, R_2 and R_3 respectively.

$$\Delta = abc \begin{bmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b}+1 & \frac{1}{b}+2 & \frac{1}{b} \\ \frac{1}{c}+1 & \frac{1}{c}+1 & \frac{1}{c}+3 \end{bmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$,

$$\Delta = abc\left[3 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right] \begin{bmatrix} 1 & 1 & 1 \\ 1+\frac{1}{b} & 2+\frac{1}{b} & \frac{1}{b} \\ 1+\frac{1}{c} & 1+\frac{1}{c} & 3+\frac{1}{c} \end{bmatrix}$$

Applying $C_3 \to C_3 - C_2$
and $C_2 \to C_2 - C_3$ and expand,

$$\Delta = 2ab\left(3 + \Sigma\frac{1}{a}\right) = 0$$

$$\therefore \Sigma\frac{1}{a} = -3$$

as $a \neq 0, b \neq 0, c \neq 0$
i.e., $a^{-1} + b^{-1} + c^{-1} = -3$

16. (C)

17. (B) $x^2 - x + 1 = 0$

$$x = \frac{-1+\sqrt{1-4}}{2} = \frac{-1\pm\sqrt{3}i}{2} \Rightarrow w, w^2$$

Now, $\begin{bmatrix} 1 & w^2 \\ w & w \end{bmatrix}\begin{bmatrix} w & w^2 \\ 1 & w \end{bmatrix}$

$$\begin{bmatrix} w+w^2 & w^2+w^4 \\ w^2+w & w^3+w^3 \end{bmatrix} = \begin{bmatrix} -1 & w^2+w \\ -1 & 1+1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

18. (B) The maximum number of triangles
$= {}^5C_3 - {}^3C_3 = 10 - 1 = 9$

19. (A) We have, $\dfrac{dy}{dx} = \dfrac{y^2}{1-3xy}$

$$\Rightarrow \frac{dx}{dy} = \frac{1-3xy}{y^2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{y^2} - \frac{3}{y}x$$

$$\therefore \frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2} \quad \text{........... (i)}$$

The above is a linear differential equation of the form

$$\frac{dx}{dy} + P(y)\,x = Q(y)$$

Here, $P = \dfrac{3}{y}$ and $Q = \dfrac{1}{y^2}$

If = $e^{\int \frac{3}{y}dy} = e^{3\log y} = y^3$

The solution of Eq. (i) is given by

$$xy^3 = \int \frac{1}{y^2}\cdot y^3\,dy + C$$

$$\therefore\quad xy^3 = \frac{y^2}{2} + C$$

20. (A) Let n (E) = 75, n (M) = 60 and n (E ∩ M) = 45
∴ Exactly one of them occurs = n (E) + n (M) – 2n (E ∩ M)
= 75 + 60 – 90 = 45

21. (C) 5 Mathematics books can be arragned in 5 ! ways
4 physics books can be arragned in 4! ways
3 chymistry book can be arragned in 3! ways
4 literature book can be arragned in 4! ways
also the are 4 different types of books, so they can be arrangedin 4! ways.
∴ Total possible ways = 5! · 4! · 3! · 4! · 4!

22. (B) Given equation of hyperbola is

$$4x^2 - 9y^2 = 1 \Rightarrow \frac{x^2}{(1/4)} - \frac{y^2}{(1/9)} = 1$$

Here, $a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{9}$

∴ Foci of the hyperbola = (+ ae,0)

$$= \left(\pm a\frac{\sqrt{a^2+b^2}}{a}, 0\right)\ (+\ \sqrt{a^2+b^2}, 0)$$

$$= \left(\pm\sqrt{\frac{1}{4}+\frac{1}{9}}, 0\right) = \left(\pm\frac{\sqrt{13}}{6}, 0\right)$$

23. (B) Given that,
$(1 + x)^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$
Put $x = 1$,
$(1 + 1)^n = a_0 + a_1 + a_2 + \ldots + a_n$
$\Rightarrow a_0 + a_1 + a_2 + \ldots + a_n = 2^n$

24. (D)

25. (A) The given equation is ,
$x^4 - 26x^2 + 25 = 0$
$\Rightarrow x^4 - 25x^2 - x^2 + 25 = 0$
$\Rightarrow x^2(x^2 - 25) - 1(x^2 - 25) = 0$
$\Rightarrow (x^2 - 25)(x^2 - 1) = 0$
$\Rightarrow (x - 25)(x + 25)(x - 1)(x + 1) = 0$
$\therefore\quad x = -5, -1, 1, 5$
So, the solution set is {– 5, –1, 1, 5}.

26. (b) ∴ cos 60°

$$= \frac{1\times1 + 0\times0 + (\cos\alpha)(-\cos\alpha)}{\sqrt{1^2+(0)^2+\cos^2\alpha}\sqrt{1^2+(0)^2+(-\cos\alpha)^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{1-\cos^2\alpha}{\sqrt{1+\cos^2\alpha}\sqrt{1+\cos^2\alpha}}$$

$$\Rightarrow \frac{1}{2} = \frac{1-\cos^2\alpha}{1+\cos^2} \Rightarrow \frac{1+2}{1-2} = \frac{2}{-2\cos^2\alpha}$$

(applying componendo and dividendo)

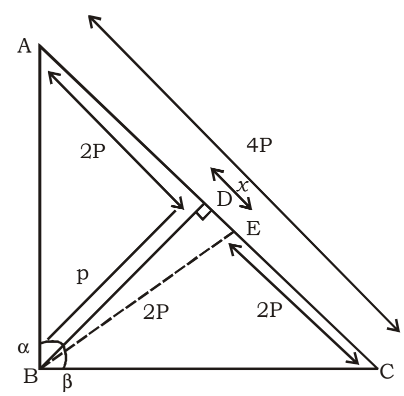
$$\Rightarrow \frac{3}{-1} = \frac{1}{\cos^2\alpha}$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

$$\therefore\quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

27. (D)

28. (A) Let BD = p, DE = $x \Rightarrow$ AC = 4 p
Let E is the mid-point of AC,
then AE = EC = BE = 2p



Now, in ΔBDE,
$(BE)^2 = (BD)^2 + (ED)^2 \Rightarrow (2p)^2 = (p)^2 + (x)^2$
$\Rightarrow 4p^2 = p^2 + x^2 \Rightarrow x^2 = 3p^2$
$\Rightarrow x = \sqrt{3}\cdot p$

Now, AD = $2p - x = 2p - \sqrt{3}\,p$
DC = $2p + x = 2p + \sqrt{3}\,p$

$$\text{In } \Delta BAD,\ \tan A = \frac{p}{2p-\sqrt{3}p} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\Rightarrow \tan A = \frac{2+\sqrt{3}}{1} = \tan 75° \Rightarrow A = 75°$$

$$\Rightarrow \tan\alpha = \frac{2p-\sqrt{3}p}{p} = 2-\sqrt{3} = \tan 15°$$

$\Rightarrow \alpha = 15°$

$$\text{In } \Delta BDC,\ \tan C = \frac{p}{2p+\sqrt{3}p} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$$

$= 2 - \sqrt{3}$

$\Rightarrow$ tan C = tan15° $\Rightarrow$ C = 15°

$\Rightarrow$ $\tan\beta = \frac{2P+\sqrt{3}P}{P} = 2+\sqrt{3} = \tan 75°$

$\Rightarrow$ β = 75°

So, the acute angle in ΔABC is ∠C = 15°

29. (A)

30. (B) $\frac{AD}{DC} = \frac{2p-\sqrt{3}p}{2p+\sqrt{3}p} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\Rightarrow \frac{4+3-4\sqrt{3}}{4-3} = \frac{7-4\sqrt{3}}{1}$

$\Rightarrow$ AD : DC = $(7-4\sqrt{3}) : 1$

31. (B) $\because$ $N(t) = ce^{kt}$

$\frac{dN(t)}{dt} = \frac{d}{dt} ce^{kt} = k(ce^{kt}) = k[N(t)]$

But $\frac{dN(t)}{dt} = \alpha N(t)$

$\therefore$ α = k

32. (B) $\frac{y-2}{1} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, $\frac{y-1}{3-y} = \frac{2e^x}{2e^{-x}}$

(applying componendo and dividendo rule)

$\Rightarrow \frac{y-1}{3-y} = e^{2x} \Rightarrow x = \frac{1}{2}\log\left(\frac{y-1}{3-y}\right)$

$\therefore f^{-1}(x) = \frac{1}{2}\log\left(\frac{x-1}{3-x}\right)$

33. (C) $\therefore \begin{vmatrix} p & -q & 0 \\ 0 & p & q \\ q & 0 & p \end{vmatrix} = 0$

Expand with respect to R_1,

$p(p^2-0) + q(0-q^2) + 0 = 0 \Rightarrow p^3 - q^3 = 0$

$\Rightarrow (p-q)(p^2+q^2+pq) = 0$

$\Rightarrow p - q = 0$ and $p^2+q^2+pq = 0$

$\Rightarrow p = q$ and $\frac{p^2}{q^2} + 1 + \frac{pq}{q^2} = 0$

$\Rightarrow \left(\frac{p}{q}\right) = 1$ and $\left(\frac{p}{q}\right)^2 + \left(\frac{p}{q}\right) + 1 = 0$

We conclude that $\left(\frac{p}{q}\right)$ is one of the cube roots of unity.

34. (C) Let A(a – 1, a, a + 1), B(a, a + 1, a – 1) and C (a + 1, a – 1, a) are the vertices of a ΔABC.

$\therefore AB = \sqrt{(a-a+1)^2 + (a+1-a)^2 + (a-1-a-1)^2}$

$= \sqrt{1+1+4} = \sqrt{6}$

$BC = \sqrt{(a+1-a)^2 + (a-1-a-1)^2 + (a-a+1)^2}$

$= \sqrt{1+4+1} = \sqrt{6}$

and $CA = \sqrt{(a-1-a-1)^2 + (a-a+1)^2 + (a+1-a)^2}$

$= \sqrt{4+1+1} = \sqrt{6}$

$\therefore$ AB = BC = CA

Hence, given points are vertices of an equilateral triangle for any real value of a.

35. (C) Circumfrence = 2πr

$\Rightarrow 10\pi = 2\pi r \Rightarrow r = 5$

and centre = (2, – 3)

By standard equation of circle, $(x-2)^2 + (y+3)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 4x + 6y - 12 = 0$

36. (D) Only two can be placed at one's place = 1 may

Tens's place can be filled by either 1 or 3 or 5 = 3 ways.

$\therefore$ By fundamental principal of country,

Total possible ways to form a two digit even no = 1 × 3 = 3

37. (A) $I = \int \frac{dx}{\sqrt{\tan^3 x \cos^4}} dx = \int \frac{\sec^2 x}{(\tan x)^{3/2}} dx$

Put tan x = t $\Rightarrow$ sec² dx = dt

$\therefore I = \int t^{-3/2} dt = \frac{-2}{\sqrt{\tan x}} + C$

38. (*) Put $x^2 = \cos 2\theta$, $1 + \cos 2\theta = 2\cos^2\theta$, $1 - \cos 2\theta = 2\sin^2\theta$

$\therefore y = \tan^{-1}\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \tan^{-1}\tan(\pi/4 + \theta)$

$= \pi/4 + \theta$

$y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot \frac{-1}{\sqrt{(1-x^4)}} \cdot 2x = -\frac{x}{\sqrt{(1-x^4)}}$

39. (A) $\frac{dy}{dx} = 3x^2 - 2x - 1 \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 3 - 2 - 1 = 0$

The equation of tangent is $y - 1 = 0(x-1) \Rightarrow y = 1$ i.e., parallel to x-axis

Therefore, both A and R true and R is the correct explanation of A.

40. (A) A. We know that,

Work done = F · d = |F|· |d| cos θ

Since, θ = 90° = F · d = |F|·|D|cos 90° = 0

There, both A and R R are true but R is the correct explanation of A.

41. (B) Required probability

$$= \frac{4}{52}+\frac{4}{52}+\frac{1}{13}+\frac{1}{13}=\frac{2}{13}$$

Therefore, both, A and R are true but R is not the correct explanation of A.

42. (C)

43. (A) Given expression $\frac{(1+4+9+\ldots+n^2)\log x}{(1+2+3+\ldots.+n)\log x}$

$$= \frac{\Sigma n^2}{\Sigma n} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}$$

44. (B)

45. (C) $\tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-3\cdot 2\log x}\right)$

$\tan^{-1} 1 - \tan^{-1}(2\log x) + \tan^{-1} 3 + \tan^{-1}(2\log x)$

$= \tan^{-1}1 + \tan^{-1}3$

$\therefore$ y = constant

$\therefore \frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$

46. (C) $(1+x)^m (1-x)^n = \left(1+mx+\frac{m(m-1)}{2}\cdot x^2+\ldots\right)$

$$\times\left(1-nx+\frac{n(n-1)}{2}\cdot x^2+\ldots\right)$$

Coefficient of x = (m – n) = 3 (given) ..... (i)

Coefficient of $x^2 = \frac{m(m-1)}{2} - mn + \frac{n(n-1)}{2}$

= –6 (gives)

$\Rightarrow m^2 - m - 2mn + n^2 - n = -12$

$\Rightarrow m^2 + n^2 - 2mn - (m+n) = -12$

$\Rightarrow (m-n)^2 - (m+n) = -12$

$\Rightarrow (3)^2 - (m+n) = -12 \Rightarrow m+n = 21$ .... (ii)

On solving Eqs. (i) and (ii) we get m = 12

47. (B) $\int_0^1 x^m (1-x)^n\,dx = \int_0^1 (1-x)^m x^n\,dx$

(using property)

But $\int_0^1 x^m (1-x)^n\,dx$ = 'K $\int_0^1 x^n (1-x)^m\,dx$

$\therefore$ K = 1

48. (A) I = $\int \frac{x\left(1-\frac{1}{x^3}\right)}{x^5}dx$

Put $1-\frac{1}{x^3} = t \Rightarrow \frac{3}{x^4}dx = dt$

$\therefore$ 1 = $\frac{1}{3}\int t^{1/4}\,dt = \frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4} + C$

49. (C) Given function, $f(x) = |x| + x^2$

Again, defining the function $f(x)$,

$$f(x) = \begin{cases} x^2 - x, & x < 0 \\ x^2 + x, & x \ge 0 \end{cases}$$

At $x = 0$,

LHL = $f(0-0) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} (-h)^2 - (-h) = 0$

RHL = $f(0+0) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} (h)^2 + (h) = 0$

Also $f(0) = 0$

$\because$ LHL= RHL = $f(0) = 0$

So, function is continuous at $x = 0$

Now,

$$Rf'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{h(h+1)-0}{h} = 1$$

$$Lf'(0) = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$= \lim_{h\to 0}\frac{(h^2+h)-0}{-h} = -1$$

$\because$ $Rf'(0) \ne Lf'(0)$

So, $f(x)$ is not differentiable at $x = 0$

50. (B) Are of the Δ AMB = $\frac{1}{2}\begin{vmatrix} x & x & 1 \\ -2 & 0 & 1 \\ 0 & 4 & 1 \end{vmatrix}$

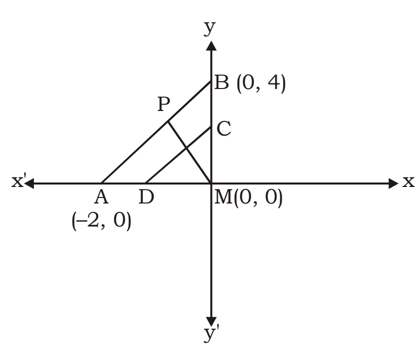
$$= \left|\frac{1}{2}(-4x+2x-8)\right|$$

$= |-(x-4)|$,

Which is minimum for x = 0 and thus the coordinates of M are (0, 0).

51. (A) As p divides AB in the ratio 2 : 1. The base of the Δ's APM and BPM are in the ratio 2 : 1 and the length ofthe perpendicular from the vertex M on the base is same.

So, the ratio of the areas of the Δ's APM and BPM is also 2 : 1



52. (B) ABCD is a quadrilateral with AD = 1, BC = 2

$$DC = \frac{1}{2}AB = \frac{1}{2}\sqrt{2^2+4^2} = \sqrt{5}$$

So, the required perimeter is

$1 + 2 + \sqrt{5} + 2\sqrt{5} = 3 + 3\sqrt{5}$

53. (A) For the singular matrix.

$$\begin{bmatrix} 2-x & 1 & 1 \\ 1 & 3-x & 0 \\ -1 & -3 & -x \end{bmatrix} = 0$$

(exapand with respect to R_1)

$\Rightarrow$ $(2-x)\,[x\,(x-3)] - [-x] + [-3 + (3-x)] = 0$

$\Rightarrow$ $x\,(x-3)(x-2) = 0 \Rightarrow x = 0, 2, 3$

So, the solution set is, S = {0, 2, 3}.

54. (C) $0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$

$$\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$$

55. (A) Since, the line passes through the point (0, 1) and making an angle with Y-axis which is equivalent ot the slope of the line $y = x - 4$.

i.e, $\theta = 45° \Rightarrow \tan\theta = 1\ m$

$\therefore$ Equation of line is

$(y - 1) = m\,(x - 0) = 1(x)$

$\Rightarrow$ $y = x + 1$

56. (C) Let $I = \int_{-a}^{a}(x^3 + \sin x)\,dx$

Here, $f(x) = x^3 + \sin x \Rightarrow f(-x) = (-x)^3 + \sin(-x)$
$= -x^3 - \sin x = -(x^3 + \sin x) = -f(x)$

i.e., $f(x)$ is an odd function

$$\therefore \int_{-a}^{a}(x^3 + \sin x)\,dx = 0$$

57. (A) since, $(\lambda a + b)\cdot(a - \lambda b) = 0$

$\Rightarrow$ $\lambda|a|^2 + (1-\lambda)^2\, a\cdot b - \lambda|b|^2 = 10$

$\Rightarrow$ $(1-\lambda)^2\,|a||b|\cos 60° = 0 \quad (|a| = |b|)$

$\therefore$ $\lambda = \pm 1$

58. (C) Given that, $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$

and $P(A \cup B) - P(A \cap B) = \frac{2}{5}$ ..... (i)

$[P(A) + P(B) - P(A \cap B)] - P(A \cap B) = \frac{2}{5}$

(by addition theorem of probability)

$\Rightarrow$ $P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$\Rightarrow$ $\frac{2}{3} + \frac{2}{5} - 2P(A \cap B) = \frac{2}{5}$

$\therefore$ $P(A \cap B) = \frac{1}{3}$

59. (C) Given, $2X - 3Y = \begin{bmatrix} -7 & 0 \\ 7 & 13 \end{bmatrix}$ ... (i)

and $3X + 2Y = \begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix}$ ..... (ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 2 and subtracting Eq. (i) from eq. (ii), we get

$$13Y = 2\begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix} - 3\begin{bmatrix} -7 & 0 \\ 7 & -13 \end{bmatrix}$$

$$\Rightarrow 13Y = 2\begin{bmatrix} 39 & 26 \\ -13 & 65 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$$

60. (D) Required sum = $(45 - 49)^4 = (-4)^4 = 256$

61. (B) $\log\left[a + \sqrt{a^2+1}\right] + \log\left\{\frac{1}{a + \sqrt{a^2+1}}\right\}$

$$= \log\left\{\left(a + \sqrt{a^2+1}\right) \times \frac{1}{\left(a + \sqrt{a^2+1}\right)}\right\} = \log 1 = 0$$

62. (C)

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	14	x	27	y	15

Given that,

Sum frequencies = 100

$\Rightarrow$ $14 + x + 27 + y + 15 = 100$

$\Rightarrow$ $x + y + 57 = 100$

$\Rightarrow$ $x + y = 43$ .... (i)

For mode, $f_m = 27$, $f_1 = x$ and $f_2 = y$, $1_1 = 20$, h = 10

Clearly, 20 – 30 is the modal class.

Since, mode lies between 20-30.

$$\therefore \text{Mode} = 1_1 + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$\text{Given, } 25 = 20 + \frac{27 - x}{54 - x - y} \times 10$$

$$\Rightarrow 5 = \frac{270 - 10x}{54 - x - y}$$

$\Rightarrow$ $x = y$ ...... (ii)

From Eqs. (i) and (ii),

$2x = 43$

$$\Rightarrow x = \frac{43}{2} = 21.5$$

$\therefore$ $x = y = 21.5$

63. (B) Use $G^2 = A \times H \Rightarrow G^2 = 27 \times 12 = 324$

$\Rightarrow G = 18$

64. (C) Given A = [a, b, c]

Number of subset of A = $2^n = 2^3 = 8$

$\therefore$ Proper subset of A = $2^n - 1 = 8 - 1 = 7$

65. (A)
66. (A)
67. (B) $25x^2 + 16y^2 - 150x - 175 = 0$
$25(x^2 - 6x + 9) + 16y^2 = 175 + 225$
$\Rightarrow 25(x-3)^2 + 16y^2 = 400 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$
Form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
Major axis lies along Y axis.
$\therefore\ e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{16}{25}$
$\therefore\ e = \frac{3}{5}$

68. (C) $\left((1+i)^2\right)^n = \left((1-i)^2\right)^n$
$\left(1+i^2+2i\right)^n = \left(1+i^2-2i\right)^n$
$(2i)^n = (-2i)^n$
$i^n = (-i)^n$ for n = 2

69. (A) Here, Statement III is wrong becuase construction of a frequency distribution is based on data which are both discrete as well as continuous.

70. (B) $\frac{1+x+iy}{1+x-iy} = \frac{(1+x+iy)(1+x+iy)}{(1+x-iy)(1+x+iy)}$
$= \frac{(1+x)^2 + iy(1+x) + iy(1+x) - y^2}{1+x^2+2x+y^2}$
$= \frac{1+x^2+2x-y^2+2iy(1+x)}{2(1+x)}$
$= \frac{1-y^2+2x+x^2+2iy(1+x)}{2(1+x)}$
$= \frac{2x^2+2x+2iy(1+x)}{2(1+x)}$ $(\because\ x^2 + y^2 = 1)$
$= x + iy$

71. (B) Required area $= \int_0^1 xe^x\,dx$
$= \left[xe^x - \int e^x\,dx\right]_0^1 = \left[xe^x - e^x\right]_0^1$
Use integration by parts
= (e – e) – (0 – 1) = 1 sq unit

72. (B) $\frac{dy}{dx} = \cos x - b <, 0\ \forall\ x \in R$
Since, the maxinum value of cos x is 1.
$\therefore$ $1 - b \leq 0$ or $1 \leq b$
But cos x 6 $\leq$ 1 and hence, b $\geq$ 1.

73. (A) $\sin^{-1}(3x - 4x^3) = 3\sin^{-1} x$
Let $\sin^{-1}(3x - 4x^3) = \theta$
$3\sin^{-1}(x) = \theta$
$\sin^{-1}(x) = \frac{\theta}{3},\ x = \sin\frac{\theta}{3}$
$\sin\theta = 3x - 4x^3$
We know that,
$-1 \leq \sin^{-1}(3x - 4x^3) \leq 1$
$-\frac{\pi}{2} \leq \sin^{-1}(3x - 4x^3) \leq \frac{\pi}{2}$
$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2},\ -\frac{\pi}{6} \leq \frac{\theta}{3} \leq \frac{\pi}{6}$
$-\frac{1}{2} \leq \sin\frac{\theta}{3} \leq \frac{1}{2},\ -\frac{1}{2} \leq x \leq \frac{1}{2}$
So, x lies between each = $\left[\frac{-1}{2}\frac{1}{2}\right]$

74. (A) Required equation of parabola is
$y^2 = 4ax$ ..... (i)
On differentiating w.r.t.x, we get
$2yy' = 4a \Rightarrow \frac{1}{2}yy' = a$
On putting this value of a in Eq. (i), we get
$y^2 = \frac{4}{2}yy'\,x \Rightarrow y = 2xy'$

75. (B) The appropriate number of classes while constructing a frequency distribution should be chosen such that the class frequency should cluster around the class mid point.

76. (B) Possibilities of words formed from the letters of word 'JOKE' are JOKE. KOJE, KEJO, JEKO, EJOK, EKOJ, OKEJ and OJEK.
Thus, required number of words = 8

77. (C)

78. (C) If a · b = 0 a $\perp$ b and a × b = 0 $\Rightarrow$ a | | b
But both conditions cannot be exist simultaneously. The one possibel way to existing both conditions simultaneously is that either a or b is a null vector

79. (B)

80. (C) Cot $\frac{B}{2}$ cot $\frac{C}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} \cdot \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$
$= \frac{s}{s-a} = \frac{\frac{a+b+c}{2}}{\frac{a+b+c}{2} - a}$
$= \frac{a+b+c}{b+c-a} = \frac{a+3a}{3a-a} = 2$

81. (D) Given that, a = 18, b = 24 and c = 30
Now, by cosine law,

$$\cos C = \left(\frac{-c^2+a^2+b^2}{2ab}\right) = \frac{(18)^2+(24)^2-(30)^2}{2\times18\times24}$$

$$= \frac{324+576-900}{864} = \frac{900-900}{864} = 0$$

$\Rightarrow$ cos C = cos 90° ∠C = 90°
$\therefore$ sin C = sin 90° = 1

82. (B) Last term of series $S_1 = 1 \times 2^{100-1} = 2^{99}$

83. (B) For as S_1 (i.e., GP) $T_m = 2^{n-1}$
For as S_2 (i.e., AP) T_m = 1 + (m – 1) 3
= 3m – 2
They are common, if

$$2^{n-1} = 3m - 2 \Rightarrow 2^{n-2} + 1 = \frac{3m}{2} \le 150$$

$\Rightarrow$ n ≤ 9, m ≤ 100
As, $2n^{-1}$ = 3m – 2
$\therefore$ (n = 1, m = 1), (n = 3, m = 2), (n = 5, m = 6), (n = 7, m = 22), (n = 9, m = 86) and for n = 2, 4, 6, 8, m is a fractions which is not possible.
Hence, No. of common terms = 5

84. (C) Sum of 100 terms of series $S_2 = \frac{100}{2}$
[2 × 1+ (100– 1) ×3] = 50 [2 + 99 × 3]
= 50 × 299 = 14950

85. (D) Given that,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1} x$$

$\Rightarrow$ $2\tan^{-1} a + 2\tan^{-1} b = 2\tan^{-1} x$

$$\left[\because 2\tan^{-1} x = \sin{-1}\left(\frac{2x}{1+x^2}\right)\right]$$

$\Rightarrow$ $\tan^{-1} a + \tan^{-1} b = \tan^{-1} x$

$$\Rightarrow \tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1} x$$

$$\therefore x = \frac{a+b}{1-ab}$$

where, a > 0 and b > 0

86. (C) $x^2 + y^2 + 4x - 4y + 4 = 0$
$(x+2)^2 + (y-2)^2 = 4$
$(x+2)^2 + (y-2)^2 = 2^2$
$[x-(-2)^2 + (y-2)^2] = 2^2$
$\therefore$ Centre = (– 2, 2) and radius = 2
So, circle touch has both axes.

87. (A) Given, curve, $y = x e^x$
On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = x \cdot e^x + e^x \cdot 1 = xe^x + e^x$$

For max and min of y,

$$\frac{dy}{dx} = 0 \Rightarrow e^x (x+1) = 0$$

$\Rightarrow$ $x = -1$
Again, differentiating w.r.t.x, we get

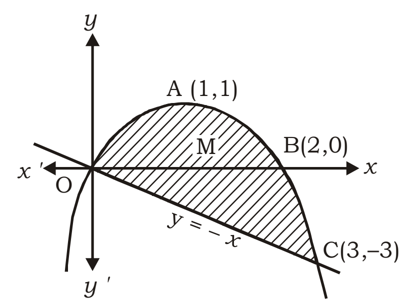
$$\frac{d^2y}{dx^2} = x \cdot e^x + e^x \cdot 1 + e^x = xe^x + 2e^x$$

$$\left(\frac{d^2y}{dx^2}\right)_{at\,x=-1} = (-1)e^{-1} + 2e^{-1} = \frac{1}{e} > 0 \text{ (minimum)}$$

$\therefore$ $f(x)$ have minimum value at $x = -1$
Hence, its minimum value is

$$y(-1) = (-1)\,e^{-1} = \frac{-1}{e}$$

88. (A) $y = 2x - x^2$ is $(x-1)^2 = -(y-1)$
It represent a parabola with vertex at (1, 1)



$$\therefore A = \left|\int_0^3 (y_1 - y_2)\,dx\right| = \left|\int_0^3 (2x - x^2) + x\,dx\right|$$

$$= \left|\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3\right| = \frac{9}{2}$$

89. (B) Given, v = s + 1

$$\Rightarrow \frac{ds}{dt} = s + 1$$

$$\Rightarrow \frac{ds}{s+1} = dt$$

On intergrating, we get
$\Rightarrow$ log (s + 1) = t
As s = 9 m, t = log (10)s

90. (A) $\frac{1}{1+3i} - \frac{1}{1-3i} = \frac{1-3i-1-3i}{(1-9i^2)}$ $(\because i^2 = -1)$

$$= \frac{6i}{10} = -\frac{3i}{5}$$

$$\therefore \text{Modulus} = \left|-\frac{3}{5}i\right|$$

$$= \sqrt{0^2 + \left(\frac{-3}{5}\right)^2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

91. (C) Let the height of the lower plane from the ground= x and PA = y

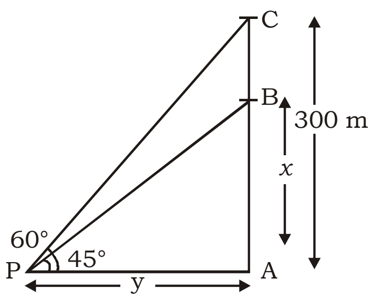
Now, in Δ ABP,

$$\tan 45° = \frac{AB}{PA} = \frac{x}{y} = 1$$

⇒ x = y

Again, in ΔAPC,

$$\tan 60° = \frac{AC}{AP} = \frac{300}{y} = \sqrt{3}$$



$$\Rightarrow y = \frac{300}{\sqrt{3}}$$

$$\Rightarrow x = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{300\sqrt{3}}{\sqrt{3}}$$

∴ $x = 100\sqrt{3}$ m

92. (D) Given that, $2A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$$\Rightarrow A = \frac{1}{2}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 3/2 & 1 \end{bmatrix}$$

Now,

$$\text{adj } A = \begin{bmatrix} 1 & -3/2 \\ -1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ 3/2 & 1 \end{bmatrix}$$

and |A| = 1 – 3/4 = 1/4

$$\therefore A^{-1} = \frac{adjA}{|A|} = 4\begin{bmatrix} 1 & -1/2 \\ -3/2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$$

93. (C) $$\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right) = \frac{(\sqrt{3}+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$$

$$= \frac{(\sqrt{3}+i)^2}{3-i^2} = = \frac{2+2\sqrt{3}i}{4}$$

$$= \frac{1+\sqrt{3}i}{2} = -\omega^2$$

Now, $$\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^6 = (-\omega^2)^6 = -\omega^{12} = 1$$

94. (A) Given line are

$$\frac{x-2}{1} = \frac{y+1}{-2} = \frac{z+2}{1} \quad \ldots\ldots \text{(i)}$$

and $$\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$$

$$\frac{x-1}{1} = \frac{2\left(y+\frac{3}{2}\right)}{3} = \frac{z+5}{2}$$

$$\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2} \quad \ldots\ldots \text{(ii)}$$

If θ be the accute angle between lines (i) and (ii), then

$$\cos\theta = \left|\frac{1\times 1+(-2)\left(\frac{3}{2}\right)+1(2)}{\sqrt{1+(-2)^2+1^2}\sqrt{1^2+\left(\frac{3}{2}\right)^2+2^2}}\right|$$

$$= \left|\frac{1+(-3)+2}{\sqrt{1+4+1}\sqrt{1+\frac{9}{4}+4}}\right| = \left|\frac{0}{\sqrt{6}\sqrt{\frac{29}{7}}}\right| = 0$$

∴ cos θ = 0

$$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

95. (B) One's digit in three-digit number can be filled in 3 ways

Ten's digit in three-digit number can be filled in 6 ways

Hundred's digit in three-digit number can be filled in 6 ways

∴ Total possible ways to form 3 digit numbers = 3 × 6 × 6 = 108

96. (B) $$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1}$$

$$= 1 + \frac{n}{2} + \frac{n(n-1)}{6} + \ldots + \frac{1}{n+1}$$

$$= \frac{1}{n+1}\left[(n+1) + \frac{(n+1)n}{2} + \frac{(n+1)n(n-1)}{3!} + \ldots + 1\right]$$

$$= \frac{1}{n+1}\left({}^{n+1}C_1 + {}^{n+1}C_2 + \ldots + {}^{n+1}C_{n+1}\right) = \frac{2^{n+1}-1}{n+1}$$

97. (B) ∴ Reqquired probability = $\frac{^{25}C_3}{^{26}C_3} = \frac{23}{26}$

98. (D) $\left(\sqrt{3}+i\right) / \left(1+\sqrt{3}i\right)$

$$\frac{\left(\sqrt{3}+i\right)\left(1-\sqrt{3}i\right)}{1-3i^2} = \frac{\sqrt{3}-3i+i-\sqrt{3}i^2}{4}$$

$$\frac{2\sqrt{3}-2i}{4} = \frac{\sqrt{3}-i}{2}$$

99. (B) Here, r = Distance between (4, 5) and (2, 2)

∴ $r^2 = 4 + 9 = 13 \Rightarrow (x-2)^2 + (y-2)^2 = 13$

$\Rightarrow x^2 + y^2 - 4x - 4y - 5 = 0$

100. (D) A hyperbola never meet/intersect conjugate axis in real points.

101.(A)

102.(D) Let a = i + j + k

Let any vector normal to a, then dot produt of both vector should be zero.

(A) (i + j +k) · (i + j – k) = 1 + 1 – 1 = 1 ≠ 0

(B) (i + j + k)· (i – j + k) = 1 – 1 + 1 = 1 ≠ 0

(C) (i + j + k)· (i – j – k) = 1 – 1 – 1 = – 1 ≠ 0

103.(B) $y = \frac{x^2}{2} - \frac{1}{x} + C$, where $C = \frac{29}{6}$ as it passes through the points (3 ,9)

104.(A)

105. (C) Let the AP is

a, a + d, a + 2d, ..., a + (2n – 1) d, a + 2nd

Series of even terms,

a + d, a + 3d, ..., a + (2n –1)d has n terms

∴ Sum = $\frac{n}{2}$ [(a + d) + {a + (2n – 1)d}]

= $\frac{n}{2}$ [2a + 2nd]= n [a + nd]

series of odd terms

am, a + 2d, a + 4d, ..., a + 2nd has (n +1) terms.

∴ Sum = $\frac{n+1}{2}$ [a + (a + 2nd)]

= $\frac{n+1}{2}$ (ma + 2nd) = (n + 1) (a + nd)

so, the ratio = $\frac{n+1}{n}$

106.(C) $y = \frac{(x-2)(x-1)}{(x+3)(x-1)} = \frac{(x-2)}{(x+3)}$, $x \neq 1,\ x \neq -3$

or $y = \frac{x+3-5}{x+3} = 1 - \frac{5}{x+3}$

$\frac{dy}{dx} = \frac{5}{(x+3)^2}$ = Positive

Always for all values of x in its domain.

So, y = $f(x)$ is an increasing function in its domain.

107.(B) ∵ Total ways = $^6C_2 = 15$

and favourable ways = $^3C_1 \times {}^3C_1 = 9$

∴ Required probability = $\frac{9}{15} = \frac{3}{5}$

108.(B) Given that d is the number of degrees contained in angle, m is the number of minutes and s the number of seconds.

i.e., m = 60 d and s = 60 m

Now, $\frac{s-m}{m-d} = \frac{60m-60d}{m-d} = \frac{60(m-d)}{(m-d)} = 60$

109. (C) Given, quadratic equation is :

$(x - a)(x - b) = c,\ c \neq 0$

$\Rightarrow x^2 - (a + b)x + (ab - c) = 0$

The roots of this equation is (α , β)

Then, α + β = – [– (a + b)] = a + b .... (i)

and αβ = ab – c .... (ii)

Now, consider the equation,

$(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow x^2 - (\alpha + \beta)x + (\alpha\beta + c) = 0$

From Eqs. (i) and (ii)

$x^2 - (a + b)x + (ab - c + c) = 0$

$\Rightarrow x^2 - (a + b)x + ab = 0$

So, the roots of this equation is (a, b).

110.(B) ∵ $x = e^x y$ and $y = xe^{-x}$

∴ $\frac{dy}{dx} = e^{-x}(1-x)$

Now, $\frac{dy}{dx} = e^{-x}(1-x) = \Rightarrow x = 1$

∵ $\frac{d^2y}{dx^2} < 0$

∴ Maximum value at $x = 1$

and $y = 1 \cdot e^{-1} = e^{-1}$

111.(C) Equation of curve is $y^2 = 12x$

At y = 6, 36 = 12 $x \Rightarrow x = 3$

∴ Required area = $\int_0^6 x\,dy = \int_0^6 \frac{y^2}{12}\,dy$

$= \frac{1}{12}\left[\frac{y^3}{3}\right]_0^6 = \frac{1}{36} \times (6)^3 = 6$ sq units

112. (A) Put $xy = v \Rightarrow y + x\frac{dy}{dx} = \frac{dv}{dx}$

∴ $\frac{dv}{dx} = x\frac{\phi(v)}{\phi'(v)} \Rightarrow \frac{\phi'(v)}{\phi(v)}\,dv = x\,dx$

$\Rightarrow \log\phi(v) = \frac{x^2}{2} + \log k$

$\Rightarrow \log \frac{\phi(v)}{k} = \frac{x^2}{2}$

$\Rightarrow \phi(v) = ke^{x^2/2}$

$\Rightarrow \phi(xy) = ke^{x^2/2}$

113. (A) Given, $ax\cos\phi + by\sin\phi - ab = 0$

At point $(+\sqrt{b^2-a^2}, 0)$

$$d_1 = \frac{a\sqrt{b^2-a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

At point $(-\sqrt{b^2-a^2}, 0)$,

$$d_2 = \frac{-a\sqrt{b^2-a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

$$\therefore\ d_1 d_2 = -\frac{\left[a^2(b^2-a^2)\cos^2\phi - a^2b^2\right]}{a^2\cos\phi + b^2\sin^2\phi}$$

$$= -\frac{a^2(-b^2\sin^2\phi - a^2\cos^2\phi)}{a^2\cos^2\phi + b^2\sin^2\phi} = a^2$$

114.(B) We have, $y = \frac{x+1}{x-1}$

Now, differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

115. (B) The equation of the circle of radius 6 and centre at (3,5) is

$(x-3)^2 + (y-5)^2 = (6)^2$

Let $S = (x-3)^2 + (y-5)^2 - 36 = 0$

At point $(-2, -1)$

$S = (-2, -3)^2 + (-1-5)^2 - 36 = 25 + 36 - 36 = 25 > 0$

Which represents outside the circle.

At point (0, 1),

$S = (0-3)^2 + (1-5)^2 - 36$

$= 9 + 16 - 36 = -9 < 0$

which represents inside the circle.

Hence, point (0, 1) lies inside the circle.

116.(B) $\lim_{x\to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1} = \lim_{x\to 0} \frac{(2x-1)\{(1+x)^{1/2}+1\}}{\{(1+x)^{1/2}-1\}\{(1+x)^{1/2}+1\}}$

$$= \lim_{x\to 0} \frac{2^x - 1}{1+x-1}\{(1+x)^{1/2}+1\}$$

$$= \lim_{x\to 0} \frac{2^x-1}{x} \lim_{x\to 0}\{(1+x)^{1/2}+1\}$$

$= (\log 2)\cdot 2 = 2\log 2$

$$\left[\text{because} \lim_{x\to 0}\frac{2^x-1}{x}\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{2^x\log 2}{1} = \log 2\right]$$

117.(C) Given that, $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{7}{10}$

Alos, A and B are independent events, that $P(A \cap B) = P(A)\cdot P(B)$

$\Rightarrow P(A) + P(B) - P(A\cup B) = P(A)\cdot P(B)$

(by addition theorem of prbability)

$\Rightarrow \frac{1}{5} + P(B) - \frac{7}{10} = \frac{1}{5} \times P(B)$

$\Rightarrow P(B) + \frac{2-7}{10} = \frac{P(B)}{5}$

$\Rightarrow P(B) - \frac{P(B)}{5} = \frac{5}{10}$

$\Rightarrow \frac{4P(B)}{5} = \frac{1}{2} \Rightarrow P(B) = \frac{5}{8}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{5}{8} = \frac{3}{8}$

118.(D) LHL $\lim_{h\to 0} e^{-\frac{1}{(0-h)}} = \lim_{h\to 0} e^{1/h} = e^{\infty} = \infty$

RHL $= \lim_{h\to 0} e^{-\frac{1}{(0+h)}}$

$= \lim_{h\to 0} e^{\frac{-1}{h}} = e^{-\infty}$ = Does not exist

119. (C) $\frac{y^2}{4} - \frac{x^2}{9} = 1$

Here, the coefficient of y^2 is positive and that of x^2 is negative and hence it represents a hyperbola, whose transerse axis is vertical.

i.e, $a^2 = 4$, $b^2 = 9$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow \frac{9}{4} + 1 = e^2$

$\therefore\ e = \frac{\sqrt{13}}{2}$

120.(B) Put $nx = t$ and adjust the limits and change into sin and cos,

$$\therefore\ I = \frac{1}{n}\int_0^{\pi/2} \frac{\sin^n t}{\sin^n t + \cos^n t}dt$$

On applying property and then adding, we get

$$2I = \frac{1}{n}\int_0^{\pi/2} dt = \frac{1}{n} \times \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{4n}$$

NDA (MATHS) MOCK TEST - 39 (Answer Key)

1.	(B)	21.	(C)	41.	(B)	61.	(B)	81.	(D)	101.	(A)
2.	(B)	22.	(B)	42.	(C)	62.	(C)	82.	(B)	102.	(D)
3.	(A)	23.	(B)	43.	(A)	63.	(B)	83.	(B)	103.	(B)
4.	(D)	24.	(D)	44.	(B)	64.	(C)	84.	(C)	104.	(A)
5.	(D)	25.	(A)	45.	(C)	65.	(A)	85.	(D)	105.	(C)
6.	(B)	26.	(B)	46.	(C)	66.	(A)	86.	(C)	106.	(C)
7.	(A)	27.	(D)	47.	(B)	67.	(B)	87.	(A)	107.	(B)
8.	(B)	28.	(A)	48.	(A)	68.	(C)	88.	(A)	108.	(B)
9.	(A)	29.	(A)	49.	(C)	69.	(A)	89.	(B)	109.	(C)
10.	(B)	30.	(B)	50.	(B)	70.	(B)	90.	(A)	110.	(B)
11.	(B)	31.	(B)	51.	(A)	71.	(B)	91.	(C)	111.	(C)
12.	(A)	32.	(B)	52.	(B)	72.	(B)	92.	(D)	112.	(A)
13.	(B)	33.	(C)	53.	(A)	73.	(A)	93.	(C)	113.	(A)
14.	(B)	34.	(C)	54.	(C)	74.	(A)	94.	(A)	114.	(B)
15.	(B)	35.	(C)	55.	(A)	75.	(B)	95.	(B)	115.	(B)
16.	(C)	36.	(D)	56.	(C)	76.	(B)	96.	(B)	116.	(B)
17.	(B)	37.	(A)	57.	(A)	77.	(C)	97.	(B)	117.	(C)
18.	(B)	38.	(*)	58.	(C)	78.	(C)	98.	(D)	118.	(D)
19.	(A)	39.	(A)	59.	(C)	79.	(B)	99.	(B)	119.	(C)
20.	(A)	40.	(A)	60.	(D)	80.	(C)	100.	(D)	120.	(B)

Note:- If you face any problem regarding result or marks scored, please contact 9313111777

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003